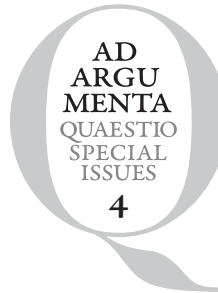
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Fallacies in the Arabic, Byzantine, Hebrew and Latin Traditions

Edited by
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Tony Street and Michele Trizio

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Fallacies in the Arabic, Byzantine, Hebrew and Latin Traditions. Introduction*

> "In some way or another error is the usual manifestation itself of the truth – the paths of the truth are essentially the paths of error" (Lacan 1954, p. 290)

1. Prolegomena

As suggested time and again[1], fallacies are no trifling matter both in their own right and as a cautionary tale about everything that can go amiss when we reason on our own or argue with one another. As a matter of fact, not only the paths of error are every bit as worth exploring as those of truth – and so much more entertaining at that – but flawed, biased, mistaken and outright deceptive arguments also provide pristine evidence about good argumentative practices insofar as illegitimate argumentative moves reveal the principles our arguments abide by to the same extent that they themselves are revealed by the standards and rules they break or twist.

The great abundance of current literature on argumentative failures is a testament to the interest fallacious arguments – that is, arguments which appear to be sound but, in fact, are not – have elicited in recent years among specialists and laymen alike. Even if we leave aside bad arguments hit parades[2], ludicrous social media's digests[3], as well as record-breaking collections of poor reasoning's speci-

* Research leading to the publication of this volume was funded by the Swedish Research Council (2018-04239_VR), as one of the three main axes of Christina Thomsen Thörnqvist's Project "Filling the Gap: Medieval Aristotelian Logic 1240-1360" (namely, "Mediaeval Theories of fallacies"). Sten Ebbesen's paper, as well as Irene Caiazzo's, Yehuda Halper's, Charles H. Manekin's, Shahid Rahman / Walter Edward Young's, Aviram Ravitsky's and Hassan Rezakhany's were presented at the "Logic in Reverse" Conference (Lille, European Center for the Humanities and Social Sciences, May the 24th- May the 26th 2021). Financial support for the publication has been provided by the Swedish Research Council, the University of Lille (UMR 8163 STL) and the MESHS-Lille-Nord de France.

1. See first and foremost Ebbesen 1981, I, p. 1 and, more recently, Biro / Siegel 1997; Bolton 2012, p. 270; van Eemeren et alii 2014, p. 544; Oswald / Herman 2020, p. 41.

2. Despite its promising title, Arp / Barbone / Bruce 2019 mention very few actual arguments from actual philosophers… a redeeming quality the educated reader had come to expect, if nothing else because the authors' previous endeavour to provide the argument industry with raw materials, namely Bruce / Barbone 2011, relied on several genuine examples.

3. *Ab uno disce omnes*: Cliff 2018 will disappoint neither friends nor foes of twittering former

mens[4], and we confine ourselves to a quick inventory of scholarly literature alone, the result is telling. Fallacies have been for some time a well established field of argumentation theory which comes in many varieties, all of which seem to pay a great deal of attention to the topic of faulty arguments.

With so few exceptions as to make little difference[5], the ever-growing number of essays, book-chapters, book-length studies, reviews special issues and even handbooks on the subject – which are easily counted by the hundreds – share a peculiar feature: namely, all these works suffer from a definite lack of interest in Mediaeval theories of fallacies – arguably the most creative stage in the history of argumentation theories. In fact, both specialists and non-specialists have been working under either one of two questionable assumptions[6]:

1. for all practical purposes, fallacy studies have come to prominence in the early 1970s, courtesy of a most influential book by computer science pioneer and distinguished logician Charles Leonard Hamblin[7]

2. for no apparent reason, after Aristotle got off to a tentative start, the discipline barely held its own until – different men tell different tales – either people at Port-Royal and John Locke or Richard Whately first and John Stuart Mill soon afterwards, revived it in spectacular fashion[8]

When their logical merits are acknowledged at all, Mediaevals are rarely granted more than a grudging recognition for their efforts to tackle fallacies – as Woods bluntly but effectively put it: "there is no deep theory of fallacious inference to be found in Aristotle. Although over the centuries fallacies have remained part of

President @realDonaldTrump. Readers interested in some of the weirdest lines of argument on recent record will also appreciate the authoritative *New York Times*' list of the 598 (as of his banishment in January 2021) people, places and things Donald Trump has been drumming his fingers about (https://www.nytimes.com/interactive/2016/01/28/upshot/donald-trump-twitter-insults.html).

4. As far as accumulating huge amounts of disparate items goes, one can hardly go to greater lengths than Bennett 2012, whose three hundred findings (and then some) deserve praise (as he himself candidly points out p. 18). Of course, hard work along these lines leaves one open to the old but still devastating Aristotelian analogy with those who pretend to teach the science of relieving foot pain by presenting their pupils with a whole array of different shoes rather than introducing them to the art of shoe-making (Arist., *De Sophisticis Elenchis*, 34, 183b36-184a7).

5. Every generalisation has its unsung heroes, but in this particular instance they are hard to come by and far between (e.g., Read 1988; Tabarroni 2002; Hintikka / Spade 2020).

6. For obvious reasons, we only mention the most recent titles for each trend. Through the usual bibliographical threads follow up routine, interested readers will trace back previous relevant works.

7. The Hamblin-connection has become very popular amongst hard-core logicians and argument theorists who, if they look back at all, seldom refer to anything older than Hamblin 1970 (cf., e.g., Visser / Budzynska / Reed 2018; Hansen 2023).

8. The Aristotle through either Port-Royal and John Locke or Richard Whately and John Stuart Mill gap has long become the conventional wisdom and requires little comment here (cf., e.g., Woods / Walton 1989; van Eemeren / Garssen / Meuffels 2009; Dufour 2019; etc.), apart from pointing out that, on occasion, the disregard for elementary source-checking simply defies belief (for what is worth, Tamarkin 2017, p. 4 is a dispiriting case in point: opium's sleepifying virtue was a gibe of Molière's well before Jeremy Bentham used it as an example of begging the question).

the project of logic, this lack of theoretical depth has persisted, albeit with some rare exceptions. Although there was much logical sophistication in the Middle Ages, Mediaeval logicians made comparatively little headway with the fallacies. John Locke (1690), etc."[9]

2. For present purposes (and future reference)

If not altogether false, both pictures are inaccurate to say the least and should be dismissed accordingly. "Fallacies in the Arabic, Byzantine, Hebrew and Latin Traditions" will not attempt to replace them with a brand new narrative which would ideally bring to bear the full resources of mediaeval treatment of fallacies – nothing short of a comprehensive, fine-grained and transferable reconstruction of Mediaeval fallacy theories would adequately fill the gaps of the standard story[10]. The volume will not confine itself to the casual rescue of some overlooked areas in current scholarship either. As its title implies and the scope and multiple focus of the various contributions attest, "Fallacies in the Arabic, Byzantine, Hebrew and Latin Traditions" will explore in some details how Mediaeval authors (logicians for the most part, but also jurists and theologians) discussed fallacies within and across the Latin West and the Greek East, as well as in the Arabic and the Hebrew traditions – that is, the volume will probe and get its bearings without boundaries or limitations imposed by differences in discipline, language and culture. For – we surmise – this is where both the potential for novelty and the rightful place of mediaeval theories of fallacies lies within contemporary argumentation studies. By working its way from the inside out within each mediaeval tradition, the volume will not only bear witness to mediaeval ingenuity and sophistication when it comes to reckon if an argument fails and why, but it will also compare mediaeval findings and lessons with contemporary views and trends. Whether or not

9. Woods 2010, p. 164.

10. Let us put this a bit more into perspective. What do we mean by saying that a reconstruction of Mediaeval fallacy theories worthy of the name should be 'comprehensive', 'fine-grained' and 'transferable' and why would it be so hard to achieve? In so many words: 'comprehensive' insofar as any such reconstruction should cover the whole range of the four major Mediaeval traditions (Arabic, Byzantine, Hebrew and Latin) and their argument-focused disciplines (logic, juridical disputation and theology to begin with), 'fine-grained' insofar as one should take into account context-related specificities and peculiarities to avoid anachronisms and hasty generalisations, 'transferable' insofar as it should confront and compare Mediaeval standards of fallacious reasoning to those of contemporary Fallacy Studies. All of which would require a formal identification and cogent explanation of both similarities and differences between the way argumentation specialists in the Byzantine, Hebrew, Islamic and Latin tradition came to terms with error and deception. A tall order, by any standard – all the more so since we are by and large in uncharted territory here. As a matter of fact, in this particular instance Mediaeval traditions did not come together or influenced one another in the way they did in other cases – metaphysics and psychology immediately spring to mind as examples of cross-cultural Mediaeval interaction.

and to what extent "Fallacies in the Arabic, Byzantine, Hebrew and Latin Traditions" actually lays the groundwork for new and better ways of describing and assessing the laws and flaws of argumentation may well remain, for the time being, an open question. What the volume does provide is ample and unambiguous record of the exegetical proficiency, technical expertise and argumentative savoir-faire typically displayed by mediaeval authors on issues whose complexity we either choose to ignore or underestimate to some degree – such as defining what a fallacy is in the first place or asking what the pitfalls of linguistic expression are and how they compare to those of other symbolic notations, etc.

3. Latin Tradition

As John Buridan – faithful to a long and illustrious tradition – aptly put it at the beginning of his *Summulae logicales*, "logica est" – among other things but first and foremost – "exstirpativa falsarum rationum [logic's job is to root out false arguments]"[11]. To be sure, Latin logicians took their job very seriously. It would be a bit of an overstatement to claim that everything they adapted from their Roman, Late Ancient, Byzantine and – to a degree – Arabic sources, let alone the logical novelties they came up with by themselves (e.g., the theory of the properties of terms, most notably *suppositio*[12]) was prompted or motivated by a concern with fallacious arguments. That being said, it is unquestionably true that they took fallacious arguments at least as seriously as legitimate ones. It is also true that they used each and every tool in their logical arsenal to cope with the problem of flawed argumentation and even added a few devices of their own invention[13].

11. Iohannes Buridanus, *Summulae logicales*, *Prooemium*, ed. van der Lecq, p. 7, ll. 11-12.

12. Characteristically, Gerald Odonis introduced his study of 'supposition' by quoting Aristotle's tract on fallacies: Giraldus Odonis, *Logica*, II, ed. de Rijk, p. 233: "quoniam 'qui nominum virtutis sunt ignari, de facili paralogizantur, et ipsi disputantes et alios audientes': ne studiosos scientiae ac veritatis amicos ex huius virtutis ignorantia faciliter paralogizari in veritatis inquisitione contingat, de suppositionibus notitiam aliqualem tradere studui, in quibus ultimate consistit virtus et ultimum de potentia terminorum [since 'those who know little to the properties of words, are easily mislead by fallacious reasoning both when they take part into a discussion and when they listen to others', in order to avoid that those who seek knowledge and the friends of truth be easily deceived by their ignorance of the power of words, I set out to teach them to an extent what supposition is, for this is what the properties and the power of words ultimately boil down to]" (Cf. Arist., *Sophistici Elenchi*, 1, 165a15-17 [ed. Dod, p. 6, ll. 11-13]). On Gerald Odonis and *suppositio*, *cf.* Brown 2009; on the force of linguistic items in general, see Cesalli 2014 and on the prologue of Aristotle's *Sophistici Elenchi* in the Latin commentary tradition, Gazziero's contribution to this volume.

13. The distinction between the 'matter' and the 'form' of an argument (especially in its syllogistic shape) – introduced, by the way, in the Latin tradition through a scholium to the prologue of Aristotle's *Sophistici elenchi* as a convenient way of setting apart sophisms whose premisses are false (*peccantes in materia*) and sophisms whose deductive fabric is at fault (*peccantes in forma*) – is an excellent example of the standard tools category (cf. Biard 1989; Barnes 1990; Spruyt 2003; Thom 2013; Brumberg-Chaumont 2017), the distinction between the way an argument goes wrong (*causa*

For, by and large, Latin logicians conceived their contributions to logic – even at its most original – as a continuation of Aristotelian lore rather than as a departure from it[14], it does not come as a surprise that Aristotle's *Sophistici elenchi* provided the most fertile ground for their keen interest in fallacies. Three of the four papers dealing here with the Western Middle Ages actually focus on issues directly related to Aristotle's work on fallacies and its fortune in the Latin commentary tradition. The fourth paper shifts its attention to theology, another field which early on relied upon the analysis of fallacies as a formidable tool for ferreting doctrinal error out and defeating heretics.

Sten Ebbesen, *Are the Fallacies Topoi?* – Fallacies are said in many ways. It then stands to reason to address the issue of defining what a fallacy is to begin with. The way Latin commentators tackled the problem involved asking a question contemporary scholarship has largely passed over, namely whether or not the thirteen types or varieties of fallacies Aristotle introduced to tell spurious arguments apart are, in fact, as many topical patterns. While a closer look to the manuscript tradition reveals that the textual evidence is less clear-cut than modern editions of the *Topics* and the *Sophistici Elenchi* seem to suggest, the role *fallaciae* play in sophistical arguments is essentially the same as the role *loci* hold in dialectical ones. This appears to be a safe assumption as far as Aristotle is concerned. More to the point, it was a basic tenet of mediaeval Latin understanding of fallacies. If we go along with the results of Sten Ebbesen's extensive survey of ancient and mediaeval sources, then we might as well make the most of another largely neglected feature of mediaeval fallacy theories and reintroduce in the ongoing discussion about what fallacies are and how the play out the mediaeval notion of sophistical pseudo-maxims as opposed to (but closely related with) genuine dialectical maxims. If nothing else, fallacies understood in terms of sophistical axioms will help us with figuring out one of the most elusive feature fallacies confront us with, namely the hidden link between the way an argument fools us (what gives an argument its respectable appearance) and the way it actually goes sideways (what makes an argument defective, despite its good looks that is).

Costantino Marmo, *The* Fallacia Consequentis *between Term Logic and Sentence Logic in its Medieval Reception* – It's a commonplace – albeit one Costantino Marmo disputes here – that a main difference between Modern logic, on the one hand, and Ancient as well as Mediaeval logic, on the other hand, has to do with the latter being essentially a logic of terms while the former deals first and

defectus or *non existentiae*) and the way it fools us (*causa apparentiae*) is an excellent example of the *ad hoc* devices Latins came up with on their own (Ebbesen 1987 and his contribution to this volume; Huelsen 1988; Tabarroni 1994; Gazziero 2015).

14. As Ebbesen 1992, p. 167 and Ebbesen 2011, pp. 100-101 put it, the perceived continuity of the Aristotelian logical tradition is a 'basic fact' and is generally acknowledged as such – cf., e.g., Brumberg-Chaumont 2015, pp. 253-254; Storck 2015, p. 135; Mora-Marquez 2021, p. 148.

foremost with propositions. A few well-known exceptions confirm the rule: the Stoics developed a propositional logic of sorts, Boethius' tract on hypothetical syllogisms went in the same direction, so did a handful of medieval thinkers such as Peter Abelard in the first half of the twelfth century, Walter Burley at the turn of the fourteenth, as well as later authors of tracts on *consequentiae*. Costantino Marmo's paper tells a different story, one which advocates further and more extensive revision of the standard narrative by following a thread from Aristotle to the late thirteenth century Latin reception of Aristotle's work on fallacies, namely the question about what the *fallacia consequentis* is to begin with and how Mediaevals understood it. To contemporary ears, this fallacy conveys the idea of transgressing one rule or another of *modus ponens* (either by affirmation of the consequent, or by negation of the antecedent). However, as Costantino Marmo shows, this is only one of two possible interpretations of this kind of fallacies: while Aristotle definitely understood "consequent" as an accident or a property whose possession by a given subject is supposed to follow from the possession of another property or accident (consequent as a predicate), Boethius' interpretation of the same fallacy paved the way for another reading focusing on conditional inferences between propositions instead (consequent as a proposition). The evolution of the alternate view may be traced back as early as the first reception of Aristotle's *Sophistici Elenchi* and is well attested throughout the twelfth to the fourteenth century. Accordingly, Costantino Marmo's survey strongly supports the idea that propositional logic was not as foreign to medieval minds as it has been previously suggested.

Leone Gazziero, *"Qui imperitus est vestrum, primus calculum omittat". Aristotelis Sophistici Elenchi* 1 *in the Boethian Tradition* – The prologue of the *Sophistici elenchi* is about as close an Aristotelian text gets to dealing with language as a subject matter in its own right, only in reverse. Language and its features bear consideration to the extent that they account for some major predicaments discursive reasoning land itself into. That being said, the linguistic pitfalls that trick us into thinking that whatever goes for words and word-compounds is also the case for the things and facts linguistic expressions stand for reveal as much about good linguistic etiquette as they themselves are revealed by the principles and rules we abide by when arguing and discussing. In this connection, Aristotle resorted to a peculiar analogy between words and pebbles which plays a major role in explaining why language is both a tool we cannot dispense with and a powerful source of illusion and deception. In a nutshell, the untrained and the unwary are as easily misled by words as they are by counters, insofar as there's no guarantee that both words, in the course of the same conversation, and counters, in the course of the same calculation, are always worth the same. As it happened, Aristotle's *ψῆφοι* disappeared from Boethius' translation where one reads 'calculations' (*compoti*) and 'numbers' (*numeri*) instead. This lead Latin commentators, who

trusted Boethius' translation implicitly and were well aware of Boethius' views on disputational hazards as opposed to computational reliability, to understand Aristotle's comparison as if it was an analogy in name only: while, on the one hand, reckoning numbers stand in an unambiguous relation with the reckoned things whose numbers they are and one has to work hard to get off track when crunching figures, many a word, on the other hand, stands in an ambiguous relation to the things it signifies and one has got to work hard to keep on track when dealing with linguistic items. This is still the standard story, but it is neither the only narrative nor the most compelling one Mediaeval commentators came up with. As a matter of fact, despite Boethius' translation put them at a considerable disadvantage, at least two of them built a strong case in favour of another understanding of Aristotle's pebble analogy, one which explains why Aristotle brought them together in the first place. This is how Anonymus Bavaricus and William of Ockham's story goes: being two sets of symbolic variables which are neither entirely free nor entirely bound, words and counters are every bit as tricky. In fact, Aristotle's analogy has no silver lining: everybody and everything fails – those who reckon and what they reckon with no less than those who argue and what they argue with. Simply put, failure is the whole point here, failure to spot subtle and yet disruptive shifts in the worth of counters and in the meaning of words which plague discussions and calculations alike.

Irene Caiazzo, *Theology, Fallacious Reasoning and Heresy on the Borders of the Twelfth and Thirteenth Centuries: Some Remarks on the* Fallaciae *in theologia and Amalricians* – In her paper, Irene Caiazzo studies the relationship between logic and theology in the *Fallaciae in theologia*, a late twelfth century anonymous tract discovered by Jean Leclerq in 1945, unedited to this day. Her survey of the manuscript tradition adds three witnesses to the seven known to date, one of which, a fifteenth-century Italian copy, attests that the treatise circulated well beyond the milieu of Parisian theological schools and faculies. As Irene Caiazzo shows, the *Fallaciae in theologia* are a remarkable text for the history of theories of argumentation on at least two counts. For one thing, it is an example of an independent treatise on fallacies – namely, a text that, although showing a certain familiarity with Aristotle's *Sophistici elenchi*, is neither a commentary on the Aristotelian text nor a work in the vein of the contemporary growing logical literature on fallacies. Rather, the *Fallaciae in theologia* are a well reasoned transfer – as opposed to a mere compilation – of theoretical findings about the flaws of argumentation to another field, that of theological debate. For another, the *Fallaciae in theologia* help us better understand the way logic and theology interacted in the late twelfth and early thirteenth century: Aristotle analysis of argumentative failures, as expounded by earlier and contemporary commentators and master dialecticians, is called upon to explain, in fact refute on logical grounds the errors of the heretics, in this particular instance those of the sect of the Amalricians. For instance, the

Anonymous author – in all likelihood a Parisian master, possibly from the circle of Peter the Chanter – saddled his Amalrician foes with two charges of fallacy of equivocation when they argue that charity is in this stone for the Holy Spirit is everywhere (to begin with, the noun 'charity' means in turn a virtue and its bearer; next, the preposition 'in' refers indiscriminately to essence and inherence).

4. Byzantine Tradition

The Byzantine philosophical and theological tradition is rife with references to fallacies, especially with allegations of spurious reasoning and sophistical argumentation. As a matter of fact, despite being of a mixed mind as to whether logical and philosophical training were an asset or a liability, Byzantine scholars were very keen on accusing each other of misusing arguments and syllogisms – as made clear by the like of Michael Psellos (d. after 1076), Nicholas of Methone (d. 1160/1166), Theodore II Doukas Laskaris (d. 1258), Nicephorus Blemmydes (d. 1269-1272), Barlaam the Calabrian (d. 1348), Gregory Palamas (d. 1359), Philotheus Kokkinos (d. ca. 1377), etc. who time and again aptly used and even explicitly praised fallacy theory as a powerful tool against questionable views in matters of reason and faith. Unsurprisingly, Aristotle's treatment of fallacies provided the main framework and most of the tools for dismantling bad arguments. As it happens, while the materials related to the *Sophistici elenchi* have been thoroughly studied[15], other influential texts of the Aristotelian corpus, as reflected in the Aristotelian exegetical tradition, have not yet received the attention they deserve. A case in point is Aristotle's discussion of fallacies in *Rhetoric* II, 24 and its interpretation in two twelfth century commentaries – one anonymous, the other by the learned bishop of Trebizond, Stephanos Skylitzes.

Melpomeni Vogiatzi, *Byzantine Treatments of Fallacy: The Reception of Aristotle's Account* – Both commentaries as well as other contemporary and later sources (Anonymous Heiberg's logical compendium, John Italos' treatise on syllogistic matter and construction, etc.) are discussed by Melpomeni Vogiatzi's paper in this volume. Vogiatzi shows that the Byzantine commentary tradition focused primarily on the analysis of fallacies as material defects and paid special attention to the identification of the sources of deception (how a given fallacy escape our notice), which they sometime acknowledged as distinct from the sources of error (why a given fallacy is a flawed piece of argumentation). In this connection, Byzantine contribution to the theory of fallacies can hardly be dismissed as derivative

15. And – one might add – selectively edited by Sten Ebbesen who has pieced together the sequence of texts which stemmed from the early corpus of glosses and developed into the later full-scale commentaries of Michael of Ephesus, Leo Magentinus and Sophonias. See Ebbesen 1981, especially the first volume (*The Greek Tradition*) and Pars I of the second volume (the edition of Greek texts).

or pedantic: Byzantine commentators had an exegetical agenda of their own and did much more than merely reproduce earlier lines of questioning and solutions.

5. Arabic Tradition

In his monumental new book on the formation of Post-Classical philosophy in Islam, Frank Griffel confirms and elaborates upon the fact that Islamicate thought in the twelfth century gave birth to a new, original philosophy – one which was 'post-classical' in the sense that it had moved well beyond the translated Greek materials, critically engaging with and building upon the foundational work of Muslim philosophers who would become the new 'Ancients': al-Fārābī (d. 950) and Avicenna (d. 1037). In scale and influence, post-classical Islamic philosophy must be seen in terms similar to those used to account for Rationalism, German Idealism and British Empiricism. Griffel[16] adopts the name commonly used by the practitioners of this new discipline, *ḥikma* (wisdom), a discipline that cannot be dismissed merely as rational theology (*kalām*) under another name; it is, rather, an autonomous though syncretic body of systematic thought with its own concepts and perspectives.

Given that concepts and perspectives are constituted by methods of argument, it is no surprise that these, too, show significant evolution in post-classical Islamic philosophy. Griffel's insights should be completed by a thorough study of another post-classical development of the twelfth century which took place in – among other places – the eastern school of Raḍī al-Dīn al-Nīsābūrī (d. 544/1149), whose students, especially Rukn al-Dīn al-ʿAmīdī (d. 615/1218), promoted a fusion of Avicennian logic and a distinctly Islamic juristic dialectic called *jadal* or *khilāf*[17]. The streamlining of this fusion takes on – somewhat later – a characteristic inflection in the "protocols of dialectical inquiry and disputantion" (*ādāb al-baḥth wa-l-munāẓara*) of Shams al-Dīn al-Samarqandī (d. 722/1322), who writes in his *Qisṭās al-afkār*:

> "A custom of earlier scholars has been to append a section on dialectic (jadal) to the end of their logic books. But since, in our times, the science of juristic disagreement (*ʿilm al-khilāf*) has made this superfluous, I have put in its place rules for the protocols of dialectical inquiry (*ādāb al-baḥth*) and its proper ordering (*tartīb*), and for directing and cultivating proper discourse. <These protocols> are, for establishing and refining a position (*al-taqrīr wa-l-taḥrīr*), like logic (*manṭiq*) is for deliberation and reflection"[18].

16. Griffel 2021.
17. See Young 2019, pp. 207-208 and Young 2021, 2.2.
18. Shams al-Dīn Muḥammad al-Samarqandī, *Qisṭās al-Afkār fī l-Manṭiq*, ed. Fallāḥī, p. 601.

The second part of al-Samarqandī's concluding section is devoted to error and its causes (*fī l-ghalaṭ wa asbābihi*, *Qisṭās al-afkār*)[19]; it corresponds in large part to the sections on fallacy theory in post-Avicennian logic texts. These post-Avicennian texts are already some distance from Aristotle's *Sophistical Refutations*, not least in leaving the intention of the interlocutor out of the equation; and the rules offered by al-Samarqandī help the solitary thinker avoid error in her reflection. Al-Samarqandī extends the coverage to deal with, among other things, a version of the Liar Paradox[20]. In short, by al-Samarqandī's time we have come to a second current in Islamic philosophy, this time focused on argument theory, that draws on two genetically distinct traditions to produce a new, original and systematic body of thought about proper dialectical method. The two contributions relating to the Islamicate tradition in this volume bear witness to this kind of syncretic convergence, although one deals with a species of causal fallacy before *ḥikma* was born, while the other deals with a species of paradox after *ḥikma*'s formation and across subsequent centuries of its evolution.

Shahid Rahman / Walter E. Young, *Outside the Logic of Necessity: Deontic Puzzles and 'Breaking' Compound Causal Properties in Islamic Legal Theory and Dialectic* – Shahid Rahman and Walter E. Young discuss legitimate and illegitimate moves in arguments involving compound causal properties within Islamic juristic dialectic by two of the most important legal theorists of the 11th century CE. Their study examines an objection in Islamic juristic dialectic called *kasr*, or 'breaking', as treated by the dialecticians and legal theorists Abū Isḥāq al-Shīrāzī (d. 1083 CE) and Abū l-Walīd al-Bājī (d. 1081 CE). The authors focus on the discourse of *kasr*, namely, the proper and improper paths to challenging and defending the causal components of a correlational argument (*qiyās*) in which the *ratio legis* (*ʿilla*) of the root-case's ruling is a compound of two or more properties. The developmental history of this dialectical objection is complicated; long and heated controversies centered on which modes of *kasr* (and responses to *kasr*) were fallacious and which were not. There were even those who rejected *kasr* from the outset, some with arguments paralleling classical and medieval Latinate claims that one cannot refute an argument whose premises have a meaning *in sensu composito* by the blunt separation of its parts.

Hassan Rezakhany, *A Forgotten Mereological Paradox* – In one of the first studies of its kind in the burgeoning literature devoted to paradoxes in the Islamicate tradition, Hassan Rezakhany offers an example of how a mereological paradox involving concepts of totality – such as the totality (*majmūʿ*) of all relations (*nisab*), discussed from the thirteenth to the nineteenth century by philosophers

19. Shams al-Dīn Muḥammad al-Samarqandī, *Qisṭās al-Afkār fī l-Manṭiq*, ed. Fallāḥī, pp. 625 sqq.

20. Shams al-Dīn Muḥammad al-Samarqandī, *Qisṭās al-Afkār fī l-Manṭiq*, ed. Fallāḥī, p. 645.

of the eastern Islamic world (Iraq to India) – provides yet further evidence against the old cliché that post-Avicennian Islamicate thought suffered a decline. Hassan Rezakhany's paper (which includes an appendix with a number of passages never before translated) documents the most influential formulations of the paradox, solutions thereto, and objections to those solutions. In the 'Historical Development' section, it is argued that the paradox is 'functionally equivalent' to Russell's. By functionally equivalent, the author means that "both paradoxes upset the same intuition: both show that not every object (or property) can be used to form a sound totality (or set, in Russell's case)".

6. Hebrew Tradition

Jewish concern with fallacious reasoning is as old as the Talmud, but the systematic analysis of fallacies begins with the appropriation of Aristotelian logic by Jewish thinkers, first in Arabic in the lands of Islam, and then in Hebrew in Southern Europe and the Byzantine/Ottoman empires. "The art of logic", writes Moses Maimonides, "guards the rational faculty from committing errors, and guards speech from fallacy, through providing universal rules that enable external speech to conform to internal thought"[21]. Perhaps because of Maimonides's warm recommendation of al-Fārābī's logical writings to his Hebrew translator in Provence, al-Fārābī's *Fallacious Topics* was one of the first philosophical works translated from the Arabic into Hebrew. It found a readership among the Jews of Southern France, Spain, and Italy during the high and late Middle Ages and was subsequently used by the Provençal intellectual, Joseph ibn Kaspi in his compendium of logic, *A Bundle of Silver* (1332). A much larger treatment of the fallacies was translated into Hebrew in 1313 by Kalonymos b. Kalonymos of Arles, Averroes's Middle Commentary, or paraphrase, of the *Sophisticis Elenchis*. Few of Aristotle's actual works were translated into Hebrew, so Averroes's paraphrase became the standard presentation of the fallacies. Shortly after it was translated, an extensive commentary was written on the work by the fourteenth century polymath Levi b. Gershom (Gersonides), who wrote commentaries on almost of all of Averroes's paraphrases available to him. Not only did Gersonides explain in detail Averroes's treatment of the fallacies for budding Jewish intellectuals, but he also criticized and expanded that treatment.

At around the same time as Gersonides wrote his commentary, another Provençal Jewish scholar, Hezekiah bar Ḥalafta composed a work on logic that made use of a gloss-commentary of Peter of Spain's *Summule Logicales*, which marked the entrance of scholastic treatment of the fallacies into Jewish philosophical cir-

21. Moses Maimonides, *Millot ha-Higgayon*, ed. Efros, p. 38, ll. 17-21.

cles. This encounter with scholastic logic increased with additional translations of the Peter of Spain's work into Hebrew, often replacing Peter's *De fallaciis* with the Pseudo-Aquinas's, *De fallaciis ad quosdam nobiles artistas*, a common substitution in Italy. Finally, one should mention the treatment of the fallacies in the work of a mid-fifteenth century Italian Jewish savant, Judah Messer Leon. Judah wrote for his students in Hebrew a compendium of logic, *The Perfection of Beauty*, similar to those studied at the universities in Bologna and Padua, which he may have attended, devoting two chapters to insolubles and fallacies. In short, the medieval Hebrew tradition of fallacies in the Middle Ages, though modest in comparison to the Arabic and Latin traditions, is interesting less, perhaps, for its originality than for it appropriation and adaption of the fallacies to a Jewish intellectual context, whether that be scriptural exegesis, philosophical argument, or Talmudic exegesis.

The articles in this collection are devoted to these three topics.

Charles H. Manekin, *Fallacies and Biblical Exegesis – The Case of Joseph ibn Kaspi* – The application of the doctrine of fallacies to scriptural exegesis was undertaken by Joseph Ibn Kaspi, who claimed that the Bible cannot be understood correctly without a proper grounding in grammar and logic. Charles H. Manekin shows how Kaspi drew upon his treatment of the fallacies in his logical compendium to explain difficult passages in the Bible. For example, in explaining how the Israelites were deceived by the Gibeonites into entering into a treaty with them (Joshua 9), he shows that the deception was based on the fallacy of the concomitant accident, i.e., the fallacious inference from the Gibeonites worn clothes and crumbling provisions that they had come from afar. Even Moses committed well-known fallacies (intentionally) in his speeches to the Israelites for the sake of their welfare. In an appendix to the article Manekin provides an English translation of Kaspi's section on fallacies from his logic compendium.

Aviram Ravitsky, *Fallacies in Rabbinical Thought, in Medieval Jewish Philosophy, and in the Treatise on Talmudic Methodology by Abraham Elijah Cohen* – As in the Islamic tradition, Jewish jurists discuss the place of legal fallacies and fallacious argumentation. In his contribution on the subject, Aviram Ravitsky provides examples of how the Talmud views fallacies as a useful heuristic device for sharpening the mind but bans them from practical legal reasoning and ruling. After considering the impact of the Aristotelian concept of fallacies in two Andalusian Jewish philosophers, Ravitsky turns to a late medieval commentary on the thirteen hermeneutical rules of R. Ishmael, the rules of inference by which rabbis were said to derive laws from scripture. With the penetration of philosophy and science into the intellectual world of the rabbis of Southern Europe, attempts were made to find parallels and even identities between these thirteen hermeneutical rules and Aristotelian rules of inference. The commentary, written by one Abraham Eliyahu Cohen, provides a formal analysis of the principles, which reveals his knowledge of Aristotelian logic. A large part of his treatment of *a fortiori*

(*qal va-ḥomer*) arguments is devoted to instructing the student on how to distinguish fallacious from valid arguments, and he constructs his analysis using terms and concepts taken from Aristotelian logic. Although this approach did not become a trend, much less a school, it is a testament to the impact of Aristotelian logic on Jewish legal writings.

Yehuda Halper, *Are Zeno's Paradoxes of Motion Fallacies? Evidence from the Hebrew Aristotelian Logical Tradition* – One of the best-known fallacies in philosophy, at least according to Aristotle and his school, had to do with Zeno's paradoxes of motion. Although Aristotle's main treatment of the paradoxes is found in the *Physics*, the paradoxes were considered in the Arabic and Hebrew peripatetic traditions also in the *Topics* and the *Sophistici Elenchi*. Yehuda Halper suggests in his contribution that al-Fārābī's connected the paradoxes with widely held opinions, thus placing them in the context of dialectic, and this might have been due to the atomism that some of the Kalām theologians considered to provide a solution to the paradoxes. Halper traces the place of the paradoxes as dialectical fallacies in subsequent Arabic and Jewish thought, While Al-Fārābī focused on the physical theory that would refute Zeno's paradoxes, Averroes in his Middle Commentary on the *Topics* focused on the logical argumentation for making and refuting inductions; at least one later Jewish commentator preferred to revert to al-Fārābī's treatment.

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Abstract: Bad arguments have never been in short supply. Much the same holds for the scholarly interest they have elicited both in their own right and as a cautionary tale about everything that can go amiss when we reason. This is, of course, hardly surprising. Asking what is wrong with flawed arguments is tantamount to investigating the very fabric of argumentation along with the norms that make some argumentative moves more legitimate than others – only in reverse. Any overview of the scholarly literature of the last fifty years will provide ample evidence that fallacies Studies have simply escalated. Without notable exception, however, the ever-growing literature on argumentative failures suffer from a conspicuous lack of interest in Mediaeval fallacy theory – arguably the most creative stage in the whole history of argumentation theories. The standard story is that after Aristotle got off to a tentative start, the study of fallacies laid dormant until people at Port Royal and John Locke revived it in spectacular fashion. The volume will show that this picture is both inaccurate and misleading. By working its way from the inside out within each mediaeval world, "Fallacies in the Arabic, Byzantine, Hebrew and Latin Traditions" will provide ample and unambiguous record of the exegetical proficiency, technical expertise and argumentative savoir-faire typically displayed by mediaeval authors on issues which are all too often our own.

Keywords: Fallacies; Paradoxes; Argumentation; Logic; Rhetoric; Theology; Arabic Philosophy; Islamic Juristic Dialectic; Latin philosophy; Byzantine Philosophy; Hebrew Philosophy; Talmudic Logic; Aristotelian Tradition.

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Sten Ebbesen

Are the Fallacies *Topoi*?

1. Introduction

Ever since the twelfth century, the thirteen causes or types of apparent refutation presented in *Sophistical Refutations* (*Soph. el.*), chapters 4-5 (and again treated in chapters 19-30 with a view to how to neutralize them) have been known as the thirteen *fallaciae* or *fallacies*. The word 'fallacy' is also used (2) about particular fallacious arguments (paralogisms), and (3) in the sense of fallaciousness of an argument, or (4) the deception produced by it, but in this essay I shall only use it in the sense of a class of fallacious arguments or the principle that establishes membership of the class[1]. It does not occur in either of the first two senses in Boethius' translation of *Soph. el.*, but it *is* used: seven times to render *ἀπάτη* and once to render *μοχθηρία*[2]; moreover, in connection with a mention of *Soph. el.* Boethius in his first commentary on *Peri hermeneias* proposes to call sophists *fallaces argumentatores* in Latin[3], so there was some authoritative support for chosing the word to name something that Aristotle had no clear terminology for, *τόποι*/*loci* and *τρόποι*/*modi* being the best candidates in his text.

Everybody agrees that the *Sophistical Refutations* is a sort of companion or appendix to the *Topics*, both because it clearly states that it is concerned with dialectic – or para-dialectic, as we might call it –, and because the epilogue in chap-

1. For medieval comments on the several meanings of *fallacia*, see, e.g., *Anonymi Aurelianensis I Commentarium in Sophisticos Elenchos* [SE13], ed. Ebbesen, pp. 75-76, and *Anonymi Cantabrigiensis Commentarium in Sophisticos Elenchos* [SE15], ed. Ebbesen, pp. 67-68. These two anonymi both distinguish between *fallacia* in the sense of *causa fallendi* and *deceptio quae provenit ex tali causa fallendi.* Numbers like [SE13] refer to the list of *Elenchi*-related texts in Ebbesen 1993a.

2. *fallacia* ~ *ἀπάτη* Arist., *Soph. el.*, 5, 167b4; 7, 169a22, 169a37, 169b2, 169b11, 169b15; 15, 174a29. ~ *μοχθηρία* 24, 179b18; at 179b17 *μοχθηρίας* is rendered *fallendi occasiones.*

3. Boethius, *In librum Aristotelis Περὶ ἑρμηνείας commentarium*, *Pars Prior*, ed. Meiser, pp. 81-82: "sed diligentius haec in libro quem *σοφιστικῶν ἐλέγχων* inscripsit edisserit. illic enim sophistarum, quos fallaces argumentatores Latine possumus dicere, [...] argumenta distinxit".

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ter 34 must be understood as an epilogue to both works, as commentators have realized ever since the Middle Ages, and no doubt earlier. Theodor Waitz (1864) and David Ross (1958), in their editions of the two texts even went so far as to label *Soph. el.* book I, i.e., 9, of the *Topics* in spite of a complete lack of manuscript warrant for so doing.

So, are the thirteen fallacies as many *topoi*? In the Latin Middle Ages, this was a standard assumption, and for the medievals this had consequences for the way they interpreted the *Sophistical Refutations*.

In this essay I will first examine the textual evidence for holding that Aristotle considered the fallacies to be *topoi*, and next – supposing he did so – see what consequences this could have for the way to read the *Sophistical Refutations.* In trying to work out such consequences I shall draw inspiration from some medieval predecessors.

2. Are the Fallacies *topoi*? The Textual Evidence

The answer to the question whether the fallacies are *topoi* seems straightforward in view of the following passage[4]:

> 1. "Οἱ μὲν οὖν παρὰ τὴν λέξιν ἔλεγχοι ἐκ τούτων τῶν τόπων εἰσίν, τῶν δ' ἔξω τῆς λέξεως παραλογισμῶν εἴδη ἔστιν ἑπτά"[5] ("So, refutations that depend on expression proceed from these *topoi*. Expression-independent paralogisms come in seven types"[6]).

The *τόποι* clearly are the six fallacies *in dictione*. For their non-linguistic counterparts Aristotle uses *εἴδη* instead of *τόποι*. The obvious implication is that proceeding from the same *topos* is what unites the members of one type (*eidos*) of paralogism.

A less certain passage is:

> 2. "οἱ δὲ τοῦτο ποιοῦσι μὲν οὔ, δοκοῦσι δὲ διὰ πολλὰς αἰτίας· ὧν εἷς τόπος εὐφυέστατός ἐστι καὶ δημοσιώτατος, ὁ διὰ τῶν ὀνομάτων"[7] ("Some syllogisms and refutations, while not being actually conclusive, appear to be so for several reasons. One most

4. As will appear, in some passages the manuscript tradition of the *Sophistical Refutation* exhibits a *τόπος - τρόπος* variation. Several years ago, Louis-André Dorion (Université de Montréal) was so kind as to check his collations of a great number of mss of the Greek text for this sort of variation. Likewise, Pieter Sjoerd Hasper in 2015 kindly sent me a draft of his forthcoming edition of the Greek text, for which he has collated more manuscripts than used in Ross's 1958 edition. I am deeply grateful to both Dorion and Hasper. Finally, I have looked up a few mss myself. When, in the following, I quote the *Soph. el.* without mentioning which edition I use, this means that there is agreement between the editions of Ross and that of Hecquet. I also occasionally refer to Immanuel Bekker's 1831 edition.

5. Arist., *Soph. el.*, 4, 166b20-22.

6. All translations from Greek or Latin in this essay are by the author.

7. Arist., *Soph. el.*, 1, 165a3-6.

apt and popular topos for such syllogisms and refutations is the one depending on the names". Or alternatively: "Some syllogisms and refutations, while not being actually conclusive, appear to be so for several reasons, among which is one most apt and popular topos, the one depending on the names"[8]).

Many commentators have seen this as a reference to equivocation or to equivocation and amphiboly[9], and their interpretation is well supported by the sequel (1, 165a6-13) in which Aristotle argues that we are easily led to believe in a one-to-one correspondence between names and things, although, in fact, there must be cases in which one name or phrase signifies several things.

Also not quite clear is

3. "Δῆλον οὖν ὅτι οὐ πάντων τῶν ἐλέγχων ἀλλὰ τῶν παρὰ τὴν διαλεκτικὴν ληπτέον τοὺς τόπους· οὗτοι γὰρ κοινοὶ πρὸς ἅπασαν τέχνην καὶ δύναμιν"[10] ("So it is clear that what we should look for is not the *topoi* of all [types of] refutations, but of those pertaining to dialectic, for they are common with respect to every discipline and faculty").

Quite possibly, the *τόποι* here are the thirteen fallacies.

In the passages adduced above the manuscript tradition is unanimous in having the word *τόπος*, and modern editors follow them. In the following two there is vacillation between *τόπος* and *τρόπος*.

4. "ὥστε πάντες οἱ *τόποι* πίπτουσιν εἰς τὴν τοῦ ἐλέγχου ἄγνοιαν, οἱ μὲν οὖν παρὰ τὴν λέξιν, ὅτι φαινομένη <ἡ> ἀντίφασις, ὅπερ ἦν ἴδιον τοῦ ἐλέγχου, οἱ δ' ἄλλοι παρὰ τὸν τοῦ συλλογισμοῦ ὅρον"[11] ("Thus all the *topoi* fall under ignorance of refutation, the ones depending on expression because the contradiction – which is, we said, the characteristic of a refutation – is only apparent, the rest because they do not comply with the definition of a syllogism").

The *τόποι* here are the thirteen fallacies, which Aristotle has just reduced to one fundamental type, and which at the beginning of the reduction were referred to as *τρόποι*:

8. I have chosen 'to be conclusive' to render *ποιεῖν* in this passage. For a more precise interpretation, see, e.g., *Anonymi Aurelianensis I Commentarium in Sophisticos Elenchos* [SE13], ed. Ebbesen, p. 25: "bene dicit *illi hoc non faciunt,* quod tamen faciunt veri syllogismi vel elenchi, id est ex positis aliquid aliud inferunt, quantum ad syllogismos; vel contradictionem ex intentione probant, quantum ad elenchos".

9. Equivocation: *Anonymi Aurelianensis I Commentarium in Sophisticos Elenchos* [SE13], ed. Ebbesen, p. 25: "Referendum est ergo quod dicit *unus locus* ad aequivocationem". Equivocation, amphiboly or both: Ps. Alexander Aphrodisiensis (Michael Ephesius), *In Aristotelis Sophisticos Elenchos Commentarius*, ed. Wallies, p. 11.

10. Arist., *Soph. el.*, 9, 170a34-36.

11. Arist., *Soph. el.*, 6, 169a18-21.

4bis. "ἔστι γὰρ ἅπαντας ἀναλῦσαι τοὺς λεχθέντας *τρόπους* εἰς τὸν τοῦ ἐλέγχου διορισμόν"[12] ("For all the above-mentioned *tropoi* may be resolved into [breaches of] the definition of refutation").

No modern editor registers any variant on either occasion, but in his 1923 edition Wallies created uniformity between the two passages by emending *τόποι* in nº 4 into *τρόποι*. He claimed no support for the reading from mss of the Aristotelian text, but indicated that he was following the lead of the early twelfth-century commentator Michael of Ephesus and the fourteenth-century paraphrast Sophonias[13].

Michael must have used a manuscript that actually read *τρόποι*. The form occurs both in his lemma and in his paraphrase, and there is no difference between his first, unedited, version of the commentary and the final one, which Wallies edited in *Commentaria in Aristotelem Graeca* II.3[14]. Michael's lemma and paraphrase run as follows:

4ter. "**Ὥστε πάντες οἱ *τρόποι* πίπτουσι**] Δείξας ἐκ τῆς ἐπαγωγῆς, ὅτι πάντες οἱ τρόποι εἰς τὴν τοῦ ἐλέγχου ἄγνοιαν ἀνάγονται, καὶ πάλιν ἡμᾶς ἀναμιμνήσκει τῆς τούτων ἀναγωγῆς, καὶ σαφέστερον ἀπαγγέλλει πῶς οἱ δεκατρεῖς τρόποι εἰς τὴν τοῦ ἐλέγχου ἄγνοιαν ἀνάγονται"[15] ("**Thus all the *tropoi* fall**] Having shown by deduction that all the *tropoi* are reducible to ignorance of the refutation, he again reminds us of this reduction, and declares more clearly how the thirteen *tropoi* are reduced to ignorance of the refutation").

Interestingly, in one of the mss of the first edition, in which the commentary is written in the margins around a complete text of the *Sophistical Refutations*, the Aristotelian text actually has *τρόποι*[16]. Since the archetype of the tradition of Mi-

12. Arist., *Soph. el.*, 6, 168a19-20.

13. The apparatus says "τρόποι Π M¹ τόποι libri Bo Str." (Aristoteles, *Topica cum libro De sophisticis elenchis e schedis Ioannis Strache*, ed. Wallies, p. 202), which means that the reading τρόποι enjoys the support of Sophonias' paraphrase and Michael's commentary, while the mss of the Aristotelian text and Boethius' translation offer τόποι. The inserted <ἡ> in quotation 4 is also due to Wallies; it has been accepted by both Ross and Hecquet.

14. Cf. n. 9. For his edition of the final version of Michael's commentary (Ps.-Alexander-1) Wallies did not use ms *C* = Vat. gr. 269, which combines being the best ms with belonging to a different family from Wallies' *dux*. *C*, however, agrees with the printed text in this scholium (*C* 32v). As for the first version (Ps.-Alexander-2) I have checked on the readings of ms *229* = ms Madrid, Escorial, Φ.III.10 (f. 5v), ms *150* = ms Jerusalem, St. Sepulchri, 150 (f. 191v) and ms *1770* = ms Vat. gr. 1770 (f. 30r). The Ps.-Alexander-2 mss have longer lemmata for this scholium, but otherwise the same text as Wallies' edition (Ps. Alexander Aphrodisiensis (Michael Ephesius), *In Aristotelis Sophisticos Elenchos Commentarius*, ed. Wallies), and – most importantly – have *τρόποι* both in the lemma and in the paraphrase.

15. Ps. Alexander Aphrodisiensis (Michael Ephesius), *In Aristotelis Sophisticos Elenchos Commentarius*, ed. Wallies, p. 65, ll. 21-25.

16. Ms. Jerusalem, St. Sepulchri, 150, f. 191v.

chael's first version had the same format, it seems possible that the ms in question reproduces not only Michael's scholia but also his Aristotelian text[17].

The fact that Sophonias' paraphrase also presupposes *τρόποι* is less interesting. He may owe it to Michael rather than to a manuscript of the Aristotelian text[18]. By contrast, Leon Magentenos (late twelfth or early thirteenth centuries) in a paraphrasing scholium has *τόποι*[19].

Wallies may have been anticipated by Julius Pacius, whose 1597 edition also has *τρόποι* on both occasions (*Ἀριστοτέλους Ὄργανον / Aristotelis Stagiritæ Peripateticorum principis Organum*, 799 and 802), but it is unclear whether this is the result of conjectural emendation or reflects one or more of the manuscripts he used.

In the next passage, Bekker, Ross and Hecquet all print

> 5. "*Τρόποι* μὲν οὖν εἰσιν οὗτοι τῶν σοφιστικῶν ἐλέγχων"[20] ("These, then are the *tropoi* of sophistical refutations").

but Boethius' early 6th-century translation has

> 5bis. "*Loci* ergo de sophisticis elenchis hi sunt"[21] ("These, then are the *topoi* of sophistical refutations").

– and, in fact, *τόποι* is attested in several Greek manuscripts (including the venerable ms *Barberinianus gr. 87* [Vatican] and ms *Gudianus gr. 24* [Wolfenbüttel]) and it is supported by three Arabic translations[22]. Whichever reading one prefers, the thirteen fallacies must be meant.

Besides, there is one more passage in which Boethius' manuscript must have had *τόπος* instead of the *τρόπος* found in the Greek mss:

> 6. "διὸ καὶ τῶν παρὰ τὴν λέξιν οὗτος ὁ *τρόπος* θετέος"[23] – "Quare in his quae sunt secundum dictionem hic *locus* ponendus"[24] ("Therefore, this *topos/tropos* must also be counted among those depending on speech").

17. For the transmission of Michael's commentary, including the filiation of the mss, see Ebbesen 1981, III, Appendix 4, pp. 12-63.

18. Sophonias = *Anonymi (Sophonias) in Aristotelis Sophisticos Elenchos Paraphrasis*, ed. Hayduck, p. 20, l. 12. For Sophonias' dependence on Michael, see Ebbesen 1981, I, p. 333-336.

19. Ms. Vat. gr. 244, f. 602v: "ὥστε πάντες οἱ τόποι τῶν σοφισμάτων τῶν παρὰ τὴν λέξιν καὶ τῶν ἐκτὸς τῆς λέξεως πίπτονται καὶ ἀνάγονται εἰς τὴν τοῦ ἐλέγχου ἄγνοιαν". The Aristotelian text of the ms has τόποι, but with a tiny ρ added over the τ.

20. Arist., *Soph. el.*, 11, 172b5.

21. Arist., *Soph. el.*, 172b5, ed. Dod, p. 27.

22. According to the apparatus of P.S. Hasper's forthcoming edition.

23. Arist., *Soph. el.*, 7, 169a36-37.

24. Arist., *Soph. el.*, 169a36-37, ed. Dod, p. 18.

The *τ(ρ)όπος* in case is the fallacy of figure of speech. Boethius' reading seems not to be attested in any surviving Greek manuscript.

But the thirteen fallacies are not the only *topoi* mentioned in the *Sophistical Refutations*. Indeed, just as the *Topics* lists *topoi* under each of its four predicate types (definition, proprium, genus, accident), so the *Elenchi* lists *topoi* under each of the five goals (lat.: *metae*) of the sophists: refutation, blatant falsehood, paradox, solecism, babbling (*ἔλεγχος, ψεῦδος, παράδοξον, σολοικισμός, ἀδολεσχία*). *Topoi* for the last four *metae* are explicitly mentioned in the following places (though in one of them there is again a *τόπος/τρόπος* variation:

> 7. "Πρὸς δὲ τὸ ψευδόμενον δεῖξαι ἴδιος *τ(ρ)όπος* ὁ σοφιστικός, τὸ ἄγειν πρὸς τοιαῦτα πρὸς ἃ εὐπορεῖ λόγων"[25] ("A *t(r)opos* peculiar to showing that the opponent is speaking a falsehood is the sophistical one that consists in leading him [or: the discussion] toward subjects concerning which one has a good supply of arguments").

This *t(r)opos* is said to be peculiar to the aim of *falsehood* in order to distinguish it from some previously mentioned strategies that can also be used lead the opponent to paradox. By implication, the strategies mentioned just before this passage (in 12, 172b9-24) are also *topoi* or *tropoi*, according to which reading one prefers. *τόπος* (adopted by Bekker and Ross) seems to be the majority reading, and the ms. used in the twelfth century by James of Venice for his Latin translation must have sided with the majority (ed. Dod, p.64, l. 16). Michael's brief paraphrase of the passage (pp. 100-101, ll. 33-1) does not reveal which reading his ms. had[26]. According to Hasper, the ms. *Gudianus 24* (Wolfenbüttel), ms. *Barbarinianus 87* (Vatican) and the late ms. *C.I.15* (Durham) read *τρόπος*, which was also what Boethius read. Hecquet opted for *τρόπος* in her edition, and claims the reading for ms. *Urbinas 35* (A) and ms. *Marcianus gr. 201* (B), while she claims *τόπος* for ms. *Barb. 87* (V), but this is surely wrong. I have myself checked that mss A and B actually have *τόπος*, as claimed by all other editors, and that ms. V has *τρὸπος*, as claimed by Haspers.

Now, this particular passage has a close parallel in the *Topics*:

> 7[bis]. "Ἔτι ὁ σοφιστικὸς *τρόπος*, τὸ ἄγειν εἰς τοιοῦτον πρὸς ὃ εὐπορήσομεν ἐπιχειρημάτων"[27] ("Moreover, the sophistical *tropos* that consists in leading our opponent [or: the discussion] toward a subject concerning which we shall have a good supply of arguments").

25. Arist., *Soph. el.*, 12, 172b25-26.

26. The scholium *ad loc.* in ms. Paris, gr. 2019, f. 228r, which may represent Michael's first version, is a shorter version of the printed text, with which it agrees in not including either τόπος or τρόπος. For a discussion of the scholia in the Paris ms., see Ebbesen 1981, I, p. 292-294.

27. Arist., *Top.*, II, 5, 111b32-33.

Most of the mss used by Brunschwig for his authoritative 1967 edition of the *Topics* read *τρόπος*, and according to the apparatus in Minio-Paluello and Dod's 1969 edition of Boethius' Latin translation, the mss unanimously offer *modus* (ed. Minio-Paluello / Dod, p. 38, l. 8). Brunschwig, however, records the reading *τόπος* for two respectable mss: **C** (=Paris, BnF, ms Coislin gr. 330) and **c**$^{a.c.}$ (= ms Vat. gr. 1024).

In this case we have a text witness that is even older than Boethius' translation, viz. Alexander of Aphrodisias's commentary. But his comment on the passage exhibits a strange fluctuation between the two readings:

> 7ter. "Ὁ τόπος, ὃν νῦν παραδίδωσιν, ἐστὶ τὸ μεταφέρειν τὸ πρόβλημα καὶ τὸν λόγον πρὸς τοῦτο πρὸς ὃ εὐποροῦμεν ἐπιχειρημάτων. ὃν (p. 167, ll. 26-27) **σοφιστικὸν** εἶπεν εἶναι ***τρόπον***, ἐπεὶ τοῖς σοφισταῖς σύνηθες τὸ καταλιπόντας τὸ περὶ τοῦ προκειμένου ποιεῖσθαι τοὺς λόγους μετάγειν αὐτοὺς καὶ μεταφέρειν πρὸς τοῦτο πρὸς ὃ εὐποροῦσιν ἐπιχειρημάτων, ὡς ἐν τῷ Πρωταγόρᾳ τῷ Πλάτωνος ὁ Πρωταγόρας ποιεῖ, [...] γίνεται δέ ποτε καὶ ἀναγκαία ἡ μετάληψις καὶ μετάβασις, ὅταν εἰς τοιοῦτον γίνηται ὃ κατασκευασθὲν χρήσιμόν ἐστι πρὸς τὸ ἐξ ἀρχῆς κείμενον. διὸ καίτοι (p. 168, l. 8) **σοφιστικὸν** ὄντα καθόλου τὸν ***τόπον*** ἐν τοῖς διαλεκτικοῖς τίθησι· διαλεκτικοῦ γὰρ ἡ εἰς τοιαῦτα μετάληψις δι' ὧν δειχθέντων τὸ ἐξ ἀρχῆς κείμενον συγκατασκευάζεται. ὡς γὰρ αὐτός φησι, ποτὲ μὲν εἰς *ἀναγκαῖόν* τε καὶ χρήσιμον πρὸς τὸ ἐξ ἀρχῆς κείμενον ἡ τοιαύτη μετάληψις γίνεται, ποτὲ δὲ εἰς *φαινόμενον ἀναγκαῖον* οὐκ ἀναγκαῖον δέ, ποτὲ δὲ εἰς φανερῶς ἀλλότριον καὶ *οὔτε ἀναγκαῖον* εἰς τὸ προκείμενον οὔτε φαντασίαν παρέχον ἀναγκαίου, ὅσπερ (p. 168, l. 14) ***τρόπος*** τῆς μεταλήψεως φανερῶς ἐστι **σοφιστικός**"[28] ("The *topos* that he now presents consists in shifting the problem and the argument to that concerning which we have a good supply of arguments. He calls it (p. 167, ll. 26-27) **a sophistical *tropos***, because it is a habit with sophists to leave arguing about the matter at hand and move and shift the discussion to that concerning which they have a good supply of arguments, as does Protagoras in Plato's *Protagoras* [...]. Occasionally a shift and transition even becomes necessary, when it is to something that, if established, will be useful with a view to the original thesis. This is why, for all its being utterly (p. 168, l. 8) **sophistical**, he presents this ***topos*** among the dialectical ones, for it is a dialectician's move to produce a shift to things that, if demonstrated, will help establish the original thesis. As he himself says, such a shift sometimes occurs toward something that is necessary and useful with a view to the original thesis, sometimes toward something that appears to be necessary but is not necessary, sometimes toward something that is blatantly irrelevant and neither necessary with a view to the matter at hand and does not even show an appearance of being necessary – the latter (p. 168, l. 14) ***tropos*** of shift is blatantly **sophistical**").

The *τρόπος* (in p. 168, l. 14) may not be meant as a quotation of Aristotle's text, but both *σοφιστικὸν ... τρόπον* (in p. 167, ll. 27-28) and *σοφιστικὸν ... τόπον* in (p.

28. Alexander Aphrodisiensis, *In Aristotelis Topicorum libros octo commentaria*, ed. Wallies, pp.167-168, ll. 25-14.

168, l. 8) seem to be so. At p. 168, l. 8 Wallies' apparatus records the variant *τρόπον* from **P** = Paris, ms gr. 1874 and the Aldine edition. This is certainly *lectio facilior*. To explain the *lectio difficilior* preferred by Wallies, and apparently best supported by the tradition, we might conjecture that Alexander actually read *τρόπος* in Aristotle, but took it to be, in the context, equivalent to *τόπος* and wanted his readers to do the same. But this is hardly the case, for his explanation of the expression *σοφιστικὸς τρόπος* focuses uniquely on why Aristotle claims that this *τόπος* is *σοφιστικός*, while it contains no attempt to explain why Aristotle calls it a *τρόπος*.

8. "Πλεῖστος δὲ *τόπος* ἐστὶ τοῦ ποιεῖν παράδοξα λέγειν, ὥσπερ καὶ ὁ Καλλικλῆς ἐν τῷ Γοργίᾳ γέγραπται λέγων, καὶ οἱ ἀρχαῖοι δὲ πάντες ᾠοντο συμβαίνειν, παρὰ τὸ κατὰ φύσιν καὶ κατὰ τὸν νόμον·"[29] ("A rich *topos* for making the opponent say paradoxical things is the one about by nature and by convention, as Callicles is made to say in [Plato's] *Gorgias* and all the ancients thought to be the case").

This is, in fact, the third in a list of four strategies for making the opponent say something paradoxical. By implication, the two previous ones (12, 172b29-173a6) are also *topoi,* and so is the following one (12, 173a19-30), and this is confirmed by the subsequent phrase connecting the section about paradox with the one about babbling:

9. "Καὶ τὰ μὲν παράδοξα ἐκ τούτων δεῖ ζητεῖν τῶν *τόπων*· περὶ δὲ τοῦ ποιῆσαι ἀδολεσχεῖν"[30] ("Paradoxes should be looked for on the basis of these *topoi*. Concerning babbling, however ...")

3. Are the Fallacies *topoi*? Conclusion

There is, then, a great deal of confusion in the manuscript tradition about the distribution of *τόπος* and *τρόπος* in the relevant passages of both *Sophistical Refutations* and *Topics*, and apparently the confusion goes all the way back to antiquity. It is best explained, I propose, by assuming that Aristotle himself sometimes used *τρόπος* for what he would elsewhere call a *τόπος*. Whether the distribution of the two words in modern editions exactly reproduces the original is another matter.

In spite of the vacillation between *τόπος* and *τρόπος*, and of the fact that even on the highest count the word *τόπος* has rather few occurrences in the *Sophistical Refutations*, I consider it reasonable to conclude that the work is fundamentally an analysis of *topoi* conducive to the five aims of the sophists (*metae sophistarum* in scholastic parlance) laid out in chapter 3.

29. Arist., *Soph. el.*, 12, 173a7-10.
30. Arist., *Soph. el.*, 13, 173a31-32.

This may not come as a big surprise, but the fact has not received quite the attention from modern commentators that it deserves.

Paolo Fait in his annotated Italian 2007 translation of the *Sophistical Refutations* correctly, I think, proposes to see the structure of the work as fundamentally a miniature version of that of the *Topics*, and takes *Soph. el.* chapters 2-5 and 12-14 to correspond to *Topics* II-VII. Those parts of the two works "elencano gli elementi costitutivi dell'arte", he says, and to support his claim of correspondence, he adds: "Si noti che anche i tredici paralogismi e gli espedienti per indurre al falso e al paradosso sono pensati da Aristotele come *topoi*"[31]. Fait did not in 2007 draw any further consequences from his recognition that the *Elenchi* is a book about *topoi* – or at least not explicitly.

Confirmation that Aristotle did consider the fallacies to be *topoi* is provided by *Rhetoric* II.23-24, where a list of *topoi* for genuine enthymemes includes some known also from the *Topics*, such as the *locus a casibus (ἐκ τῶν πτώσεων)*, whereas the parallel list of those for apparent enthymemes includes items that also occur in the *Sophistical Refutations*: homonymy, composition and division, consequent, *secundum quid et simpliciter.*

4. Consequences of Assuming the Fallacies are *topoi*

If we assume that the fallacies are *topoi*, we should in the interpretation of the *Sophistical Refutations* use whatever we think we know about *topoi* from our study of the *Topics.* But first we have to look at how a sophistical paralogism works, because a fallacy is a class of such paralogisms, and a sophistical paralogism is not just a bad argument, but one that has some appearance of being a good one while also having some flaw. For anyone to be deceived by a paralogism, he must – whether consciously or subconsciously – accept a flawed proposition which does not, normally, appear in the paralogism, because spelling it out would make it easy to see that it is false, but which, if true, would have validated the move from premisses to conclusion.

Paolo Fait[32] calls such a proposition a "false validating premiss", and finds good support for the notion in Aristotle's text, notably at *Soph. el.*, 8, 170a12-19, where Aristotle stresses that for a sophistical refutation to succeed it presupposes an unstated proposition which the answerer tacitly accepts, such as "*x* signifies only one thing" if the refutation depends on equivocation, or "*x* signifies only a this" if it depends on figure of speech[33].

31. Fait 2007, p. l.

32. See Fait 2007, pp. xxii-xxv and the later, more detailed treatment in Fait 2013.

33. Arist., *Soph. el.*, 8, 170a12-19: "Ἔστι δ' ὁ σοφιστικὸς ἔλεγχος οὐχ ἁπλῶς ἔλεγχος ἀλλὰ πρός τινα· καὶ ὁ συλλογισμὸς ὡσαύτως. ἂν μὲν γὰρ μὴ λάβῃ ὅ τε παρὰ τὸ ὁμώνυμον ἓν σημαίνειν καὶ ὁ παρὰ τὴν

To see how the validating premiss works, we can consider a paralogism depending on equivocation. Consider this argument[34]:

All ashes are trees,
some ashes are human remains,
therefore some human remains are trees.

This would be a valid syllogism if the terms had the same signification in all occurrences. And this is what the validating premiss *'Ash' has exactly one sense* claims for the problematic term on whose double sense the paralogism hangs.

But if particular paralogisms depending on equivocation have each their own validating premiss in which *x* is a term actually occurring in the paralogism in question, one would expect there to be a universally validating premiss corresponding to the universal notion of the fallacy of equivocation, or else, what is the point of gathering a number of paralogisms under that species of fallacy? Aristotle never spells such a universally validating premiss out, just as he does not spell out any particular validating premiss, leaving his readers with a quick hint[35].

Here Theophrastus, the very first exegete of Aristotle's *Topics*, comes to our rescue[36]. According to Alexander of Aphrodisias he had noticed a common structure in Aristotle's presentation of his *topoi*:

> 10. "δεῖ δὲ μὴ ἀγνοεῖν ὅτι Θεόφραστος διαφέρειν λέγει παράγγελμα καὶ τόπον· παράγγελμα μὲν γάρ ἐστι τὸ κοινότερον καὶ καθολικώτερον καὶ ἁπλούστερον λεγόμενον, ἀφ' οὗ ὁ τόπος εὑρίσκεται· ἀρχὴ γὰρ τόπου τὸ παράγγελμα, ὥσπερ ὁ τόπος ἐπιχειρήματος. οἷον παράγγελμα μὲν τὸ οὕτως λεγόμενον, ὅτι δεῖ ἐπιχειρεῖν ἀπὸ τῶν ἐναντίων, ἀπὸ τῶν συστοίχων, τόπος δὲ οἷον 'εἰ τὸ ἐναντίον πολλαχῶς, καὶ τὸ ἐναντίον', ἢ 'εἰ τῷ ἐναντίῳ τὸ ἐναντίον ὑπάρχει, καὶ τὸ ἐναντίον ὑπάρχει τῷ ἐναντίῳ', καὶ πάλιν 'ὡς ἓν τῶν συστοίχων, οὕτως καὶ τὰ λοιπά'"[37] ("Notice that Theophrastus says that exhortation and *topos*

ὁμοιοσχημοσύνην τὸ μόνον τόδε, καὶ οἱ ἄλλοι ὡσαύτως, οὔτ' ἔλεγχοι οὔτε συλλογισμοὶ ἔσονται, οὔθ' ἁπλῶς οὔτε πρὸς τὸν ἐρωτώμενον. ἐὰν δὲ λάβωσι, πρὸς μὲν τὸν ἐρωτώμενον ἔσονται, ἁπλῶς δ' οὐκ ἔσονται· οὐ γὰρ ἓν σημαῖνον εἰλήφασιν ἀλλὰ φαινόμενον, καὶ παρὰ τοῦδε" ("The sophistical refutation is not a refutation in an unrestricted sense, but relative to some person, and the same holds for the deduction; for if the one that relies on equivocation does not assume that [a given term] signifies just one thing, or the one that relies on similar form [does not assume] that it signifies only a 'this', and so on, there will be no refutations or deductions, whether in an unrestricted sense or relative to the answerer. But if they do assume it, there will be [refutations and deductions] relative to the answerer, although not in an unrestricted sense").

34. Fait 2007, pp. xxii-xxiii used an example exploiting the equivocation of Italian *rombo* "turbot, rhombus", Fait 2013, pp. 243-244 used "The Vienna Circle is a circle. A circle is a geometrical figure. Therefore: The Vienna Circle is a geometrical figure" with the false validating premiss "'Circle' means only one thing". The latter is an elegant modernization of a paralogism hinted at in *Soph. el.*, 10, 171a10, which, when spelled out, would run "ἡ Ὁμήρου ποίησις κύκλος, ὁ δὲ κύκλος σχῆμα, ἡ ἄρα Ὁμήρου ποίησις σχῆμα".

35. See quotation in n. 33.

36. I have discussed Theophrastus' views about *topoi* on a couple of earlier occasions. See Ebbesen 1981, I ch. IV.3.5, especially p. 107; Ebbesen 1993b, pp. 29-31.

37. Alexander Aphrodisiensis, *In Aristotelis Topicorum libros octo commentaria*, ed. Wallies, p. 135, ll. 2-10.

> are different things. An exhortation is the more commonly, universally and simply expressed statement, starting from which one finds the *topos*, for the exhortation points to a *topos* just as the *topos* points to an argument. For example, a statement like 'One should argue from opposites' or 'from conjugates' is an exhortation, while [a statement like] 'If one of a pair of opposites is said in many ways, so is the other' or 'If one of a pair of opposites applies to one of another pair, then the other of the first pair applies to the other of the second pair' or 'As one of the conjugates, so the others' is a *topos*").

Theophrastus does not quote Aristotle *verbatim*, but there can be little doubt that he is thinking of passages like *Topics* I, 15, 106a36-b12 (opposites said in many ways) and II, 8-9 (opposites and conjugates). A passage that lends itself very neatly to a Theophrastean analysis occurs in *Topics* IV, where Aristotle says that if somebody is claiming that *A* is the genus of some species *B*, one should

> 11. "Ὁρᾶν δὲ καὶ εἰ ἔν τινι τῷ αὐτῷ πέφυκεν ἄμφω γίνεσθαι· ἐν ᾧ γὰρ τὸ εἶδος, καὶ τὸ γένος, οἷον ἐν ᾧ τὸ λευκόν, καὶ τὸ χρῶμα, καὶ ἐν ᾧ γραμματική, καὶ ἐπιστήμη. ἐὰν οὖν τις τὴν αἰσχύνην φόβον εἴπῃ ἢ τὴν ὀργὴν λύπην, οὐ συμβήσεται ἐν τῷ αὐτῷ τὸ εἶδος καὶ τὸ γένος ὑπάρχειν· ἡ μὲν γὰρ αἰσχύνη ἐν τῷ λογιστικῷ, ὁ δὲ φόβος ἐν τῷ θυμοειδεῖ· καὶ ἡ μὲν λύπη ἐν τῷ ἐπιθυμητικῷ (ἐν τούτῳ γὰρ καὶ ἡ ἡδονή), ἡ δὲ ὀργὴ ἐν τῷ θυμοειδεῖ. ὥστ' οὐ γένη τὰ ἀποδοθέντα, ἐπειδὴ οὐκ ἐν τῷ αὐτῷ τοῖς εἴδεσι πέφυκε γίνεσθαι"[38] ("Look and see if both are such as to be in one and the same [subject], for in whatever the species is, its genus is. For instance, in whatever white is, colour is, and in whoever there is literacy there is knowledge. So, if someone says that shame is fear or that anger is grief, the species and the genus will not turn out to be in the same, for shame is in the rational faculty while fear is in the emotional faculty, and grief is in the appetitive faculty (for that is where [its opposite] pleasure is, while anger is in the emotional faculty. So, the alleged genera are not genera since they are not such as to be in the same as the species").

Thus, according to Theophrastus, the *topos* proper is a statement like "If one of a pair of opposites is said in many ways, so is the other" or "In whatever the species is, its genus is". And he, if anyone, ought to know what was essential to an Aristotelian *topos.* As reported by Alexander, Theophrastus concentrated on the heuristic function of such statements – they suggest to us how to formulate an argument –, but the reason why they can help us in that way is that they are general rules, which, supposing they are true, will validate a number of arguments.

Sophistics is perverted dialectics, which depends on appearances, so a sophistical *topos* in the Theophrastean sense ought to be a pseudo-rule that will not stand close scrutiny. In short, the generalized false validating premiss we have been looking for.

38. Arist., *Top.*, IV, 5, 126a3-12.

Admittedly, reducing an Aristotelian *topos* to merely being a logical rule or pseudo-rule will not work. Thus the *topos* about nudging the discussion in the direction of matters about which one has a store of arguments (item 7, above) seems to be a strategy, and Aristotle's presentation of it contains just an exhortation and no rule. The concept underlying Aristotle's use of *τόπος* in connection with argumentation is rather fuzzy, and is perhaps best explained by the equally imprecise "source of arguments" or "starting point for arguments" – *ἀφορμὴ ἐπιχειρήματος*, as the ancients said[39]. It can thus be the strategy or the principle that unites a class of arguments. Yet, it seems to me that Theophrastus had grasped something important: one central ingredient in those *topoi* that are not mere procedural strategies is a logical rule or pseudo-rule, and except, perhaps, for the case of the strange "ignorance of refutation", it does make sense to look for pseudo-rules in the thirteen fallacies.

5. *Causae apparentiae*, *causae defectus* and *maximae*

An Aristotelian fallacy is a type of argument that is not only bad but also appears to be good, at least at first glance. To catch this double aspect, Latin exegetes of the early thirteenth century introduced a new pair of concepts. Each of the thirteen fallacies was assumed to have its own *causa apparentiae* and it own *causa non existentiae*, (also known as *principium motivum* and *causa defectus*, respectively), one explaining why paralogisms falling under the fallacy in case appear to be good arguments, the other why they are no good anyway. By introducing those two sorts of *causa* and trying to spell them out for each of the thirteen fallacies, the scholastics explicitated something that is implicit in Aristotle's text. Which is precisely the job of a good commentator[40].

From Manlius Boethius' *De topicis differentiis* the scholastics had inherited the notion that each dialectical *topos (locus)* consists of two parts, *locus differentia* and *locus maxima*, the first being a label identifying the key logical notion involved, as in l*ocus a specie* (with *species* being the *differentia*), the second being a rule like *De quocumque praedicatur species et genus* "Of whatever a species is predicated, its genus is also predicated"[41]. What Boethius and the medievals called dialectical maxims were, of course, the same sort of rules that Theophrastus had claimed were the *topoi* proper. *Maxima (propositio)* was Boethius' translation of *ἀξίωμα*[42],

39. See sources in Ebbesen 1981, I, p. 111.

40. About the medieval notions of causes of appearance and non-being, and of sophistical maxims, see Ebbesen 1987. See also Fait 2007, pp. xx-xxi.

41. See, e.g., Petrus Hispanus, *Tractatus*, V, 13, ed. de Rijk, p. 64. For more about medieval topics, see Green-Pedersen 1984.

42. If proof of this is needed, it is provided by Boethius' translation of *Top.* VIII, 1, 155b14-15:

and the scholastics said that dialectical arguments *decurrunt super* such maxims, i.e. they proceed in accordance with them[43].

Realizing the thirteen fallacies are as many *loci*, the scholastics transferred this terminology to their exegesis of the *Elenchi*. This results in each fallacy having (a) a name like *locus ab aequivocatione*[44], (b) a *differentia* like *aequivocatio*, (c) a sophistical maxim. Cashing out the contents of (b) is stating the cause of appearance, which in the case of equivocation could, for instance, be formulated as "the material and formal identity of two or more words on the phonetic level" (*unitas vocis incomplexae secundum materiam et formam*)[45], while the associated maxim could run "One name always signifies just one thing" (*Omne nomen unum significat unum*)[46] or "The word is one, both materially and formally, so its meaning is the same" (*Vox est eadem secundum materiam et formam, ideo significatio est eadem*)[47]. The cause of not being could then be stated as "lack of identity etc." accompanied by a denial of the rule.

The maxim is the generalized version of the false validating premiss. As one thirteenth-century commentator puts it[48]:

> 12. "nullus enim artificialiter opponit quin causam apparentiae et defectus cognoscat, et sic opponens paralogismum aequivocationis istam propositionem cognoscit 'omne nomen unum significat unum'; per istam autem subintellectam decipitur respondens"[49] ("Nobody can perform the task of an opponent in a skilful way without knowing the cause of appearance and defect, and so one who employs a paralogism of equivocation knows the proposition 'every one name signifies one thing', and it is this implicit proposition that causes the respondent to be deceived").

The sophistical maxims had to be elicited from a text in which they are not explicitly stated. The notion of a *causa apparentiae* has a good basis in the Aristotelian text, though the technical term is not there. Thus in ch. 6 he says about

μάλιστα γνώριμα καὶ σύνεγγυς εἶναι τὰ ἀξιώματα: maxime notae et propinquae sunt *maximae propositiones* (Aristoteles, *Topica. Translatio Boethii*, ed. L. Minio-Paluello / B.G. Dod, p. 156).

43. The metaphor is briefly discussed in Fait 2013, p. 254. Fait suggests we are to imagine water running in a river-bed; this may, indeed, be the origin of the metaphor, but I doubt if it conjured up such a picture to the scholastics, for whom it was rather just a technical term.

44. Sometimes, for short, the medievals use *aequivocatio, amphibolia* etc. as names of the fallacies, but for them, as for Aristotle, these terms referred primarily to the phenomenon exploited in the paralogisms falling under the fallacy. Just as he had talked about arguments or paralogisms *παρὰ τὴν ὁμωνυμίαν* (*Soph. el.*, 4, 165b30; 17, 175b40; etc.), so they would talk about *orationes paralogisticae secundum amphiboliam* etc.

45. *Anonymi SF quaestiones super Sophisticos Elenchos* [SE64], q. 51, ed. Ebbesen, p. 114.

46. *Anonymi SF quaestiones super Sophisticos Elenchos* [SE64], q. 33, ed. Ebbesen, p. 68.

47. *Anonymi Marciani Commentarium in Sophisticos Elenchos Aristotelis* [SE45], ed. Ebbesen, p. 226.

48. The same texts with minimal variants in ms. Paris, B. Mazarine, 3489, ff. 20vb-21ra.

49. Robertus de Aucumpno, *Commentarium in Sophisticos Elenchos* [SE48], ms. Cambridge, Peterhouse, 206, f. 160va.

secundum quid et simpliciter that it is reducible to ignorance of what a refutation is because it violates the stipulation that the refutation should be the negation of exactly the same as was originally affirmed, and he concludes that the person producing an apparent refutation *simpliciter*:

13. "οὐ ποιεῖ ἔλεγχον, φαίνεται δὲ διὰ τὴν ἄγνοιαν τοῦ τί ἐστιν ἔλεγχος"[50] ("does not produce a refutation, but he appears [to do so] because of [the answerer's] ignorance of what a refutation is").

Indeed, it may be claimed – and some medieval commentators did claim – that in his presentation of the fallacies in chapters 4-5 Aristotle does, in fact, spell out the causes of appearance in the case of the non-linguistic fallacies. Thus *secundum consequens* is introduced as follows:

14. "Ὁ δὲ παρὰ τὸ ἑπόμενον ἔλεγχος διὰ τὸ οἴεσθαι ἀντιστρέφειν τὴν ἀκολούθησιν· ὅταν γὰρ τοῦδε ὄντος ἐξ ἀνάγκης τόδε ᾖ, καὶ τοῦδε ὄντος οἴονται καὶ θάτερον εἶναι ἐξ ἀνάγκης"[51] ("The [apparent] refutation that depends on what follows is due to believing that the [relation of] following is convertible, for when *p* being the case implies by necessity that *q* be the case, they believe that *q* being the case implies by necessity that *p* be the case").

In the words of a late-thirteenth century commentator,

15. "Determinat de fallacia consequentis dans primo causam apparentiae, et est eo quod putamus consequentiam converti quae non convertitur. Unde sicut ad antecedens necessario sequitur consequens, sic arbitramus via versa, sc. quod ad consequens sequitur antecedens"[52] ("<Aristotle here> deals with the fallacy of consequent, and first of all he states its cause of appearance, which is that we think a consequence is convertible when it is not. Thus we believe that just as the consequent necessarily follows from its antecedent, so also the other way round, i.e. that the antecedent follows from the consequent").

Similarly, *secundum quid et simpliciter* is introduced with the words

16. "Ὁ δὲ παρὰ τὸ <τὸ> μὴ αἴτιον ὡς αἴτιον, ὅταν προσληφθῇ τὸ ἀναίτιον ὡς παρ' ἐκεῖνο γινομένου τοῦ ἐλέγχου"[53] ("The apparent refutation which depends on treating as a cause what is not a cause occurs when a non-cause is inserted and treated as though it was what the refutation depended on").

50. Arist., *Soph. el.*, 6, 168b15-16.
51. Arist., *Soph. el.*, 5, 167b1-3.
52. *Anonymi e Musaeo 133 Expositio Sophisticorum Elenchorum* [SE39], ms Oxford, Bodleian Library, e Musaeo 133, f. 3va.
53. Arist., *Soph. el.*, 5, 167b21-22.

But then a question arises: Why does Aristotle not do the same for the linguistic fallacies, except in the case of figure of speech? This problem is discussed by a thirteenth-century commentator whom I have baptized Anonymus Monacensis. His solution is not entirely convincing, but he may have been on the right track. According to Anonymus Monacensis, figure of speech excepted, the linguistic fallacies generally carry their causes of appearance on their sleeves, as it were, since they depend on their phonetic shape which is available for direct inspection in spoken language, whereas it is not so easy to see what does the trick in the extra-linguistic fallacies, and so it was more important for Aristotle to make the causes of appearance clear in the latter case[54].

The notion of sophistical maxims has not enjoyed great popularity in recent Aristotelian scholarship. In 1866 it was mentioned with approval by Edward Poste in his annotated translation of the *Sophistical Refutations*[55], and then there was silence till Paolo Fait took up the theme again in 2013. Fait has several objections

54. *Anonymi Monacensis Commentarium in Sophisticos Elenchos* [SE34], ms München, BSB, clm 14246, f. 10va-b: "Hic primo quaeritur propter quid in docendo vel in ponendo paralogismos extra dictionem semper tangit causam generalem sive principia motiva in qualibet fallacia extra dictionem antequam ponat pralogismos, in fallaciis autem in dictione, cum ponat paralogismos non tangit prius causam generalem vel principium motivum ipsarum excepta fallacia figurae dictionis. § Item quaeritur propter quid potius in fallacia figurae dictionis tetigit huiusmodi principum motivum sive causam generalem omnium paralogismorum, in aliis autem fallaciis in dictione numquam fecit sic. § Ad solutionem huius notandum quod numerus et distinctio fallaciarum omnium, sc. tam in dictione quam extra dictionem, debet sumi penes principia motiva; sed quia Auctor in qualibet fallacia extra dictionem tangit specialiter principium et locum, sicut patet generaliter considerando in generatione paralogismorum cuiuslibet fallaciae, ex ipsis principiis motivis specialiter tactis patet differentia et numerus fallaciarum extra dictionem, et sic patet quod non oportet probare numerum fallaciarum extra dictionem. Sed in syllogismis in dictione non tetigit principia motiva in generatione paralogismorum, ergo ex ipsis principiis motivis non potuit apprehendi numerus earundem, cum ea non tangat, et ob hoc numerum {numerum: numerus M} fallaciarum in dictione prius probavit. Ad illud ergo quod quaeritur dicendum quod principia motiva in fallaciis in dictione sunt a parte sermonis exterius prolati. Ergo patet quod ex ipsis paralogismis et ex eorum formatione et generatione, cum ibi exprimantur voces vel propositiones, patet principium motivum, neque oportebat tangere, sed in fallaciis extra dictionem non sunt principia motiva a parte sermonis, sed rei, ergo nec ex ipsa formatione paralogismorum neque ex ipso sermone exterius prolato patet principium motivum, et ob hoc oportebat in huiusmodi fallaciis principia motiva in generatione paralogismorum <tangere>. § Ad illud quod ultimo quaeritur dicendum quod inter omnes fallacias in dictione minime eius principum motivum <est> a parte sermonis. Unde quidam posuerunt eam esse extra dictionem, et propter hoc Aristoteles probat eam esse de fallaciis in dictione in illo capitulo <c.7> Fallacia autem fit in hiis. Item, principium motivum figurae dictionis non totaliter causatur a parte terminationis dictionis, sed a modo significandi, ut patuit prius. Et ex hoc patet quod sicut in paralogismis extra dictionem non patet principium motivum ex ipso sermone exterius prolato, et ob hoc oportuit tangere in qualibet fallacia extra dictionem principium motivum; similiter in figura dictionis, quamvis aliquo modo patet principium eius motivum ex ipsa prolatione sermonis, non totaliter, et ita plane sicut in aliis fallaciis in dictione, et ob hoc eius principium motivum tetigit Auctor ubi docuit generare paralogismos, non autem sic fecit in aliis fallaciis in dictione. Et sic patet solutio ad obiecta". The same text, but with several variants in ms Admont, Stiftsbibliothek, 241, f. 25rb-va.

55. Poste 1866, p. 120, n.3. Quoted in Fait 2013, p. 253. Poste's work includes the Greek text of *Soph. el.*, but he had not done any manuscript work, so his text reproduces Bekker's with a few changes.

to accepting such maxims. A key stumbling-block is a difficult passage in which Aristotle claims that the same types of un-asked premisses that make it appear to bystanders that the respondent has been refuted would actually also appear acceptable to the respondent himself, for even if he has not been asked them, he believes he has been so, and would concede them if they were added to the questions asked, although with some types of pseudo-refutation asking the missing question immediately reveals the fault in the argument – for example, in the case of the language-dependent types and solecism[56].

Apparently, then, the respondent would assent not only to the false validating premiss but also to the corresponding maxim, but who would happily concede that every word means just one thing? And who would concede that "ashes" means just one thing, unless the question was asked before the other premisses, and Aristotle's phrasing does not suggest such an ordering. Then again, Aristotle apparently excepts all the fallacies *in dictione*. But who would grant the proposition "Whatever is said of something in some respect is said of it absolutely"?

It may not be possible to make Aristotle's position completely coherent. As also noted by Fait, he is somewhat experimenting, trying more than one approach to fallacious reasoning in the *Refutations*. Still, I think the medieval notions of *causae apparentiae* and false maxims are useful extensions of lines of thought that one finds in Aristotle, and without something of the sort it becomes a problem to explain why one should take any interest in the fallacies at all, and one may be forced to agree with Petrus Ramus (Pierre de la Ramée, 1515-1572) that the whole doctrine of fallacies is superfluous once you have laid down the rules for what constitutes correct reasoning, bad arguments being simply such as fail to satisfy the criteria for being a good argument[57]. In his reduction of all the fallacies to ignorance of refutation in *Soph. el.* ch. 6 Aristotle had flirted with such a formalistic approach, finding for each fallacy a clause in the definition of a refutation that is

56. Arist., *Soph. el.*, 8, 169b30-37: "Ὅτι δ' ἔχομεν αὐτοὺς τῇ αὐτῇ μεθόδῳ, δῆλον· παρ' ὅσα γὰρ φαίνεται τοῖς ἀκούουσιν ὡς ἠρωτημένα συλλελογίσθαι, παρὰ τοσαῦτα κἂν τῷ ἀποκρινομένῳ δόξειεν, ὥστ' ἔσονται συλλογισμοὶ ψευδεῖς διὰ τούτων ἢ πάντων ἢ ἑνίων· ὃ γὰρ μὴ ἐρωτηθεὶς οἴεται δεδωκέναι, κἂν ἐρωτηθεὶς θείη. πλὴν ἐπί γέ τινων ἅμα συμβαίνει προσερωτᾶν τὸ ἐνδεὲς καὶ τὸ ψεῦδος ἐμφανίζειν, οἷον ἐν τοῖς παρὰ τὴν λέξιν καὶ τὸν σολοικισμόν". Fait 2013, p. 243 translates the final clause "for example in some of those that depend on expression and of those that depend on solecism", which is a possible interpretation, but hardly the most natural.

57. See Petrus Ramus, *Scholae in liberales artes*, XIX, 1, ed. Paris, col. 563: "Syllogismi sophistici doctrina in Elenchis posita est, sed eadem reliquis antecedentibus libris sæpe iterata & repetita: Cui tamen in vera descriptione artis locus esse non potest, quia nullum de vitiis præceptum est in virtutis explicatione homogeneum: Imó rectum ipsum, index *(sic!)* est sui & obliqui". Ramus nevertheless goes on to speak at length about the matters dealt with in *Soph. el.*, and so did several later Ramists, but never without stressing that strictly speaking this should be superfluous. See, e.g., Heizo Buscher, *De ratione solvendi sophismata solide et perspigue* (sic!) *ex P. Rami Logica deducta et explicata libri duo*. In his brief *Dialecticæ libri duo* (1556 and later) Ramus included no section on fallacies.

violated. But he did not give up the idea that one must also account for the cause of appearance.

In my opinion, Petrus Ramus was a false prophet, also on this point. I think any worth-while account of how to construe good and valid arguments should be supplemented with one explaining why some bad ones may at first glance look OK, and in some cases actually deceive people.

And I think future commentators on Aristotle's *Sophistical Refutations* should take seriously that he sometimes talks of the fallacies as *topoi*, and also, by the way, does the same in connection with the other *metae* – babbling, solecism, blatant falsehood and paradox[58].

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58. I would like to record a debt to my audiences at three oral presentations of earlier versions of this paper. The first was presented at a symposium in Durham in 2011, the second in Geneva some years later, and a third version at a Lille 'zoominar' in 2021. An objection raised by Laurent Cesalli on the last occasion proved particularly useful.

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Abstract: By general agreement, Aristotle's *Sophistical Refutations (Soph. el.)* is a sort of companion or appendix to the *Topics*. This raises the question whether its thirteen types of fallacious refutation (traditionally called "the thirteen fallacies") are as many *topoi*. In the Latin Middle Ages this was a standard assumption, and for the medievals this had consequences for the way they interpreted the *Soph. el.* Modern commentators have shown little interest in the question. The essay will first examine the textual evidence for holding that Aristotle considered the fallacies *topoi*. The evidence is less clear than appears from modern editions, but it will be concluded that Aristotle did, indeed, think of the fallacies as *topoi*. Next, it will be argued that this being presupposed, we ought to consider re-introducing the medieval notion of sophistical pseudo-maxims corresponding to the genuine dialectical maxims of the *Topics*.

Key words: Aristotle; Fallacy; *topos*; *locus*; maxim.

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The *Fallacia Consequentis* between Term Logic and Sentence Logic in its Medieval Reception

Introduction

Charles L. Hamblin, in the first chapter of his influential study on fallacies, after quoting Aristotle's text (*Sophistical Refutations* [*Soph. El.*] 5, 167b1-3) under the title "Affirming the Consequent", describes the fallacy of the consequent as follows:

> "The ordinary form of reasoning *S implies T and S is true* to *T is true* is commonly called *modus ponens*; and the Fallacy of the Consequent is generally regarded as a backwards version of it, from *S implies T and T is true* to *S is true*"[1].

In the following page he admits that Aristotle doesn't actually use the phrase "affirming the consequent"; he also acknowledges that there is a difference between Aristotle's treatment of the fallacy and that of "the Stoic and modern logicians": Aristotle doesn't use the hypothetical "if...then..." formulation, but rather examples inspired by his categorical syllogistics; and he wonders why Aristotle didn't provide a treatment of this fallacy as a *formal* fallacy, as the modern logicians do, but deals with it in the framework of a *material* fallacy. In the following chapter, discussing Aristotle's list of fallacies, Hamblin appears to be deeply disturbed by Aristotle's "bewildering statement that Consequent is a variety of Accident". And he is not completely satisfied by the solution of this puzzle that considers the examples of consequent as "cast in syllogistic form rather than propositional"[2]. What remains untouched is the general picture of the fallacy as a violation of the *modus ponens*, in particular the one described by the phrase "Affirming the Consequent" (from now on AC).

1. Hamblin 1970, p. 35.
2. Hamblin 1970, p. 86.

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In some recent articles devoted to the fallacy of the consequent,[3] various scholars follow Hamblin's footsteps in their reading of Aristotle's *Soph. El.* 5 (167b1-3). The authors of these papers accept the common interpretation of the fallacy according to which it is a violation of two basic rules of propositional logic known as *modus ponens* (MP) and *modus tollens* (MT), in the forms of Affirming the Consequent (AC) or Denying the Antecedent (DA). They interpret Aristotle's text either as a suggestion to get rid of arguments of the type there described, since they do not provide any (new) information, or as an explanation of why many people are inclined to accept these kind of wrong arguments[4]. Luciano Floridi, in particular, comments on *Soph. El.* 5 (167b1 sqq.), saying that people "mistake 'only if' for 'if and only if', treating 'if it is a square, then it has four sides' as the same as 'if it is water, then it is H_2O'. This was already Aristotle's view"[5]. Furthermore, just before this passage, the same author says that this kind of *Formal Logical Fallacy* is dismissed by many authors as "providing zero information" and that on this point "no significant advancement has been made since Aristotle condemned logical fallacies to the dustbin in his *De Sophisticis Elenchis*".

Here, I see two problems in all these interpretations:

1. the description of this fallacy as a violation of basic propositional rules of inference does not correspond at all to Aristotle's view, since – as Hamblin acknowledges – he didn't have a propositional logic as the Stoics had[6];

2. the condemnation of the fallacy to the dustbin doesn't correspond either to Aristotle's view or to a fair description of its medieval reception: like various medieval philosophers after him, Aristotle presented fallacious inferences as dialectically or rhetorically acceptable and usable (just as Floridi tries to maintain, but following a different line of argumentation).

In this paper I would like to show first (very briefly) how Aristotle described the fallacy of the consequent, and how his Greek commentators started to link it to his syllogistics. Second, I shall present some of the medieval interpretations of the fallacy of the consequent, focusing on the first period of the reception of Aristotle's *Sophistici Elenchi* from the 1160s to the beginning of the following century: in this context, an interpretation emerges which corresponds to modern treatments of the fallacy of the consequent and of its two basic types (AC and DA). Third, I shall show how at the end of the thirteenth century the link with Aristotle's syllogistic (and Boethius' *Topics*) was commonly accepted.

3. See, for instance, Floridi 2009 and Godden / Zenker 2015.

4. Cf. Floridi 2009, p. 96.

5. Floridi 2009, p. 320.

6. As Hitchcock 2000 shows it does not correspond to his later syllogistics either. See also Schreiber 2003, pp. 113-139, for a partially different interpretation of Aristotle's treatment of the fallacy of the consequent.

1. Aristotle's text and its Greek commentators

1.1. *Aristotle, consequent and conversion*

Aristotle describes the fallacy of the consequent, one of the fallacies "independent of language" (*ἔξω τῆς λέξεως*, *extra dictionem*), in the following terms:

> "The refutation which depends upon the *consequent* (*παρὰ τὸν ἑπόμενον*) arises because people suppose that *the relation of consequence is convertible* (*διὰ τὸ οἴεσθαι ἀντιστρέφειν τὴν ἀκολούθησιν*). For whenever, if this is the case, that necessarily is the case, they then suppose also that if the latter is the case, the former necessarily is the case"[7].

The terms *ἑπόμενον* and *ἀκολούθησις* quite correctly are rendered into English respectively as *consequent* and *consequence*, leaving unaltered their ambiguity, though. Aristotle certainly didn't refer to what nowadays logicians call that way, respectively, the consequent (or apodosis) of a conditional proposition and the conditional proposition itself. If one looks at other passages of his logical works where these terms (or their corresponding verbs) are used, such as *Topics*, II, 8, 113b15 sqq., it is clear that the relation of 'sequence' (this is the English word translating here *ἀκολούθησις*) has to do with terms rather than with propositions:

> "Seeing that the modes of opposition are four in number, you should look among the *contradictories* of your terms (*ἐπὶ τῶν ἀντιφάσεων*), reversing the order of their *sequence* (*ἀνάπαλιν ἐκ τῆς ἀκολουθήσεως*), both when demolishing and when establishing a view [...]. E.g., if man is an animal, what is not an animal is not a man; and likewise also in other instances of contradictories. For here the *sequence* is reversed (*ἐνταῦθα γὰρ ἀνάπαλιν ἡ ἀκολούθησις*); for animal follows upon man, but not-animal does not follow upon not-man, but the reverse not-man upon not-animal"[8].

As a matter of fact, here, the contradictories (*αἱ ἀντιφάσεις*) are terms such as "man" and "not-man", or "animal" and "not-animal". This is also clearly implied in *Soph. El.* 6, where Aristotle makes the "bewildering statement" that the fallacious arguments from the consequent are part of the fallacy of the accident[9]: consequents are signified by terms just like accidents are, e.g. "white", "swan" or "snow" (168b30-31, 34-35). The same holds for the explanation of how both fallacies work in *Soph. El.* 7 (169b3-9), where it is clear that the consequent is a kind of accident that follows a thing (*πράγμα*): in both cases the error arises "because we cannot distinguish what is the same and what is different... [as for the con-

7. Arist., *Soph. El.*, 5, 167b1-2 (translations in Barnes 1984, I, p. 283).
8. Arist., *Top.*, II, 8, 113b15-21 (Barnes 1984, I, p. 189).
9. See Schreiber 2003, ch. 7, pp. 113-130, for a thorough discussion of the relationship between these two fallacies.

sequent] in many cases it seems and it is claimed that if this is inseparable from that so also is that from this". In this passage sameness (inseparability) and difference (separability) are equivalent to affirming or denying a predicate of a subject[10]. This is the predicative sense of *ἀκολούθησις* (*consequence*) that one finds in *Soph. El.* 5: it is a relation between terms and not between the clauses of a conditional. Furthermore, in Aristotle's *Prior Analytics* (I, 28, 44a11 sqq.), the couple *ἑπόμενον* / *ἕπομαι* is used as equivalent to *ὑπάρχον* / *ὑπάρχω*, so that "consequent" in these texts is also to be understood as "predicate": therefore, again, as a term, and not as a proposition.

Consequently, in *Soph. El.* 5 when Aristotle talks about *τὸ ἑπόμενον*, he refers to a predicate, such as 'yellow' in "honey is yellow" (*διὰ τὸ ἕπεσθαι το ξανθὸν χρῶμα τῷ μέλιτι*), or "smartly dressed" in "the adulterer (*μοιχός*) is smartly dressed (*καλλωπιστής*)", or "observed to wander around at night" in "the adulterer is observed to wander around at night (*νύκτωρ ὁρᾶται πλανώμενος*)": all these predicates indicate properties that 'flow' from things but do not convert with them, in the sense that the proposition in which they occur as predicates cannot be converted, i.e. their subjects and predicates cannot switch their positions. The reason why they do not convert is in these cases that they have not the same extension: even if that predication in some cases might be true, the converse is not true in many other cases (167b11-12). This sense of conversion or being convertible (*ἀντιστρέφειν*) corresponds to Aristotle's theory of simple conversion of universal negative and affirmative propositions as explained at the beginning of *An. Pr.*:

> "It is necessary then that in universal attribution the terms of the negative proposition should be convertible, e.g., if no pleasure is good, then no good will be pleasure; the terms of the affirmative must be convertible, not however universally but in part, e.g., if every pleasure is good, some good must be pleasure[11]".

As for the second point, Aristotle himself suggests, in the same chapter 5 of his *Soph. El.*, that argumentations based on this kind of fallacious reasoning are used in rhetoric, to build what he calls "demonstrations through signs" (*κατὰ τὸ σημεῖον ἀποδείξεις*), such as those quoted above (the adulterer case). The treatment of this fallacy comes before the elaboration of Aristotle's syllogistics, so that his explanation of the reason why the fallacy can deceive only makes appeal to the wrong opinion of those who believe that the consequence (i.e. predication) is convertible. Even though there is no direct link between this treatment and his discussion on enthymeme based on signs (*An. Pr.* II 27 and *Rhet.* I 2), conversion might play

10. Arist., *Soph. El.*, 7, 169b3-9 (Barnes 1984, I, p. 287). Cf. Arist., *Top.*, I, 18, 108a29-37 (Barnes 1984, I, p. 180) for this sense of sameness, applied to accidents.

11. Arist., *An. Pr.*, I, 2, 25a6-7 (Barnes 1984, I, p. 40).

a role there, too. There is no room here for analysing the whole theory of enthymemes based on signs worked out by Aristotle in these passages[12]; Aristotle distinguishes there two senses of 'sign' (*σημεῖον*), one linked to the first figure of the syllogism (and therefore insoluble, also called *τεκμήριον*), and one grounded on the second and third figures (called *σημεῖον* in a narrower sense). The third-figure sign-enthymemes has two singular premises and a universal conclusion, while the second-figure sign-enthymemes has two affirmative premises and an affirmative conclusion[13]. In both cases, the syllogisms are invalid and can be easily rejected. The reason why the sign-enthymemes based on the second figure are invalid may be that, as Aristotle explains in *An. Pr.* I 5 (27a18-19), on two affirmative universal premises no syllogistic inference can be constructed. In his *Rhetoric*, however, he indicates another reason. The examples in *Rhet.* I 2 and II 24, respectively, are the following: "he's breathing fast, therefore he has fever" and "Dionysius is a vicious man, therefore he is a thief". In both cases the dependence on the second figure can be shown, making explicit the omitted major premise: in the first example "every man who has fever breathes fast", in the second "every vicious man is a thief". As Aristotle says, the first sign-argumentation is refutable because one can breath fast without having fever: the middle term, breathing fast, has a wider extension than the major term, having fever (*Rhet.* I 2). As Aristotle specifies in *Rhet.* II 24, commenting on the second example, this kind of argumentation "yields no deduction… [since] not every vicious man is a thief, though every thief is a vicious man"[14], namely the major premise cannot be converted. If converted, the major premise would transform the sign-enthymeme based on the second figure ("[every thief is a vicious man,] Dionysius is vicious, therefore Dionysius is a thief") into a sound argument in the first figure ("[every vicious man is a thief,] Dionysius is vicious, therefore Dionysius is a thief"), formally irrejectable, even if materially false.

This would have made it possible for Aristotle to link explicitly his treatment of the sign-argumentations to the fallacy of the consequent: he didn't, however. In the same chapter (*Rhet.* II 24)[15], after examining the fallacious enthymeme based on sign, he also lists the one based on the consequent, using again the example of the smart dressed maybe-adulterer he used in *Soph. El.* 5, failing to acknowledge that they both participate in the same error.

12. For a detailed analysis, see Marmo / Bellucci 2023, ch. 1.

13. Being enthymemes, one of the premises is not expressed, since this may be either well known or utterly false, and is left to be provided by the audience (see Burnyeat 1994 for a thorough examination of Aristotle's theory of enthymeme).

14. Arist., *Rhet.*, II, 24, 1401b1-14 (Barnes 1984, II, p. 101).

15. This chapter, according to the stratification-interpretation of Aristotle's *Rhetoric* (the so-called Solmsen-Barnes thesis) is probably coeval with *Soph. El.* 5 (see Burnyeat 1994, p. 31, n. 76).

1.2. *The Greek Commentators*

About the great Greek commentator, Alexander of Aphrodisia, two things are worth noticing in this short paragraph. First, in his commentary on the *Topics*, he takes the term 'consequent' (*τὸ ἐπόμενον*) in the sense of 'apodosis' or main clause of a conditional proposition, as opposed to 'antecedent' (*τὸ ἡγούμενον*)[16], maybe taking up Stoic suggestions. Secondly, he distinguishes two types of contentious or eristical syllogisms: one "which owes its contentious character to a mistake in subject-matter, not form" (*παρὰ τὴν ὕλην … οὐκέτι δὲ παρὰ τὸ σχῆμα*), and can still be called 'syllogism', since it moves from premises "which look like approved, but are not" (*τίνα δέ ἐστι τὰ φαινόμενα ἔνδοξα μὴ ὄντα δέ*)[17]; another one "which is faulty in form" (also *παρὰ τὸ εἶδος*), such as the following

Every human being is an animal
Every horse is an animal
therefore
Every human being is a horse

which is not syllogistic, even though its premises are true, "since it comprises two affirmative statements in the second figure"; it cannot be simply named 'syllogism', but rather "contentious syllogism" as a whole (*ἐριστικὸς συλλογισμός*)[18]. Since his commentary on *Soph. El.* is lost we don't know if and how he applied this distinction to the thirteen Aristotelian fallacies; however, as Sten Ebbesen noticed[19], the following commentators didn't "engage in any serious attempt to classify fallacious arguments on the basis of the matter/form distinction", since none of Aristotle's examples falls in any of those fallacies, the only resemblance being in the fact that – as we saw above – the fallacy of the consequent can also be interpreted as a violation of the rules for a sound second figure syllogism[20]. The distinction between matter and form of a syllogism, however, would be applied systematically, and pedantically, in the commentaries on other Aristotelian works by the following Greek commentators on Aristotle's *Organon*, including some commentaries ascribed to Alexander but actually written by Michael of Ephesus in the twelfth century[21].

16. Alexander Aphrodisiensis, *In Aristotelis Topicorum libros octo commentaria*, ed. Wallies, p. 10, l. 30 *ad* I, 1, 100a25.

17. Alexander Aphrodisiensis, *In Aristotelis Topicorum libros octo commentaria*, ed. Wallies, p. 20, ll. 3-6 and p. 21, ll. 5-6 *ad* I, 1, 100b23 (translation is from Van Ophuijsen 2001, pp. 22-23: "Aristotle says that the contentious syllogisms which are so by their subject-matte are syllogisms too"). Cf. Ebbesen 1981a, I, p. 95.

18. Alexander Aphrodisiensis, *In Aristotelis Topicorum libros octo commentaria*, ed. Wallies, p. 21, ll. 5-6 *ad* I.1, 101b1-4 (Van Ophuijsen 2001, pp. 23-24).

19. Ebbesen 1981a, I, pp. 96-97.

20. Fait 2007, pp. xxvi-xxvii, and xxvii, n. 25.

21. On the ps.-Alexander, see Ebbesen 2008.

John Philoponus in his commentary on *An. Post.* I 2 (71b9-12) quotes some examples from *Soph. El.* 5, giving them a particular twist:

> "clearly there are other syllogisms, in between sophistical ones and scientific ones, that establish truths on the basis of likely <premises>, but in neither the demonstrative nor the sophistical way. For example, people who say 'such and such a person is a dandy, so he is an adulterer' or 'such and such a person wanders around during the night, so he is a thief' or 'the woman has milk, so she has given birth'. These are plausible signs (*πιθανὰ τεκμήρια*), but they are certainly not the causes of the conclusion (*αἴτια τοῦ συμπεράσματος*). For it is possible for someone to be a dandy but not an adulterer or to be wandering around at night but not a thief, and to have milk but not to have given birth"[22].

In *Soph. El.* 5 both fancy dressing and wandering at night are taken to be signs (in the strict sense) of being an adulterer, while here the example is split: fancy dressing is a sign of being an adulterer, while the wandering around at night is a sign of being a thief. It is in this split form that the example will return in later texts, such as the Anonymus Heiberg's compendium of logic (1007)[23], Michael Psellus' *Brevis Traditio*[24] and Michael of Ephesus' commentaries on *Soph. El.* (twelfth century)[25]. All these texts show that:

1. 'consequent' (*τὸ ἑπόμενον*) is taken as indicating the relationship between terms rather than propositions;

2. the Aristotelian example of the adulterer who is smartly dressed or wanders around at night is split in two: the one who is smartly dressed remains an adulterer, the one who goes around at night is a thief.

Let's just see the Anonymus Heiberg's text:

> "<The paralogism> dependent on the consequent <are> like this: this guy wanders about at night, whoever wanders about at night is a thief, therefore this guy is a

22. Ioannes Philoponus, *In Aristotelis Analytica Posteriora Commentaria*, ed. Wallies, p. 21, ll. 8-15: "εἰσὶ γὰρ δῆλον ὅτι καὶ ἄλλοι συλλογισμοὶ μεταξὺ τῶν τε σοφιστικῶν καὶ τῶν ἐπιστημονικῶν, ἀληθῆ μὲν καὶ ἐξ εἰκότων κατασκευάζοντες, οὐ μὴν τὸν ἀποδεικτικὸν τρόπον οὔτε τὸν σοφιστικόν, οἷον ὡς οἱ λέγοντες 'ὁ δεῖνα καλλωπιστής, μοιχὸς ἄρα'. δεῖνα νύκτωρ πλανᾶται, κλέπτης ἄρα'. 'ἡ γυνὴ γάλα ἔχει, τέτοκεν ἄρα'· ταῦτα τὰρ πιθανὰ μὲν τεκμήρια, οὐ πάντως δ' αἴτια τοῦ συμπεράσματος· δυνατὸν τὰρ καὶ καλλωπιστὴν εἶναί τινα, μὴ μοιχὸν δέ, καὶ νύκτωρ πλανώμενον, μὴ κλέπτην δέ, καὶ γάλα ἔχειν, μὴ τετοκέναι δέ" (translation in McKirahan 2008, p. 33 – slightly modified).

23. The compendium of logic includes a section on *Soph. El.* (see Ebbesen 1981a, I, pp. 262-264; a reprint of this part of the edition in Ebbesen 1981a, III, Appendix 9, pp. 90-101).

24. This is a survey on the thirteen Aristotelian fallacies, in form of a letter (see Ebbesen 1981a, III, pp. 102-110).

25. See Ebbesen 1981a, I, pp. 268-285, on Michael's different redactions of his commentary and his dependence on Psellus'. Cf. Ps. Alexander Aphrodisiensis (Michael Ephesius), *In Aristotelis Sophisticos Elenchos Commentarius*, ed. Wallies, pp. 48-49, ll. 27-3 (see Ebbesen 2018, pp. 34-38). An analysis of this passage is in Marmo / Bellucci 2023, 2.7.

thief. It is false: it is not necessary that all those who wander at night are also thieves. This <paralogism> is said depending on the consequent, because the wandering about at night follows from the thief: but you accept the converse (i.e., that the thief wanders about at night); and you argue that the converse is true"[26].

Here we can see that the verb 'follow' (*ἕπεσθαι*) is applied to terms (predicates or their meanings: being a thief or wandering around at night), and there is no mention of the adulterer[27]. Differently from the Latin commentaries, as we will see, the Greek ones do not refer to any logical rule, such as the *modus ponens*, in order to explain how the fallacy of the consequent works.

2. Some of the first commentaries on *Soph. El.*: *Anonymi Aurelianensis I* and *Cantabrigiensis*

As some of the twelfth-century commentaries show, medieval commentators used great caution in interpreting the term *consequens* (the Latin translation of *ἑπόμενον*) in the phrase *fallacia consequentis*. This attitude is justified by the polysemy of the term which warrants both a lexical interpretation (as predicate of a categorical proposition) and a propositional interpretation (as consequent of a conditional)[28].

Aristotle's *Soph. El.,* translated from the Greek by James of Venice in the 1130s together with glosses ascribed to Alexander of Aphrodisias (but actually by Michael of Ephesus who made use of his predecessors' commentaries), started to be commented upon around the middle of the century and found their places in logical treatises and handbooks in the second half of the twelfth century. Among the first commentators, the commentaries by the *Anonymus Aurelianensis I*[29] and the *Anonymus Cantabrigiensis*[30] are very interesting for my purposes.

Their discussion of the fallacy of the consequent begins with a discussion about the meaning of the term *consequens* which – in the second commentary

26. Cf. Ebbesen 1981a, III, pp. 96-97 (appendix 9): "Παρὰ δὲ τὸ ἑπόμενον, οἷον ὁ δεῖνα νύκτωρ πλανᾶται, ὁ νύκτωρ πλανώμενος κλέπτης ἐστίν, ὁ δεῖνα ἄρα κλέπτης ἐστίν. ἔψευσται οὐ γὰρ ἀνάγκη πάντα τὸν νύκτωρ πλανώμενον καὶ κλέπτης εἶναι. λέγεται δὲ τὸ τοιοῦτον παρὰ τὸ ἑπόμενον διὰ τὸ ἕπεσθαι τῷ κλέπτῃ τὸ πλανάσθαι νύκτωρ· λαμβάνεται δὲ ἀντιστρόφως, ὅτι ἡ ἀντιστροφὴ ἀληθές ἐστι". Cf. Michael Psellus, *Brevis Traditio*, ed. Ebbesen, III, p. 105 (appendix 10); Ps. Alexander Aphrodisiensis (Michael Ephesius), *In Aristotelis Sophisticos Elenchos Commentarius*, ed. Wallies, 167b1, first (and final) edition in Ebbesen 2018, p. 35.

27. It appears only in Michael of Ephesus' final version of his commentary (Ebbesen 2018, p. 36).

28. In this, they agree with Schreiber 2003, ch. 7.

29. Anonymus Aurelianensis I's is labelled SE13 in Ebbesen 1993, p. 152 who dates it around 1160/1190.

30. Anonymus Cantabrigiensis' commentary is labelled SE15 in Ebbesen 1993, p. 153 who dates it from the 1180s (see Ebbesen 2019, pp. 13-17).

– introduces the presentation of the various modes of the fallacy[31]. According to the *Anonymus Aurelianensis I* there is a variety of opinion concerning the meaning of *consequens*:

> "Concerning the consequent various people think different things: someone says that the consequent is only a proposition that follows another, someone that it is only a predicable, someone else that it is only the meaning of (that) proposition. Each of them grounds his own opinion on different Aristotelian authoritative texts, and thanks to those (authorities) can defend his faulty position. However, it must be said truthfully, that in this passage (*Soph. El.* 5, 167b1) the consequent is taken as what always or frequently accompanies something else in the same truth, therefore we say that the consequent of human being is animal, that of snow is white, that of pitch is black, that of having rained is wet, that of adulterer is smartly dressed, that of thief is wander around (at night), that of ambitious is wasteful"[32].

The Anonymous of Cambridge distinguishes five senses of *consequens*:

(i) In a first sense, *consequens* corresponds to the sense which Aristotle probably intended: it is a predicate that is said of a subject as a consequence of the fact that another predicate is said of the same subject; furthermore, its negation with respect to a subject follows the negation of another predicate with respect to the same subject; in this sense, *animal* can be called *consequens* of *homo*, because if *homo* is predicated of something, also *animal* is predicated of the same subject, and if *animal* is denied of a subject, *homo* also is denied of the same (*consequens* in this case is synonym of *totum universale*).

(ii) In a second sense, one thing is said to be *consequens* as a synonym of *causa*: thus, Aristotle claims (*Top.* IV 5, 125b28-34) that sadness is *consequens* of anger, because sadness is the cause of anger, that is, it is impossible for anger to occur without sadness, while the opposite is not true (someone can indeed be sad without being angry for that reason).

(iii) In a third sense, *consequens* is what comes after something else in temporal terms or according to nature, just as every effect is *consequens* (both temporally and according to nature) with respect to its cause.

(iv) In a fourth sense, *consequens* is the proposition that is inferred from or follows another proposition.

31. *Anonymi Cantabrigiensis Commentarium in Sophisticos Elenchos Aristotelis*, ed. Ebbesen, pp. 185-186 (see below).

32. *Anonymi Aurelianensis I commentarium in Sophisticos Elenchos*, ed. Ebbesen, p. 134: "De consequente diversa diversorum sentit opinio, nam quidam dicunt tantum esse consequens propositionem quae aliam sequitur, quidam tantum praedicabile, quidam tantum propositionum significatum. Horum singuli diversis Aristotelicis innititur auctoritatibus, erroremque suum quod tueantur habent. Dicitur autem vero: hoc loco accipitur consequens quod aliud in eadem veritate semper vel frequenter comitatur, ut consequens hominis animal, nivis album, picis nigrum, compluti madefactum, moechi comptum, latronis errabundum, ambitiosi liberale consequens dicimus esse".

(v) In a fifth and last sense, it is *consequens* what is added to something else (and this also includes the sign of that thing)[33].

The Anonymous of Cambridge derives from this list of senses, four types of fallacy of the consequent, each regarded as the result of the violation of a distinct logical rule:

A. he first type is when the rule concerning the part-whole relationship between predicate and subject[34] is violated, as in the following cases: "Every man is an animal; Brunellus is an animal; therefore, Brunellus is a man" or "Every man is an animal; Brunellus is not a man; therefore, Brunellus is not an animal" (no reference to a violation of syllogistic rules)[35];

B. the second type is when the rule concerning the relationships between cause and effect is violated,[36] as in the following cases: "If there is battle, then there is victory" (*pugna est, ergo victoria est*) or "There is no victory, therefore there is no battle" (*victoria non est, ergo pugna non est*);

C. the third type is when the laws of inference between propositions are not observed (*quando non servatur lex consequendi inter propositiones*)[37];

D. the fourth type is when one considers signs that do not always accompany what they are added to or signify, as in the following cases: "Someone wanders around at night, therefore he is an adulterer" or "Someone has dust in his shoes, therefore he has walked". In these cases, there is deception because the adulterer usually wanders around at night and this property is added to it as a sign; likewise having dust in one's shoes is a sign of having walked, but it is not necessary for everything to which the sign is assigned, that also what it is a-sign-of be assigned, rather the opposite is given, namely that the necessity of inference occurs from what something is a-sign-of to the sign itself[38].

33. *Anonymi Cantabrigiensis Commentarium in Sophisticos Elenchos Aristotelis*, ed. Ebbesen, pp. 183-184.

34. Such as "if the universal part of something is affirmed, then the universal whole is affirmed", but not the other way around; or "if the universal whole of something is denied, then the part of the same is also denied", but not the other way around (*Anonymi Cantabrigiensis Commentarium in Sophisticos Elenchos Aristotelis*, ed. Ebbesen, p. 184).

35. *Anonymi Cantabrigiensis Commentarium in Sophisticos Elenchos Aristotelis*, ed. Ebbesen, p. 185. This type also falls under the Fallacy of the Accident (p. 187).

36. Such as "if the effect is affirmed, then the cause is affirmed", but not the other way around; or "if the cause is denied, then the effect is denied", but not the other way around (*Anonymi Cantabrigiensis Commentarium in Sophisticos Elenchos Aristotelis*, ed. Ebbesen, p. 184).

37. *Anonymi Cantabrigiensis Commentarium in Sophisticos Elenchos Aristotelis*, ed. Ebbesen, p. 185. Some further divisions are omitted here for the sake of brevity.

38. *Anonymi Cantabrigiensis Commentarium in Sophisticos Elenchos Aristotelis*, ed. Ebbesen, p. 186: "'Iste est errabundus de nocte, ergo iste est adulter' vel 'Iste habet pulverem in calceis, ergo iter fecit'. Inde autem fit deceptio quia solet adulter errare de nocte et habet illud adiunctum ut signum; similiter habere pulverem in calceis signum est itineris, sed non necessarium ut cuicumque conveniat signum ei conveniat id cuius est signum, immo potius videtur econverso quod ab eo cuius est aliquod signum si<t> consequentiae necessitas ad signum".

The *Anonymus Aurelianensis I* clarifies which laws of inference between propositions the Anonymous of Cambridge had in mind when he described the third type of the fallacy. He first distinguishes the three domains in which this kind of fallacy finds application. These are: disputation, when the rules of following or anteceding are not observed; opinion, when it derives from sense perception, as in the case of the wet soil taken as indicating that it has rained (this is also called "the traders' fallacy", *fallacia mercatorum*); and persuasion, when it depends on signs, such as the fancy dressing taken as sign of being an adulterer[39]. Then he distinguishes between four types of fallacy:

A. the first type is when from affirming the consequent (AC) one derives the affirmation of the antecedent, or when from denying the antecedent (DA) one derives the negation of the consequent; this case is so obvious – the Anonymus remarks – that rarely somebody is fooled this way;

B. the second is when from affirming a further consequent of an antecedent one infers the affirmation of the consequent or vice versa (from denying the consequent one denies the further consequent of the antecedent), as in "if someone is Socrates, then he is this man – the further consequent or predicate of the antecedent "Socrates" – ; therefore if someone is a human being, then he is this man";

C. the third type is when from the affirmation of a predicate one infers the affirmation of the subject, or vice versa from the negation of the subject one infers the negation of the predicate, as in "every human being is an animal, a donkey is an animal, therefore (a donkey is) a human being", or "there is no human being, therefore there is no animal";

D. the fourth type is when from one of the concomitant properties (of a thing) one infers another concomitant property (of the same thing), as in "bile is yellow, therefore bile is honey" (because honey is yellow), or "if someone wanders around at night, then he is adulterer" and so on; this coincides with the above mentioned *fallacia mercatorum*.

A short remark on these classifications. While the Anonymous of Cambridge takes into account the various meanings of *consequens* that Aristotle might have had in mind, the Anonymous of Orléans I is more focused on its propositional sense (going apparently against what he had said at the beginning – see above): this explains why at the end of his exposition he adds a note about the types of proposition where the fallacy of the consequent can be found:

> "Notice that this fallacy can be found sometimes in categorical proposition as well as in hypothetical ones. Sometimes categorical propositions have the same value of the hypothetical ones and have to be taken as hypothetical"[40].

39. *Anonymi Aurelianensis I commentarium in Sophisticos Elenchos*, ed. Ebbesen, pp. 134-135.
40. *Anonymi Aurelianensis I commentarium in Sophisticos Elenchos*, ed. Ebbesen, p. 141: "Nota

The necessity of this note for a commentator on the *Soph. El.* 5 will become clear in the following section, where the framework of propositional logic built by Abelard on the basis of the "confused and sometimes inconsistent" materials "Boethius bequeathed to the philosophers of eleventh and twelfth centuries"[41], prevails over the term logic (or Aristotelian) approach.

3. Some treatises and commentaries of the *Logica Modernorum*: the focus on the inference (*consequentia*)

While the *Fallacie Parvipontane* interpret *consequens* in a way that is similar to those of the two above mentioned commentators[42], the *Glose super Sophisticos elenchos*, probably the oldest surviving Latin commentary on *Soph. El.*[43], and the more or less contemporaneous *Summa sophisticorum elenchorum*[44] offer an interpretation that is definitely propositional.

The *Glose* present a very short comment on the *fallacia secundum consequens*:

> "Aristotle deals with the fallacy of the consequent showing that it is a false opinion when someone for instance thinks that an inference can be converted, as in 'if the first is, then the second is, too' and they convert. They are sophistical arguments, as they happen to be like 'if the consequent is affirmed, then the antecedent is affirmed too' and 'if the antecedent is denied, then the consequent is denied too'... This fallacious argument and the one deriving from the accident can be applied to the same things, but this one uses hypothetical arguments, the other categorial ones"[45].

quod in categoricis propositionibus non minus est attendendus iste modus fallendi quandoque quam in hypotheticis. Quandoque enim categoricae habent vim hypotheticarum et pro hypotheticis sunt recipiendae". Cf. *Anonymi Aurelianensis II De paralogismis*, ed. Ebbesen, p. 82: "Et notate quod haec fallacia non dicitur [tantum] secundum consequens quia fit tantum in hypotheticis propositionibus, sed etiam in categoricis. In categoricis fit haec fallacia quando aliquid praedicatur de aliquo et putamus quod eodem modo subiectum predicatur de praedicato". In the last text clearly emerges the interpretation of *consequens* as term and of conversion as the result of the inversion of the subject-predicate positions in a proposition (also cf. p. 81).

41. Martin 2001, p. 159; also cf. Marenbon 2003, p. 55.

42. The *Fallacie Parvipontane*, ed. de Rijk, p. 603 give a large definition of the fallacy which includes both predication ("Est itaque *fallacia secundum consequens* deceptio que provenit ex eo quod aliquid falso ostenditur sequi ad aliud sive secundum rationem predicationis... sive secundum rationem comitantie") and inference ("sive secundum rationem consecutionis"); the examples are respectively: a) according to predication as when the genus or the *proprium* follows its species; b) according to permanent concomitance as when the wet soil follows the rain, paleness follows giving birth or smoke follows fire, or frequent concomitance as when being an adulterer follows wandering about at night or being smartly combed; c) according to inference as in hypothetical propositions about nature ("in naturalibus hypoteticis"). This work is more recent than both the Anonymus Aurelianensis and the Anonymus Cantabrigiensis (Ebbesen 1993, p. 153 labelled it SE17 and dated it towards the end of twelfth century).

43. See Ebbesen 1993, p. 150 (SE5).

44. See Ebbesen 1993, pp. 150-151 (SE6).

45. *Anonymi Glose in Sophisticos elenchos*, ed. de Rijk, pp. 219-220: "Tractat Aristoteles paralo-

As one can see, the anonymous commentator not only understands this fallacy as having to do with hypothetical (conditional) propositions, but underlines that the difference between the fallacy of the consequent and that of the accident lies exactly on their dealing respectively with conditional sentences and with simple categorial sentences, i.e. with terms. This might be the reason why the Anonymus Aurelianensis I (probably following Anonymus Aurelianensis II), proposing a larger definition of *consequens*, felt compelled to specify that the fallacy of the consequent doesn't apply only to relations between sentences (i.e. *in hypotheticis*) but also to relations between terms (or things) (i.e. *in categoricis*). In the *Glose*, as well as in other commentaries of the second half of the century, we also find the two fallacious arguments, AC and DA, that in modern interpretations are taken to exemplify the fallacy of the consequent, as violation of the two basic rules of propositional logic, namely MP and MT: *posito consequenti ponitur antecedens* and *destructo antecedenti destruitur consequens*[46]. All this implies that for a group of commentators, around the second half of the twelfth century, *consequens* had the unambiguous meaning of "proposition that follows from another proposition, i.e. the *antecedens*".

This correspond to the logical nomenclature used by Boethius, in his *De topicis differentiis*, where he says:

> "Of the conditional propositions, which the Greeks call 'hypotheticals', are simple propositions, in which the part that comes first is called 'antecedent' and the part that comes after is called 'consequent', as in this proposition: 'if it is round, it is able to rotate', where '(this) being round' is the antecedent, and '(this being) able to rotate' is the consequent"[47].

gismum secundum consequens demonstrando falsam opinionem, secundum quam contingit, scilicet quod quidam putant consequentiam converti, ut 'si primum est, et secundum est', et convertunt. Sunt etiam sophistice, secundum quas contingunt, ut he: '*posito consequenti ponitur antecedens*' et '*destructo antecedenti destruitur consequens*'... Et iste paralogismus et accidentis fiunt in eisdem rebus, sed iste in ypoteticis argumentationibus, ille in categoricis". The use of numbers as propositional variables, in the Stoic way, derives probably from Boethius, *De hypotheticis syllogismis*, I, iv, 4-5, ed. Obertello, p. 224 (cf. Marenbon 2003, p. 51).

46. Cf. *Anonymi Fallacie Vindobonenses*, ed. de Rijk, p. 535 (second half of the twelfth century; SE16 in Ebbesen's catalogue); *Anonymi Fallacie Parvipontane*, ed. de Rijk, p. 604 ('in ypoteticis ut si argumentemur a positione a consequentis vel a destructione antecedentis'). *Anonymi Summa Sophisticorum Elenchorum* (ed. de Rijk, p. 389) gives a more complex inferential rule, probably derived from Boethius' *De hypotheticis syllogismis* or Abelard's *Dialectica.*

47. Boethius, *De Topicis Differentiis*, 1176A, ed. Nikitas: "Conditionalium uero propositionum, quas Graeci hypotheticas uocant partes, sunt simplices propositiones, cuius quidem ea pars quae prius dicitur "antecedens", quae posterius "consequens" appellatur, ut in hac propositione quae dicit: "Si rotundum est, uolubile est", rotundum esse antecedit, uolubile esse consequitur". Cf. also Boethius, *In Ciceronis Topica*, II, 1076D, ed. Orelli, where Boethius suggests that what Cicero called *argumentum a consequentibus* should rather be called *ab antecedentibus*, opposing a commonsense use of the couple "antecedent" and "consequent" to a logical one.

Abelard uses both terms in this same meaning, working out properly a sentence logic, which Boethius could not, because – according to Chris Martin – he had "no concept of propositional operations" and operators, such as the conjunction *et*[48]. Abelard in his *Dial*. III, on the *loci*, together with the two basic rules of propositional logic, proposes some negative rules and among them one that resembles AC:

> "(7) and not if the same (i.e. the consequent) is affirmed, it (i.e. the antecedens)... is affirmed"[49].

If one looks at the handbooks of logic between the end of the twelfth and the beginning of the thirteenth centuries, one can notice a basic difference between the *Dialectica Monacensis* and some Oxonian textbooks, such as the *Logica* "Ut dicit" and the *Logica* "Cum sit nostra". The *Dialectica* has a section on the fallacies, and so constitutes a complete introduction to logic, both *vetus* and *nova*. The account of fallacies in the *Dialectica* is characterized by the distinction between *causa apparentiae* and *causa falsitatis* (which will become *causa non existentiae* or *defectus*) of the fallacy itself[50]. Yet, in the discussion of the *fallacia secundum consequens* the author makes no reference to the doctrine of the *loci*[51]. The author of the *Dialectica* introduces a twofold division of the fallacy of the consequent:

> "After this it is necessary to know that there are two general types of the fallacy of the consequent. For there are two ways of inferring with necessity in a conditional proposition, that is from the affirmation of the antecedent to the affirmation of the consequent, and from the negation of the consequent to the negation of the antecedent. According to this there are two non-necessary ways, of which one is from the negation of the antecedent to the negation of the consequent, and the other from the affirmation of the consequent to the affirmation of the antecedent"[52].

48. Martin 2001, p. 164. Cf. also Marenbon 2003, pp. 50-56.

49. Petrus Abaelardus, *Dialectica*, III (*Topica*), ed. de Rijk, p. 288: "(6) neque destructo consequenti ponitur antecedens *(7) neque eodem posito ipsum vel ponitur* (8) vel aufertur".

50. Cf. *Anonymi Fallaciae Lemovicenses*, ed. Ebbesen / Iwakuma, p. 6. On this distinction, se also Ebbesen 1987, pp. 115-117.

51. Such reference is present, on the contrary, in the *Fallaciae Londinenses* and the *Fallaciae Lemovicenses.* Cf. *Anonymi Fallacie Londinenses*, ed. de Rijk, p. 676, where common accidents are mentioned with regard to the example of the adulterer, and the *loci a simili* e *a proportione* are referred to with regard to the two other species of the fallacy of the consequent; cf. also *Anonymi Fallaciae Lemovicenses*, ed. Ebbesen / Iwakuma, p. 39: "Possunt enim illa in diversis locis esse divisim".

52. *Anonymi Dialectica Monacensis*, ed. de Rijk, p. 589: "Post hec sciendum quod duo modi generales sunt paralogismorum secundum consequens. Sunt enim duo modi arguendi necessarii in conditionali, scilicet a positione antecedentis ad positionem consequentis vel a destructione consequentis ad destructionem antecedentis. Iuxta quos sumuntur duo non-necessarii, quorum unus a destructione antecedentis ad destructionem consequentis, reliquus a positione consequentis ad positionem antecedentis".

The two fallacies (AC and DA) are therefore simply the inversions of the two valid inference schemes, i.e. *modus ponens* (affirming the antecedent) and *modus tollens* (negating the consequent)[53]. This bipartition becomes the standard classification of the species of the fallacy of the consequent.

Other treatises move from the formulation of the rules and raise doubts as to the denomination of the fallacy or the validity of the bipartition. For example, the author of the *Fallaciae ad modum Oxoniae*, a treatise which was with all probability a part of the *Logica* "Cum sit nostra" and also somehow connected to the *Dialectica Monacensis*[54], asks why the fallacy is called "of the consequent" rather than "of the antecedent", since according to its standard bipartition one species of it is from the affirmation of the consequent and the other from the negation of the antecedent[55]. The answer is quite complex and considers several examples. It can be summarized as follows: the species from the negation of the antecedent falls under the species from the affirmation of the consequent, and this justifies the choice of the name. Later commentators will make appeal to similar justifications[56]. The explanation of the anonymous author runs as follows: the fallacy from the negation of the antecedent *non homo est, ergo non animal est* is modelled after the valid inference *non animal est, ergo non homo est*, which is a sound instance of the *locus a genere* (and given the true conditional *si est homo*, *est animal*, it is also an instance of *modus tollens*). Now, in the valid inference *non animal est, ergo non homo est*, the proposition *non animal est* is the antecedent and *non homo est* is the consequent. Therefore, when in the fallacy *non homo est, ergo non animal est*, one says *non homo est* one is actually affirming the consequent of the corresponding converse (valid) inference. So when one denies the antecedent one actually affirms the consequent (of the converse valid inference)[57]. The author seems not to perceive that, *mutatis mutandis*, the same is true of the other species of this fallacy, the one leading from the affirmation of the consequent. Of this species, too, we might say that when one affirms the

53. The anonymous author of the *Tractatus De Fallaciis* preserved in ms. München, Bayerische Staatsbibliothek, clm 14763, ff. 123vb-125ra (SE28 in Ebbesen 1993 catalogue, p. 156) follows the *Dialectica* quite closely.

54. Kopp 1985, pp. xv-xxi.

55. *Anonymi Fallaciae ad Modum Oxoniae*, ed. Kopp, p. 138: "Sed quaero, quare ista fallacia dicitur fallacia consequentis et non fallacia antecedentis, cum fit a destructione antecedentis ita bene sicut a postione consequentis" (ms. N) (SE33 in Ebbesen 1993, p. 158).

56. An echo of this argumentative strategy is in one of the objections in Brito's commentary on the *Sophistici Elenchi*, as we shall see; cf. *infra*, section 4.

57. *Anonymi Fallaciae ad Modum Oxoniae*, ed. Kopp, p. 138-139: "Dicendum quod ubi est fallacia consequentis, semper a positione consequenti, verbi gratia hic est fallacia a destructione antecedentis: 'non homo est, ergo non animal est', quia sequitur econverso 'non animal est, ergo non homo est' per locum a genere, et non sic; ergo quod 'non animal <est>' antecedens est, 'non homo <est>' consequens, quia cum dicit 'non homo <est>' ponit consequens, 'non animal <est>' concludit antecedens, et sic est fallacia consequentis a positione consequentis".

consequent one actually affirms the antecedent (of the converse inference in *modus ponens*). Moreover, if one accepts the explanation in these terms one would also be forced to dismiss the bipartition itself; the author of the *Fallaciae*, however, does not go so far.

4. An overview of the thirteenth century

As will be seen in what follows, the two lines of interpretation (sentence logic vs. term logic) not only do not oppose but coexist peacefully in thirteenth-century commentaries on *Soph. El.*: the logicians of that period have not yet developed the idea that the logic of predicates (syllogistic and topics) are based on propositional logic, unlike the contemporary approach, and therefore some of them propose a reading of logic rules such as MP and MT compatible with a predicate logic. In this last part, I will examine from this point of view some commentaries on *Soph. El.*, such as the one attributed to Robert Kilwardby and those by Albert the Great, Giles of Rome and Radulphus Brito.

A commentary on *Soph. El.* attributed to Robert Kilwardby[58], and presumably coeval with Kilwardby's authentic commentaries on Aristotle's logic, offers an interpretation of *Soph. El.* 5 167b1ff. that makes again use of the example of the thief in the Philoponian split form, and adopts the standard sentence logic bipartition of the fallacy as something stemming from Aristotle's words. According to Kilwardby (?), Aristotle first presents the fallacies deriving from the affirmation of the consequent and then those deriving from the negation of the antecedent. The first species[59] is itself of two sub-species: those fallacies whose converse inference (*consequentia*) is necessary and those whose converse inference is probable[60]. Examples of the former sub-species are the inference that gall is honey because it is yellow (*Soph El.* 5, 167b5-6) and that it has rained because the soil is drenched (*Soph El.* 5, 167b6-9). These are cases in which sense perception influences opinion. The converse inferences – which Kilwardby (?) actually expresses as conditional propositions – are valid, i.e. the corresponding conditional propositions are necessarily true: *si est mel, est rubeum*; *si terra est depluta, ergo est madida*. The inference of the antecedent (*est mel*, *est depluta*) from the consequent (*est rubeum*, *est madida*) is a fallacy deriving from the affirmation of the consequent (AC).

58. SE35 in S. Ebbesen' catalogue (Ebbesen 1993, p. 158). Lewry 1982, pp. 43-46 and Brumberg-Chaumont 2016, pp. 109-113, have some doubts about the attribution to Robert Kilwardby.

59. We shall see that Kilwardby (?), like some previous commentators, considers the two arguments to be one and the same.

60. Robertus Kilwardby (?), *Commentarium in Aristotelis Sophisticos Elenchos*, ms. **C** = Cambridge, Peterhouse, 205, ff. 295rb-296rb; ms. **P** = Paris, BN, 16619, ff. 22vb-23vb. Two other witnesses have extracts only: cf. Ebbesen 1993, p. 158.

The other sub-species is when the converse inference is probable. According to Kilwardby (?), these are the *demonstrationes secundum signa* that are used in rhetoric[61]. The examples are the following:

> "Then he provides the matter of the two fallacies by saying that when they want to show that someone is an adulterer, they assume that which is added to being an adulterer, namely that he is dressed up, and from that they infer that he is an adulterer. Or in order to show that someone is a thief, they assume what is added to being a thief, namely that he wanders around at night, and from that they infer that he is a thief[62]".

The author then presents the two examples in the form of AC: (i) *si aliquis est adulter, ipse est comptus; sed iste est comptus; ergo iste est adulter*; (ii) *si aliquis est fur, est errabundus de nocte; sed iste est errabundus de nocte; ergo iste est fur*.

The other species of the fallacy is DA and is exemplified by Melissus' argument for the eternity of the world in the past:

> "Once he has discussed the (first) part, then the other follows where he presents an argument which is a fallacy of the consequent, and moves from the destruction of the antecedent. He then proceeds this way: he says that sometimes the fallacy of the consequent happens in syllogistic arguments, such as Melissus' argument for the infinity of the world. And his argument can be formed this way: everything that is produced has a beginning; the world is not produced; therefore it has no beginning; and therefore it is infinite[63]".

One last observation concerns the number of the species of this fallacy. We saw that in the *Fallaciae ad modum Oxoniae* the fallacy deriving from the negation of the antecedent falls under that deriving from the affirmation of the consequent, but the taxonomic import of this move was not appreciated in that context. The author of the commentary attributed to Kilwardby, and Roger Bacon in his *Sum-*

61. This second sub-species is differentiated from the first by Aristotle's introduction at 167b9-10 of rhetorical demonstrations from signs: "Et in rethoricis quae secundum signum sunt demonstrationes ex adiunctis sunt" (Arist., *De Sophisticis Elenchis Translatio Boethii*, ed. Dod, p. 13, l. 9).

62. Robertus Kilwardby (?), *Commentarium in Aristotelis Sophisticos Elenchos*, on *Soph El.* 5, 167b10sqq., **C** 295va; **P** 22vb: "dat materiam duorum paralogismorum dicens quod volentes ostendere quod aliquis sit adulter, accipiunt quod adiunctum est adultero, scilicet quod sit comptus, et ex hoc inferunt ipsum esse adulterum. Aut ad ostendendum quod aliquis sit fur accipiunt quod adiunctum est furi, scilicet quod sit errabundus de nocte, et ex hoc inferunt ipsum esse furem".

63. Robertus Kilwardby (?), *Commentarium in Aristotelis Sophisticos Elenchos*, on *Soph El* 5, 167b10 sqq., **C** 295va; **P** 23ra: "Hoc habito sequitur pars illa in qua dat unam orationem secundum consequens peccantem a destructione antecedentis [...]. Procedit ergo sic: dicit quod fit quandoque fallacia consequentis in orationibus factis ad modum sillogismi, sicut se habet ratio Melixi per quam uoluit ostendere mundum esse infinitum. Et potest ratio eius sic formari: omne quod est factum habet principium; mundus non est factus; ergo mundus non habet principium; est igitur infinitus". This formulation, in syllogistic form, presents Melissus' argument as a violation of first figure syllogism: Every A is B; C is not A; therefore C is not B.

mulae dialectices do consider that import. The former claims that there are two species of the fallacy, one based on necessary inferences and the other on probable inferences, and both are examples of AC. Then Kilwardby (?) asks why AC and DA are not two different species of the fallacy. His answer is that the two arguments are substantially identical (*sunt idem modus in substantia*), since in a valid inference the negation of the antecedent follows from the negation of the consequent, and therefore the denied antecedent is "consequent" in this sense[64]. Here of course the author conflates the consequent of the conditional proposition in a modus tollens ('B' in "If A, then B; but not-B; therefore not-A") with the conclusion of it ('not-A'). On a similar line of interpretation, Roger Bacon argues that there is only one species of the fallacy of the consequent, which he identifies with that deriving from AC. If given the true conditional "If Socrates is a body, he is a substance", we make the inferences "Socrates is not a body, therefore he is not a substance" (DA) and "Socrates is a substance, therefore he is a body" (AC), "we have <in both cases> the same mode <of the fallacy>, that is from the affirmation of the consequent, because just as substance is consequent of body, so 'non-body' is consequent upon 'non-substance'"[65]. As we can see Bacon superimposes the point of view of the logic of terms to that of propositional logic: the consequent in his text can be both the conclusion of the valid inference ("Socrates is a substance" or "Socrates is not a body" or "Socrates is a non -body"), and its predicate ('substance' or 'non-body').

In his paraphrase of the *Liber Elenchorum*, Albert opens the discussion about the fallacy of the consequent with some preliminary observations. In the first place, *antecedens* has to be taken in the logical sense of a proposition which, once posed (*quo posito*), allows to infer another proposition (the *consequens*) either probably or necessarily[66]. In the second place, Albert examines the distinction, which originates with Boethius and which is employed in the commentary attributed to Kilwardby, between the different kinds of inference. The classification is connected by him to the disciplines corresponding to each kind: some inferences are necessary and not probable, and these are proper of demonstrative disciplines; some are probable and not necessary, and these are proper of dialectical disciplines; some are both necessary and probable but with reference to distinct middle terms (no example of this is provided); some are neither necessary nor probable, and these are proper of rhetorical and poetic disciplines.

64. Robertus Kilwardby (?), *Commentarium in Aristotelis Sophisticos Elenchos*, dub. 5, **C** 296rb; **P** 23va: "destructio enim antecedentis est consequens ad destructionem consequentis; et ita cum prius ponitur destructio antecedentis et infertur destructio consequentis, prius ponitur consequens et deinde infertur antecedens".

65. Rogerus Bacon, *Summulae dialectices*, III, iii, 2.2, §645, ed. de Libera, p. 263.

66. Albertus Magnus, *Expositio Sophisticorum Elenchorum*, I, 3, 15, ed. Borgnet, p. 584a.

According to Albert, the first species of this fallacy is that which produces wrong opinions from sense perception. He quotes some classical examples coming from *Soph. El.* 5, 167b5-9: the inference that gall is honey because it is yellow, and that it has rained because the soil is drenched. Albert, following Kilwardby (?), claims that the valid inference, that is the converse of the fallacious argument, is necessary[67]. Probable inferences, by contrast, are used in rhetoric (*in rhetoricis demonstrationes, hoc est, probationes sive ostensiones*) and are based on common rather than on proper signs; these derive from predicates that are added to some subject (*ex adiunctis*) which Cicero calls "common accidents" (*communiter accidentia*)[68]. Albert illustrates this species of the fallacy by means of an example that re-unites the adulterer and the thief (which Philoponus had split into two distinct examples) into one single argument:

> "when rhetors want to show or prove that someone is an adulterer, they assume that common predicate which is commonly added to the adulterer, or which is a common accident of the adulterer, like that he is dressed up, embellished and fancily dressed, and often looks at other people's women, or that he wanders about at night: from which the suspect is generated that he is an adulterer or a thief"[69].

The connection of the fallacy of the consequent with both Boethius' *De differentiis topicis* (*locus a communiter accidentibus*) and *An. Pr.* II 27 is explicitly emphasized. Albert goes far beyond Boethius and his commentators in saying that the argument from *communiter accidentibus*, when the accidents or *adiuncta* are 'proper' (i.e. are as extended as, and thus convertible with, the subject in which they inhere), concludes in the first figure. The example here is the lactating woman of *An. Pr.* II 27[70]. When the accidents are common (i.e. are more extended than the subject in which they inhere) the inference is in the second figure and qualifies as a fallacy of the consequent. The example here is the pale woman of *An. Pr.* II 27[71]. Albert follows Kilwardby (?) in the indication of three types of the fallacy of the consequent. The first type is when an inference based on the relationships between terms, such as *homo* and *animal* or *mel* and *rubeum* (*secundum habitudines locales*) is converted as to conclude affirming the antecedent from affirming the

67. Albertus Magnus, *Expositio Sophisticorum Elenchorum*, I, 3, 15, ed. Borgnet, pp. 585b-586a. On the dependence of Albert the Great's logical works on Robert Kilwardby's, see Ebbesen 1981 (now in Ebbesen 2009, ch. 7).

68. Albertus Magnus, *Expositio Sophisticorum Elenchorum*, I, 3, 15, ed. Borgnet, p. 586a.

69. Albertus Magnus, *Expositio Sophisticorum Elenchorum*, I, 3, 15, ed. Borgnet, p. 586a: "volentes enim ipsi rhetores ostendere sive probare, quoniam aliquis est adulter, illud praedicatum commune quod communiter adjunctum est adultero, sive communiter accidens est adultero, accipiunt, ut quoniam compositus est et ornatus et comptus, et saepe respicit ad uxorem alterius, aut quoniam in nocte videtur errabundus: ex quo suspicio generatur, quod sit adulter et latro".

70. Albertus Magnus, *Expositio Sophisticorum Elenchorum*, I, 3, 15, ed. Borgnet, p. 586a-b.

71. Albertus Magnus, *Expositio Sophisticorum Elenchorum*, I, 3, 15, ed. Borgnet, p. 586b.

consequent. The second type is when we convert an inference which is based on the whole of the circumstances related to a subject, such as the fact that an adulterer is smartly dressed or a thief usually goes around at night; this kind of fallacy is typical of rhetorical inferences, and is different from a *locus a communiter accidentibus* since the circumstance (sign or accident) has to be taken in conjunction with other circumstances (i.e. it must be a proper sign and not a common sign or accident) in order to produce a valid inference. The third one is when we convert an inference because of an opposition, like in Melissus' argument[72]. At the end Albert follows Kilwardby(?) in his answer to the question Kilwardby posed, i.e. why AC and DA are not two different types of this fallacy: they are not two substantially different types, their difference lying only in their linguistic formulation[73].

Giles of Rome's commentary on the *Soph. El.* was composed around 1274 and is a literal commentary that follows Aristotle's text quite closely. With regard to the fallacy of the consequent, Giles focuses on two issues: its pertinence to the *Soph. El.* and its denomination[74]. After the exposition of the *divisio textus* of *Soph. El.* 5, Giles presents the *dubium* according to which an argument from the affirmation of the consequent amounts to an argument from affirmative premises in the second figure, which is one of the defects of the syllogism (*inutilis coniugatio*) that Aristotle treats in *An. Pr.* I 9; for this reason the fallacy of the consequent does not fall in the domain of the *Soph. El.*[75]. In his solution of the *dubium* Giles claims that *consequens* (or *fallacia consequentis*) may be taken either as a deviation with respect to syllogism as such (*simpliciter*), and in this sense it is of pertinence of the *An. Pr.*; or as grounded on a false maxim (*que uni et eidem sunt eadem, inter se sunt eadem*[76]: things identical to a third are identical to each other) and in

72. Albertus Magnus, *Expositio Sophisticorum Elenchorum*, I, 3, 15, ed. Borgnet, pp. 587b-588a. Cf. also p. 586b (in connection with the second type): "Et ideo si debeat esse locus a communiter accidentibus, tunc oportet quod tale adjunctum accipiatur cum circumstantiis personae: quia cum multis talibus acceptum erit proprium, et infert subjectum, quod solum unum adjunctum inferre non poterit". The expression *cum circumstantiis personae* comes from Cicero's rhetoric, whose teaching was in vogue in the twelfth century but already in decadence at the beginning of the thirteenth; cf. Fredborg 1987. This observation certainly derives from and reflects previous commentaries and treatises, in which the connection between the fallacy of the consequent, Boethius' *De Topicis Differentiis* and *APr* B 27 is made explicit. Like Kilwardby (?), Albert seeks to save this dialectical *locus* from falling within the domain of fallacies by allowing a conjunctive consequent.

73. Albertus Magnus, *Expositio Sophisticorum Elenchorum*, I, 3, 15, ed. Borgnet, p. 588b.

74. Aegidius Romanus, *Expositio super Libros Elenchorum*, ed. Venetiis, ff. 20rb-21vb.

75. Aegidius Romanus, *Expositio super Libros Elenchorum*, ed. Venetiis, f. 20va: "Dubitaret forte aliquis utrum de fallacia consequentis determinari habeat in hoc libro. Et videtur quod non: dictum est enim ea que determinantur in hoc libro esse obliquitates sillogismi dialetici; sed arguere a positione consequentis est arguere ex puris affirmativi in secunda figura; sed hec est obliquitas sillogismi simpliciter et est inutilis coniugatio, et de ea determinatur in libro Priorum, capitulo nono; non ergo debet hic tractari de ea".

76. Cf. Arist., *Soph. El.*, 6, 168b31-33: "nam quae uni et eidem eadem, et sibi invicem probamus esse eadem; propter quod fit secundum consequens elenchus" (Arist., *De Sophisticis Elenchis Translatio Boethii*, ed. Dod, p. 16, ll. 24-25).

this case it falls under the *Soph. El.*[77]. The last formulation makes the fallacy of the consequent a case of false predication.

The second *dubium*, about the classical problem of its denomination, also addresses the problem of the typology of fallacies of this kind, i.e., whether there are two species of it, one from the affirmation of the consequent and one from the negation of the antecedent. Giles follows the path opened by the *Fallaciae ad modum Oxoniae*: this fallacy always derives from the affirmation of the consequent, giving it however a term semantic twist, based on considerations about the respective extension of positive and negative (or denied) terms. The more a term is specific in its positive form, the more general it is in its negative form: if *animal* is more extended than *homo*, *non-homo* is more extended than *non-animal*, and what is more extended always follows from what is less extended ("If man, then animal," but not viceversa). Thus, the term *animal* is the consequent of *homo* and the term *non-homo* is the consequent of *non-animal*. Therefore, the inference of *non-animal* from *non-homo* (that is from the negation of the antecedent) is in fact an inference from the affirmation of the consequent, taken as a predicate. The name of the fallacy is accordingly correct[78]. The strategy here is the same as in earlier commentators: the fallacy from the negation of the antecedent is also from the affirmation of the consequent, if with "consequent" we mean the conclusion of the converse valid inference (as in Kilwardby(?)) or its predicate (as in Bacon). The fallacy is one in form (from the affirmation of the consequent), although in matter there are three species of it, according to the several matters to which it is applied (*per applicationem ad diversam materiam*): that which derives from sense perception (honey and drenched soil); moral and rhetoric sciences (the adulterer; the thief is absent, as it was absent

77. Aegidius Romanus, *Expositio super Libros Elenchorum*, ed. Venetiis, f. 20va: "Respondeo dicendum quod consequens dupliciter potest accipi: primo ut est pura obliquitas sillogismi simpliciter, et sic est inutilis coniugatio et de ea habet tractari in libro Priorum; secundo, potest considerari consequens ut habet specialem maximam cui innititur: innititur enim illi maxime 'que uni et eidem sunt eadem, inter se sunt eadem', ut patebit in illo capitulo AUT SIC DIVIDENTES [6, 168a17]. Et quia huiusmodi maxima ut ibi declarabitur non est necessaria, sed apparens et sophistica, ideo consequens ut innititur tali maxime est locus sophisticus". Cf. f. 25va, where he discusses the maxim in question.

78. Aegidius Romanus, *Expositio super Libros Elenchorum*, ed. Venetiis, f. 20va: "Ulterius forte dubitaret aliquis, cum hec fallacia non solum fiat a positione consequentis, sed a destructione antecedentis, quare potius nominata est consequens quam antecedens. Dicendum quod si bene consideramus hec fallacia semper fit a positione consequentis. Nam quanto specialius est aliquid affirmative sumptum, tanto negatum generaliius efficitur: ut si animal est in plus quam homo, non-homo erit in plus quam non-animal, et quia semper illud quod est in plus sequitur ad id quod est in minus. Sed animal est consequens ad hominem, non-homo erit consequens ad non-animal; negatio ergo facit de antecedente consequens et de consequente antecedens; antecedens ergo destructum est consequens ad antecedens negatum: arguere ergo a destructione antecedentis est arguere a positione consequentis. merito ergo hec fallacia nominata est consequens, quia quodammodo semper per eam arguitur a positione consequentis".

in Aristotle); speculative and syllogistic science (Melissus' argument for the infinity of the world)[79].

Modistic question commentaries focus roughly on the same problems (denomination[80], relationship with the *fallacia accidentis*[81] and typology[82]); they occasionally use the example of the adulterer[83]. During the last decade of the thirteenth century, one among them, Radulphus Brito, returns on the connection between *fallacia consequentis*, *locus a communiter accidentibus* and the semiotic typology of *APr* II 27 in one of the questions devoted to the fallacy of the consequent in his commentary on the *Sophistici Elenchi*.

Brito's question no. 48 of his *Quaestiones super Sophisticos elenchos* is about a problem that is also discussed by Giles of Rome in his *Expositio* and by Simon of Faversham (q. 33) in his second series of question[84]. The question is whether the fallacy of the consequent is a *locus sophisticus* and whether it should then be distinguished from other fallacies. Some of the arguments for a negative answer are similar to those used by Giles and Simon. The first – probably taken from Giles – is that if the inference is in the second figure with two affirmative premises, there is an unproductive disposition of the premises (*inutilis coniugatio*), which is not a fallacy, because it concerns a formal and not a material aspect of the inference[85].

79. Aegidius Romanus, *Expositio super Libros Elenchorum*, ed. Venetiis, f. 20va-b.

80. *Anonymi SF Quaestiones super Sophisticos Elenchos*, q. 100 (*de sua nominatione*), ed. Ebbesen, pp. 233-235; Simon de Faversham, *Quaestiones novae super libro Elenchorum*, q. 33 (*utrum sit fallacia distincta contra alias*), ed. Ebbesen et al., pp. 188-191; *Anonymi Pragensis Quaestiones super Aristotelis Sophisticos Elenchos*, q. 38 (*utrum fallacia consequentis debeat nominari antecedentis vel consequentis*), ed. Murè.

81. *Anonymi SF Quaestiones super Sophisticos Elenchos*, q. 101 (*utrum consequens sit pars accidentis*), ed. Ebbesen, pp. 235-236; Simon de Faversham, *Quaestiones novae super libro Elenchorum*, q. 34 (*utrum consequens sit pars accidentis*), ed. Ebbesen et al., pp. 191-194.

82. *Anonymi SF Quaestiones super Sophisticos Elenchos*, q. 102 (*de numero modorum*), ed. Ebbesen, pp. 236-237; Simon de Faversham, *Quaestiones novae super libro Elenchorum,* q. 35 (*utrum ubicumque est fallacia consequentis necesse sit consequentiam conversam bonam esse*), ed. Ebbesen et al., pp. 194-198; q. 36 (*utrum arguendo a positione consequentis ad positionem antecedentis sit bona consequentia*), ed. Ebbesen et al., pp. 198-200; q. 37 (*utrum arguendo a destructione antecedentis ad destructionem consequentis sit bona consequentia*), ed. Ebbesen et al., pp. 200-203; *Anonymi Pragensis Quaestiones super Aristotelis Sophisticos Elenchos*, q. 39 (*utrum possimus arguere a superiori ad inferius affirmando*), ed. Murè; q. 40 (*utrum valeat processus ab inferiori ad superius negando*) ed. Murè.

83. Cf. Simon de Faversham, *Quaestiones novae super libro Elenchorum,* q. 35, ed. Ebbesen et al., p. 194, p. 197.

84. Radulphus Brito, *Questiones super Sophisticos Elenchos*, I, 48, ms. **B** = Bruxelles, Bibliothèque Royale de Belgique, 3540-47, ff. 530va-531rb; ms. **S** = Salamanca, Biblioteca Universitaria, 2350, f. 189rb-vb.

85. Radulphus Brito, *Questiones super Sophisticos Elenchos*, I, 48, **B** f. 530va; **S** f. 189rb: "Quia ubi est inutilis coniugatio non est locus sophisticus; in fallacia consequentis est inutilis coniugatio, ergo etc. Maior patet, quia inutilis coniugatio pertinet ad librum Priorum et per consequens non est fallacia que pertinet ad librum Elenchorum. Minor patet: in secunda figura arguendo ex affirmativis fit fallacia consequentis et est inutilis coniugatio, ut patet primo Priorum" (cf. Aegidius Romanus, *Expositio super Libros Elenchorum*, ed. Venetiis, f. 20va).

The second argument is close to the one used by Simon: since there is no fallacy of the antecedent, there can be no fallacy of the consequent[86].

Quite naturally, this leads to an argument, the third, that contains a clear echo of the discussions about denomination: dialectical *topoi* are named after what infers, not after what is inferred; since it is always the antecedent that infers the consequent, there must be a *locus sophisticus* of the antecedent, not of the consequent[87]. The fourth argument, then, relies on the similarity of rhetorical and sophistical *loci*. The former are subordinated to dialectical *loci* and thus are distinguished, like the dialectical, from the sophistical. So, if it is true that whenever there is fallacy of the consequent there is a rhetorical *locus*, then this *locus* cannot be sophistical[88]. Also, since the fallacy of the consequent is obtained from common accidents, it must be a *locus a communiter accidentibus* that is dialectical and not sophistical[89]. In the *determinatio* Brito answers in the positive: the fallacy of the consequent is a *locus sophisticus.* This is confirmed by what Aristotle says in the *Sophistici Elenchi* and by the earlier tradition, which for each *locus sophisticus* has given both the cause of appearance and the cause of defect: since both causes are given for the fallacy of the consequent too, it must be sophistical[90].

Following the older tradition, Brito explains that there are two species of this fallacy: one from the affirmation of the consequent and one from the negation of the antecedent[91]. As he then clarifies in his answer to the second and third arguments, even if the species are two, the fallacy takes its name from the consequent. The reason of this is that the premises from which one infers (*inferens*) is always

86. Radulphus Brito, *Questiones super Sophisticos Elenchos*, I, 48, **B** f. 530va-b; **S** f. 189rb: "Item, non habemus fallaciam antecedentis, /**B**/ ergo nec consequentis; antecedens patet, consequens declaratur: quia sicut in consequentia est consequens ita antecedens" (cf. Simon de Faversham, *Quaestiones novae super libro Elenchorum,* q. 33, ed. Ebbesen et al., p. 133).

87. Radulphus Brito, *Questiones super Sophisticos Elenchos*, I, 48, **B** f. 530vb; **S** f. 189rb-va: "Item, locus dialecticus debet denominari /**S**/ ab inferente; sed inferens est antecedens; ergo magis debet esse locus sophisticus antecedentis quam consequentis". About this Brito has the same solution as *Anonymi Pragensis Quaestiones super Aristotelis Sophisticos Elenchos*, q. 38.

88. Radulphus Brito, *Questiones super Sophisticos Elenchos*, I, 48, **B** f. 530vb; **S** f. 189va: "Item, ubi est locus rethoricus non est locus sophisticus; sed ubi est fallacia consequentis est locus rethoricus; ergo etc. Maior patet, quia locus rethoricus et dyalecticus secundum essentiam sunt idem, nec differunt nisi sicut commune et contractum sub communi, secundum Boethium, in quarto Topicorum suorum; sed locus dyalecticus et sophisticus non sunt idem, sed similes, ergo nec rethoricus et sophisticus <sunt idem>. Probatio minoris: quia, secundum Philosophum, fallacia consequentis fit ex adiunctis; modo ex adiunctis est locus rethoricus; ideo etc."

89. Radulphus Brito, *Questiones super Sophisticos Elenchos*, I, 48, **B** f. 530vb; **S** f. 189va: "Item, locus dyalecticus est ex communiter accidentibus; ergo non erit locus sophisticus; sed fallacia consequentis fit ex communiter accidentibus; ergo fallacia consequentis non est locus sophisticus".

90. Here Brito goes back to his predecessors' discussion of the two causes; cf. Radulphus Brito, *Questiones super Sophisticos Elenchos*, I, 48, **B** ff. 530vb-531ra; **S** f. 189va.

91. Radulphus Brito, *Questiones super Sophisticos Elenchos*, I, 48, **B** f. 531ra; **S** f. 189va: "Sed notandum est quod duo sunt modi istius fallacie: unus est a positione consequentis et alius a destructione antecedentis".

the real consequent (*quod in veritate est consequens*): in the fallacy *animal currit, ergo homo currit*, the premise is the consequent (i.e. the conclusion) of the valid inference *homo currit, ergo animal currit*. Here it is evident that both the objector and Brito are assuming that a valid inference can also be expressed in conditional form, so that the premise becomes the antecedent and the conclusion the consequent of the conditional[92]. It is also clear that with *consequens* Brito means *quod in veritate est consequens*, i.e. the consequent of the *true* conditional proposition, while the objector must mean the conclusion of the fallacy (or its 'consequent' when it is expressed as a conditional proposition). In fact, the objector of the third argument simply conflates the premise of the fallacious argument with the antecedent of the corresponding conditional. If indeed the fallacious inference *animal currit, ergo homo currit* (whose major premise is the true conditional *si homo currit, animal currit*) is cast in the form of the false conditional proposition *si animal currit, homo currit*, then the premise of the fallacious inference becomes the "antecedent" of the false conditional. 'Consequent' in this context has almost invariably the meaning of *quod in veritate est consequens*, i.e. of the consequent of a true conditional proposition. *Mutatis mutandis*, the same holds of the species from the negation of the antecedent.

Like Roger Bacon, Simon of Faversham, and others, and unlike Kilwardby (?) and Albert the Great, Brito argues that it is not needed that the converse of the fallacy of the consequent be always valid (*consequentia bona*); it is sufficient that it be probable[93]. In reply to the arguments *contra*, Brito follows Giles of Rome almost verbatim and distinguishes two senses of the *fallacia consequentis*; in one sense, it is a deviation from the syllogism and in so far it falls under the *Prior Analytics*. In another sense, in so far as the cause of its appearance of validity is added to the explanation of its invalidity, it falls under the *Sophistici Elenchi*.

In answer to the fourth argument, according to which from *adiuncta* one can only produce rhetorical *loci*, Brito replies:

> "Among the things that are added, some are proper and others common. Proper are those which necessarily follow the thing to which they are added, like 'having milk' is related to 'giving birth'; and these are called *prodigia* by Aristotle at the end of the *Prior Analytics*; and in these case we have a dialectical or rhetorical *topos*. By contrast, common are those which have a greater extension than the thing to which are

92. Brito would then be assuming a sort of 'deduction theorem', i.e. the equivalence between an argument and the corresponding conditional. Some such "mediaeval deduction theorem" shaped by and large fourteenth century's debates about *consequentiae*; see King 2001.

93. Radulphus Brito, *Questiones super Sophisticos Elenchos*, I, 48, **B** f. 531ra; **S** f. 189va: "Item, notandum est quod in fallacia consequentis non oportet alteram consequentiarum semper esse bonam, sed sufficit quod sit probabilis. Et propter hoc, cum dicitur quod causa apparentie eius est ydemptitas bone consequentie ad malam, ibi accipitur consequentia large pro 'vera consequentia' vel pro 'probabili'.

added, like 'this <woman> is pale, therefore she has given birth'; and in this case we have precisely the fallacy of the consequent"[94].

However, these latter *adiuncta* may either be taken singularly, and this produces the fallacy of the consequent, or they may be taken in conjunction with others (*plura adiuncta*), and this produces a respectable rhetorical argument[95]. He has no examples of this, though.

With regard to the *locus a communiter accidentibus*, Brito also re-states the Boethian distinction between that which always follows (*penitere* from *deliquisse*) and that which sometimes follows and sometimes not: the former produces a good dialectical argument, the latter a fallacy of the consequent[96]. Brito's work on the *Sophistici Elenchi* is certainly dependent upon and influenced by the long traditions of commentaries on Boethius' *Topics*, on the *Prior Analytics*, and on the *Sophistici Elenchi*.

5. Some conclusions

From what precedes I would very briefly draw some conclusions:

1. the term 'consequent' has not always been interpreted as the apodosis of a conditional proposition, but often as a predicate (an accident or a sign expressed by a term) which follows or 'flows' from another thing (be it a substance or an accident) (large interpretation);

2. at the very beginning of its Latin reception the alternative between a larger and a narrower interpretation is evident, the narrower interpretation assuming 'consequent' as the apodosis of a conditional sentence;

3. the second view appears to prevail at the end of the 12th, and in the long run: this is the historical reason why modern treatments of the fallacy of the consequent assume that the fallacy of the consequent violates the two basic rules of

94. Radulphus Brito, *Questiones super Sophisticos Elenchos*, I, 48, **B** f. 531rb; **S** f. 189vb: "ex adiunctis quedam sunt propria et quedam communia. Propria sunt illa que necessario sequuntur rem cuius sunt adiuncta, sicut 'habere lac' se habet ad 'parere', et talia vocat Aristotiles, in fine Priorum, 'prodigia', et ibi est locus dyalecticus vel rethoricus. Communia autem sunt in plus quam res cui sunt adiuncta, sicut 'ista est pallida, ergo peperit'; et in talibus est bene fallacia consequentis".

95. Radulphus Brito, *Questiones super Sophisticos Elenchos*, I, 48, **B** f. 531rb; **S** f. 189vb: "Sed adhuc notandum est quod ista communia adiuncta dupliciter possunt accipi, quia aut accipitur unum solum adiunctum ad inferendum illud cuius est adiunctum, et tunc est fallacia consequentis, ut 'est pallida, ergo peperit'; aut accipiuntur plura adiuncta ad inferendum illud cuius sunt adiuncta, et bene tunc fit argumentum rethoricum et non fallacia consequentis" (cf. Robertus Kilwardby (?), *Commentarium in Aristotelis Sophisticos Elenchos* dub. 3.2, **C** f. 296ra; **P** f. 23va).

96. Radulphus Brito, *Questiones super Sophisticos Elenchos*, I, 48, **B** f. 531rb; **S** f. 189vb: "Per idem patet ad aliam, quia communiter accidentia quedam sunt semper consequentia, et in talibus est locus dyalecticus; alia sunt que quandoque sequuntur, et quandoque non, et in talibus est fallacia consequentis".

propositional logic, MP and MT, so as to produce two kinds of fallacious arguments, namely AC and DA;

4. as we saw above, however, some of the 13th-century commentators on *Soph. El.* left open the possibility of interpreting antecedent and consequent in terms of the logic of predicates: they often consider valid inferences (or true conditional propositions) as based on relationships between terms (the so-called *habitudines locales*, which included relations between hypernyms and hyponyms, wholes and parts, substances and accidents, and so on).

The story of the reception of the fallacy of the consequent, in conclusion, might help explain why the fallacy of the consequent was not considered a formal fallacy, but rather a material one. It also shows that the reflections about the fallacy of the consequent are intertwined with the attempts at saving its argumentative value, because of its affinity with the dialectical locus *a communiter accidentibus* and with the second-figure sign-enthymemes that – as Aristotle says – can be successfully applied in rhetoric, taking into account their reception by a general public: these attempts follow a line of interpretation that might be interesting also from a contemporary point of view.

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Abstract: The term 'consequent' in 'fallacy of the consequent' is nowadays always interpreted with reference to the rule of *modus ponens*, most of the time assuming that Aristotle – the Father of Logic – could not have had anything else in mind. In this article, after briefly recalling how Aristotle deals with this fallacy and the contribution of the Greek commentators on his works, I shall focus on the first period of the reception of the *Sophistical Refutations*, between the 1160s and the end of the century. I shall examine both the commentaries on Aristotle's work and the logical handbooks belonging to the *Logica Modernorum*. On the one hand, in these contexts, the interpretation of the term *consequens* cannot fail to take into account the meaning assigned to it by Boethius in his dialectical and rhetorical works; but, from its first reception on, several alternative interpretations are advanced, apparently more in tune with Aristotle's text. In this context, an interpretation emerges which corresponds to modern treatments of the fallacy of the consequent and of its two basic types (Affirming the Consequent and Denying the Antecedent). Third, I shall show how during the thirteenth century this interpretation was commonly adopted, together with the link to Boethius' *locus a communiter accidentibus* and Aristotle's reflection on signs proposed in *Prior Analytics* II 27.

Keywords: Consequent; Fallacy of the Consequent; Sentence/Propositional Logic; Term/Predicate Logic; Signs.

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"Qui imperitus est vestrum, primus calculum omittat". *Aristotelis Sophistici Elenchi* 1 in the Boethian Tradition*

1. Prolegomena

Aristotelian texts dealing with language are not hard to come by. In fact, they are quite common, liberally interspersed throughout the Aristotelian corpus. They occur not only in works on dialectic, rhetoric and poetry, but in a variety of other writings as well, ranging from the books on the soul to the treatises on natural history, politics and first philosophy. That being said, for all that they bear witness to Aristotle's keen interest in language and language-related issues, within and across different disciplines, these texts are for the most part digressive in nature and auxiliary in purpose. That is, however straightforward and to the point Aristotle's remarks about linguistic matters actually are, they leave the distinct impression that he never broaches, let alone studies, the topic of language for its own sake[1]. Needless to say, however, a few texts come close enough. The prologue of the *Sophistici Elenchi* ([Ur-Text^{A}] below) is an excellent case in point, on at least two counts.

First, [Ur-Text^{A}] showcases the soundness and relevance of a 'reverse approach' insofar as the upsides and downsides of language turn out to be the opposite sides of the same coin. Indeed, linguistic pitfalls and snares provide pristine evidence for the principles and standards they are judged by to begin with – the 'reverse' part being there precisely to remind us that misuse and abuse of language reveal good linguistic habits and practices to the same extent that they themselves are revealed by the rules and norms they violate. It then stands to reason to assume that the better we get to know how words can be misleading or

* For, as the saying goes, the author of the essay catches up fast but one has to explain it to him a long time, he is most grateful to Sten Ebbesen for indulging his obsession with pebbles and for showing his pupil the error of some of his ways on three different occasions, without losing patience or hinting that enough is enough.

1. This peculiar, if unsurprising, feature of Aristotle's treatment of language is expounded upon in some detail in Gazziero 2021a (with relevant literature on p. 1).

downright deceptive, the more likely we are to learn important lessons about at least some of the laws we want our reasoning and discussion to abide by. And this is precisely what [Ur-Text^{A}] is all about: if language is factored in at all, it is to the extent that dishonest contenders and unsportsmanlike fellow-dialecticians exploit some of its features to lure the unwary and the untrained into believing that whatever goes for words and word-compounds (sentences and the like) also goes for the things and facts they stand for – which, of course, is tantamount to asking for trouble and leads to all kinds of errors, confusions and mystifications. Conversely, if we manage to get the hang of how unsavoury characters equivocate their way through conversations, we might just get better at catching them in the act – and, if we feel so inclined, at giving them a taste of their own bad medicine.

Second, [Ur-Text^{A}] allows us to appreciate the exegetical proficiency and technical expertise typically displayed by mediaeval commentators whose views are still largely ignored and whose solutions to perplexities – which often coincide with our own – are usually overlooked[2]. In this particular instance, the level of sophistication the Latins achieved is nothing short of remarkable, for they managed to build compelling cases both in favour of and against the reading of [Ur-Text^{A}] that was to become – and still is – the standard story.

2. *Expositores latini nostri*[3]

As a general rule, Latin commentators on Aristotle's *Sophistici Elenchi* never grew tired of asking the same questions all over again; moreover, they seemed to par-

2. The point is altogether germane to Sten Ebbesen's suggestion that mediaeval commentaries not only bring to our attention issues we chose to ignore or whose complexity we have underestimated, but also help us push our analysis of problematic texts further than we could possibly go by simply "sticking to contemporary or near-contemporary" literature (Ebbesen 2017, p. 187).

3. The Western mediaeval reception of Aristotle's tract on fallacies is well-charted territory – courtesy of Lorenzo Minio-Paluello (Minio-Paluello 1952; 1954; 1955), Lambertus Maria de Rijk (de Rijk 1962-1967) and, most notably, Sten Ebbesen who in his monumental opus on commentators and commentaries on Aristotle's *Sophistici Elenchi* (Ebbesen 1981), as well as in a wealth of authoritative contributions (just to name a few: Ebbesen 1979, 1982, 1987a, 1993, 1996 and forthcoming, which I was most kindly allowed to peruse over the years), has almost single-handedly retraced its evolution as early as the corpus of the first Byzantine glosses and as late as the more recent Greek and Latin interpreters. Therefore, for the time being, the briefest bibliographical summary will do. Besides, no matter the amount of scholarly scrutiny one throws at the matter, some seem simply unable to get the facts right, while others are stuck in another century. Just to mention a couple of recent gems, Uckelman 2021, p. 34 seems to believe that Boethius did not translate the *Sophistici Elenchi* and adds insult to injury by suggesting that she has it on the good authority of none other than Bernard G. Dod (who, of all people, she should have left alone, since he did provided us with the standard critical edition of Boethius' translation): "the *Sophistical Refutations* were not translated by Boethius but were newly translated in the middle of the twelfth century by James of Venice (Dod 1982)". In like manner, Ramirez Vidal 2021 has debunked the standard story of fallacies in the Latin West – most notably Boethius' part in it – while relying on Migne's 'editions' of Boethius' translations, where the word *fallacia* does not appear as prominently as in the *Aristoteles Latinus* edi-

ticularly enjoy picking each other's arguments apart. As a result, they not only routinely covered – both as individuals and as a group – a lot more ground than we are used to nowadays, but they also came up, more often than not, with the right answer (usually, by working through all possible solutions). Every now and then, however, they just weren't aware of all the possibilities. When this occurred, it mostly came down to either some oddity in the Latin translation of the original Greek or some peculiar association in processing earlier scholarship. As it happens, a missing piece of information in the Latin text ([Ur-Text^{B}]) and a sound, albeit misguided, connection in the logical literature available to Latin commentators ([T7] below), made it very hard for them to read anything but the contrary of an analogy – in fact, a disanalogy – into Aristotle's well-known comparison between the way we handle (mark my words), on the one hand, pebbles or counters in our calculations and the way we misuse, on the other hand, words in our verbal dealings ([Ur-Text^{A}]).

3. *Quandoque fidus dormitat Boethius*

To begin with, although Boethius has always come highly recommended as an Aristotelian interpreter[4], his translations sometimes turned out to be a bit tricky,

tions, whose very existence Ramirez Vidal either ignored or chose to disregard. Since Boethius and the Western tradition are to provide the bulk of evidence upon which our demonstration will rely upon, Latin-oriented scholarship will here receive the lion's share, all the more so on account of the derivative nature of some at least of the post-Ebbesen 1981 and pre-Vogiatzi 2019 (especially pp. 80-142) secondary literature on Byzantine fallacies. (If there's a special circle in hell for plagiarized and self-plagiarizing abominations, they're certainly saving a nice spot for Rita Salis' Michael of Ephesus redux: that is, Ebbesen 1981, I, p. 258 and p. 269 – cf. Salis 2007, pp. 378-385; 2008, pp. 16-19; 2009, pp. 430-431. How many times wilt thou lift thy neighbour's ideas before it's one too many? I lost count at three). Since 1975, we can rely on a dependable critical edition of Boethius' Latin translation (along with James of Venice's fragments and William of Moerbeke's revision) in the *Aristoteles Latinus* series, which also provides a comprehensive (in fact virtually complete) catalogue of surviving manuscripts (around 270), including reliable information on those which were most heavily glossed (whose number lies in the vicinity of 150). We have a score of editions of Latin commentaries as well – between one third and one half of the medieval Latin production, though with notable exceptions. Honourable mentions, hopefully soon to be awarded an edition of their own, include: Robert Grosseteste (*quod fertur*), Robert Kilwardby, Nicholas of Paris, Robert of Hautecombe and Giles of Rome. When no edition is available (cf. note 24 below for an updated list), at least we know where to look for the manuscripts – courtesy again of Sten Ebbesen, who accounted for all relevant unedited sources in his SE catalogue (Ebbesen 1993), which is also an invaluable study in typology, in so far as it classifies Latin texts in four main families or groups (scholia, literary commentaries, question commentaries and deviant materials).

4. Contemporaries and posterity alike had only praise for Boethius as a connoisseur of all things Greek. One can doubt Cassiodorus' good faith (if not actual expertise) and take with a grain of salt his celebration of Boethius as the equal to the great philosophers of old, namely Plato and Aristotle, whom he taught to speak better Latin than they spoke Greek (Cassiodorus Flavius Magnus Aurelius, *Variarum Librorum libri XII*, I, 45, 3-5). That being said, there must have been some truth in Cassiodorus' self-serving homage to Boethius, lest it have the opposite effect on the latter

all the more so when they neither advertised themselves as something else besides translations (word for word translations at that) nor looked suspicious in any way. Predictably enough, it is not the obvious rewriting nor the ambiguous wording and phrasing that got Latin commentators into trouble[5]. Rather, it is the casual

(who was being asked a favour or two on behalf of king Theodoric the Great, as one might recall). One may have qualms about Roger Bacon too, who might have praised Boethius only to come down harder on the translators of his time (especially his Flemish arch-foe, William of Moerbeke). Yet, Bacon's tone is so uncharacteristically subdued in his constant commendation of Boethius' remarkable knowledge of Greek (cf. Rogerus Bacon, *Opus Maius*, III, 67; Rogerus Bacon, *Opus Tertium*, XXV, 91; Rogerus Bacon, *Compendium Studii Philosophiae*, VIII, 472), that one is tempted to take Bacon's word for it and be happy he spared us another round of verbal abuse. Gazziero 2017 surveys recent – and not so recent – literature on Boethius as both a translator and a commentator of Aristotle (Ebbesen 1987b, 1990, 2008 and 2011 being the most essential reading).

5. Following a rule he had most emphatically set himself (cf. Boethius, *In Isagogen Porphyrii Commentum*, I, ed. Brandt, p. 135, ll. 2-13), Boethius usually delivered accurate word-by-word translations. Every so often, he even stuck to the original turn of phrase too close for his own good (as well as that of his Latin readers') – as Lorenzo Valla repeatedly blamed him for: "ita Graecos adscivisti ut a Latinis descisceres, et mores linguae alienae quam nostratis apud nos valere malles? [have you consorted with the Greeks to the point of rejecting the Latins and having their ways prevail among us over our own?]" (Laurentius Vallensis, *Disputationes Dialecticae*, II, 16, 116). However, this did not prevent Boethius from getting, now and then, a bit creative. For instance, to illustrate the fallacy of accent he replaced Aristotle's Homeric examples, which would not work in translation, with verses by Horace and Vergil (cf. Arist., *Sophistici Elenchi*, 4, p. 10, ll. 9-10), a fact Latin commentators had no problem figuring out (cf. *Anonymi Summa Sophisticorum Elenchorum*, ed. de Rijk, p. 326, ll. 1-8; *Anonymi Parisiensis Compendium Sophisticorum Elenchorum*, ed. Ebbesen / Iwakuma, p. 84, ll. 23-28 (Paris) - ed. Ebbesen, p. 284, ll. 20-26 (Uppsala); *Anonymi Aurelianensis I Commentarium in Sophisticos Elenchos*, ed. Ebbesen, p. 123, ll. 26-33; *Anonymi SF Quaestiones super Sophisticos Elenchos*, q. 73, ed. Ebbesen, p. 168, ll. 20-21; Aegidius Romanus, *Expositio super Libros Elenchorum*, ed. Venetiis, f. 141rb 5-7). Truth be told, they got a little carried away themselves and devised in the process a few unconventional ideas of their own: *e.g.*, along with the right explanation (*i.e.*, Greek and Latin being two different languages, the same example cannot be expected to work in both languages; thus, the Latin translator shrewdly turned for inspiration to Latin Poets rather than Greek ones), *Anonymi Cantabrigiensis Commentarium in Aristotelis Sophisticos Elenchos*, ed. Ebbesen, p. 146, ll. 16-24 suggested that it would not come as a surprise if it turned out that the examples are the same because "Latini nostri" borrowed them from their Greek predecessors; and, as a result, the same verse has simply been picked up twice. Likewise, every once in a while, Boethius' translations were open to more than one interpretation – a fact Latin commentators were also well aware of and largely took into account as demonstrated by the way they coped with the equivocal "note" in the text we're going to deal with in a moment. "ἐπεὶ γὰρ οὐκ ἔστιν αὐτὰ τὰ πράγματα διαλέγεσθαι φέροντας, ἀλλὰ τοῖς ὀνόμασιν ἀντὶ τῶν πραγμάτων χρώμεθα συμβόλοις, κτλ." (165a 6-8) reads in Boethius' translation "nam quoniam non est ipsas res ferentes disputare, sed nominibus pro rebus utimur notis, etc." (p. 6, ll. 3-5). Amongst others, *Anonymus Aurelianensis* (ed. Ebbesen, pp. 26-27) and *Anonymus Monacensis* (ms. München, Bayerische Staatsbibliothek, Clm. 14246, f. 4ra; ms. Admont, Stiftsbibliothek, 241, f. 6vb) did not fail to notice that it is not immediately clear whether *notis* should be understood as an adjective (qualifying alternatively *nominibus* or *rebus*) or whether it should be construed as a complement of the verb *utor* (*utimur* <*ut*> *notis*, that is <*ὡς*> *συμβόλοις*). The former set out the alternative quite nicely; moreover, he supported both alternatives by quoting Aristotelian parallels. On the one hand, we have "utimur [p. 27] pro rebus nominibus notis, id est cognitis [we use names instead of things, that is names we know]" (pp. 26-27), for "vitium enim, ut dicit Aristoteles, est uti in problemate ignotis nominibus [as Aristotle states <cf. *Topica*, II, 1, 109a27-33>, the fault lies in discussing the matter at hand with words which are unknown to us]". On the other hand, we have "vel notis ipsarum rerum, ut non dicatur 'notus -ta -tum', sed 'nota -tae' [or <we use them as> symbols for

omission or the smooth, inconspicuous shift from one simple, unassuming word to another that led them astray:

> [Ur-Text^{A}] Arist., *De Sophisticis Elenchis*, 1, 164b25-165a6-17: "τὸν αὐτὸν δὲ τρόπον καὶ συλλογισμὸς καὶ ἔλεγχος ὁ μὲν [26] ἔστιν, ὁ δ' οὐκ ἔστι μέν, φαίνεται δὲ διὰ τὴν ἀπειρίαν· οἱ [27] γὰρ ἄπειροι ὥσπερ ἂν ἀπέχοντες πόρρωθεν θεωροῦσιν. ὁ μὲν [165a] γὰρ συλλογισμὸς ἐκ τινῶν ἐστι τεθέντων ὥστε λέγειν ἕτερον [2] ἐξ ἀνάγκης τι τῶν κειμένων διὰ τῶν κειμένων, ἔλεγχος δὲ [3] συλλογισμὸς μετ' ἀντιφάσεως τοῦ συμπεράσματος. οἱ δὲ [4] τοῦτο ποιοῦσι μὲν οὔ, δοκοῦσι δὲ διὰ πολλὰς αἰτίας· ὧν εἷς [5] τόπος εὐφυέστατός ἐστι καὶ δημοσιώτατος, ὁ διὰ τῶν ὀνομάτων. [6] ἐπεὶ γὰρ οὐκ ἔστιν αὐτὰ τὰ πράγματα διαλέγεσθαι [7] φέροντας, ἀλλὰ τοῖς ὀνόμασιν ἀντὶ τῶν πραγμάτων [8] χρώμεθα συμβόλοις, τὸ συμβαῖνον ἐπὶ τῶν ὀνομάτων καὶ ἐπὶ τῶν [9] πραγμάτων ἡγούμεθα συμβαίνειν, καθάπερ **ἐπὶ τῶν ψήφων** [10] τοῖς λογιζομένοις. τὸ δ' οὐκ ἔστιν ὅμοιον· τὰ μὲν γὰρ [11] ὀνόματα πεπέρανται καὶ τὸ τῶν λόγων πλῆθος, τὰ δὲ [12] πράγματα τὸν **ἀριθμὸν** ἄπειρά ἐστιν. ἀναγκαῖον οὖν πλείω [13] τὸν αὐτὸν λόγον καὶ τοὔνομα τὸ ἓν σημαίνειν. ὥσπερ οὖν [14] κἀκεῖ οἱ μὴ δεινοὶ **τὰς ψήφους φέρειν** ὑπὸ τῶν ἐπιστημόνων [15] παρακρούονται, τὸν αὐτὸν τρόπον καὶ ἐπὶ τῶν λόγων οἱ τῶν [16] ὀνομάτων τῆς δυνάμεως ἄπειροι παραλογίζονται καὶ αὐτοὶ [17] διαλεγόμενοι καὶ ἄλλων ἀκούοντες [Hasper 2013, pp. 13-14: in the same way, one argument constitutes a real deduction or a real refutation, while another does not, even though it appears to due to our lack of experience. For those without experience are like people remaining at a distance and judging from far away. For a deduction is an argument based on certain granted points, such that it states, by way of necessity, something different from the points laid down, while a refutation is a deduction together with the contradictory of its conclusion. But some arguments do not achieve this, even though they seem to on various grounds – of which one type of argumentation is very fertile and popular, the one based on words. For since it is impossible to have a discussion while adducing the things themselves, and we use words as symbols instead of the things, we assume that what follows for words, also follows for the things (just as with stones for those who do calculations). It is not the same, however, since the words are limited, just like the number of sentences, whereas the things themselves are unlimited in number. It is then inevitable that the same sentence or a single word signify several things. Just as in calculation, those who are not versed in moving stones around are tricked by the experts, so too those without experience of the possibilities of words are deceived by means of fallacies, both when themselves participating in a discussion and when listening to others]".

the things themselves, as in 'nota -tae' (symbol), as opposed to 'notus -ta -tum' (known)]", for this is what *nota* means in a well-known Aristotelian text, that is "iuxta illud 'ea quae sunt in voce sunt notae eorum quae sunt in anima' [according to what Aristotle says in *Peri Hermeneias*, 1, 16a3-4: 'what we put into words is a symbol of what we have in our mind']" (*Anonymus Aurelianensis*, ed. Ebbesen, p. 27). *Anonymus monacensis* settled the question as permanently as the nature of the subject permits: "patet ad hoc solutio, quia notis idem est quod signis et sic eo utitur auctor, alio modo notis idem est quod cognitis et sic non utitur auctor eo [thus, the solution to our problem is clear, namely 'notis' is the same here as symbols and this is what Aristotle means by it; in another sense, 'notis' would mean known, but this is not the meaning it has here]" (*Anonymus Monacensis*, f. 4ra [Munich], f. 6vb [Admont]).

[Ur-Text^{B}] Arist., *De Sophisticis Elenchis Translatio Boethii*, ed. Dod, pp. 5-6, ll. 14-13: "eodem autem modo et syllogismus et elenchus [15] hic quidem est, ille vero non est quidem, videtur autem propter [16] imperitiam; nam imperiti velut distantes longe speculantur. [17] Nam syllogismus quidem ex quibusdam positis est ut dicatur [18] diversum quid ex necessitate ab his quae posita sunt, elenchus [6.1] autem syllogismus cum contradictione conclusionis. Illi vero hoc [2] quidem non faciunt, videntur autem ob multas causas, quorum [3] unus locus aptissimus est et publicissimus per nomina. Nam [4] quoniam non est ipsas res ferentes disputare, sed nominibus pro [5] rebus utimur notis, quod accidit in nominibus in rebus quoque [6] arbitramur accidere, velut **in compotis** ratiocinantibus. Hoc [7] autem non est simile. Nam nomina quidem finita sunt et [8] orationum multitudo, res autem **numero** infinitae. Necesse est ergo [9] plura eandem orationem et nomen unum significare. Quemadmodum [10] igitur illic qui non sunt prompti **numeros ferre** a scientibus [11] expelluntur, eodem modo et in orationibus qui nominum [12] virtutis sunt ignari paralogizantur et ipsi disputantes et alios [13] audientes".

4. Can't we just call it a pebble?

While literal, or very close to literal, Boethius' translation parted ways with its source in at least one respect, and it happened to do so twice[6]. First, "καθάπερ ἐπὶ τῶν ψήφων τοῖς λογιζομένοις" ([Ur-Text^{A}] 165a9-10) became "velut in compotis ratiocinantibus" ([Ur-Text^{B}] p. 6, l. 6). Secondly, "οἱ μὴ δεινοὶ τὰς ψήφους φέρειν" ([Ur-Text^{A}] 165a14) endured a similar and – if anything, more radical – transformation, insofar as it read in Boethius' translation: "qui non sunt prompti numeros ferre" ([Ur-Text^{B}] p. 6, l. 10). On the face of it, both deviations from the original involved what we would call – in the parlance of our times – an "abstraction change" of sorts[7]. To be sure, Boethius shifted on both occasions from a rather concrete term (*ψῆφος*) to a more abstract one (*compotus*, first; *numerus*, next).

6. Aristotle's prologue and its Boethian translation have been recently compared by Crialesi 2020, who however has chosen a different path: "in translating this controversial passage of the *Sophistical Refutations*, Boethius follows the *verbum de verbo* method, reproducing the text with mirror-like symmetry. His argumentative patterns also follow the Aristotelian text, etc." (Crialesi 2020, p. 115). It is a bit odd, then, to collate the 'argumentative patterns' of the original and of its word-for-word translation: insofar as the translation is literal, the 'argumentative pattern' (whatever 'argumentative pattern' means here: presumably, the order or sequence of claims and the reasons that support them) is simply the same – they call it *verbum de verbo* for a reason. Accordingly, nothing meaningful will emerge from comparing the argumentative flows of the text, on the one hand, and of its word-for-word Latin translation, on the other hand. On the contrary, asking whether or not the right Latin word stands for the original Greek word, that's a different story altogether, worth telling in its own right... even if – God forbid – we get it wrong.

7. 'Abstraction change' is a loan from Chesterman 1997, p. 103. Last time we checked, his system of thirty-odd translation 'strategies' had won widespread acceptance amongst armchair translators and professional alike. Those interested in its peculiar contribution to traductology's metalanguage may consult Gambier 2008, who has a few suggestions of its own as to how implement Chesterman's strategy with proper 'tactics' (it figures).

Before arguing the merits and demerits of such move from one "level of abstraction" to another – mostly its demerits, insofar as a translation along Boethius' lines quite simply takes the symbolic dimension of Aristotle's analogy between counters and words out of the equation – one might wonder whether or not and to what extent Boethius' departure from the obvious Latin translation for *ψῆφος* (that would be *calculus*, as attested in [T1] below and the glossaries mentioned note 9), was more deliberate than a mere slip of the mind or the tongue would allow. Admittedly, the smart money is never on proving a negative and, indeed, short of asking Boethius himself, we'll never know for sure whether he deliberately avoided introducing the word *calculus* as a Latin equivalent for *ψῆφος*. If we go out on a limb and give it a shot here, it is simply because, whatever the result is going to be, there is a lesson to be learned. After all, textual interpretation being an exact science and all, when one tackles exegetical issues, a passably near miss is not as good as a mile.

To begin with, how likely, if at all, is it that Boethius simply ignored the basic meaning of *ψῆφος* and had to improvise? Not very likely. In fact, it is hard to believe that someone as fluent in classical Greek as he was – and a weight and currency expert to boot[8] – would have drawn this particular blank in the first place. It is even harder to think that he would not have set the record straight and checked whether or not his translations of *ψῆφος* made good linguistic sense – be it on his own or with the help of, say, either Symmachus, a native Greek tutor in his service or in Symmachus' household, a common friend or any Roman or Byzantine acquaintance of his with any training in literary Greek. All the more easily – one might add – since the association between *ψῆφος* and *calculus* was a matter of course in late ancient and early medieval Greek and Latin sources. Its only remarkable feature, as noted time and again, is that the gender shifts from feminine (in Greek) to masculine (in Latin)[9]. In fact, it was so trivial that no one pos-

8. At least on one occasion (Cassiodorus Flavius Magnus Aurelius, *Variarum Librorum libri XII*, I, 10), king Theodoric the Great called upon Boethius as a monetary consultant to inquire into the debasement which his palace guard's pay had – allegedly – suffered at the hands of the master of coin. Clipping, adulterating or counterfeiting coins were no trifling matter under Amal rule: they were all capital offences, punishable by death; and to be successful, the criminal investigation required some computational skills and the ability to crosscheck, on the one hand, non-decimal monetary exchange rates, and, on the other hand, non-decimal weight standards. (By the way, it is hard to say whether finger reckoning alone would have done the trick or not; an abacus would certainly have come in handy, but we know next to nothing about the way Boethius' investigation unfolded, if there ever had been one to start with). While the date of Boethius mission is controversial (cf. Shane Bjornlie 2013, p. 174), its importance is not, as suggested by Cassiodorus himself, for whom tampering with state coins becomes a threat to the whole order of Creation (you can hardly fault him for that: if you cannot trust the king's coin, everything else falls apart before you know it). On the manifold ramifications of the counterfeiting affair and its gravity in the Amal administration's eyes, cf. Pizzani 1978; della Corte 1981; Cuppo Csaki 1987; and, more recently, Lafferty 2013 (especially pp. 208-209).

9. 'Calculus ψῆφος' occurs in the *idiomata generum* section of the anonymous Bobiensis fragment (*Anonymi Ars Bobiensis*, ed. de Nonno, p. 32, l. 8), along with several scores of other "nomina quae

sessing the minimal education and slightest interest in learning Greek could have missed it and proven unable to tell apart – accordingly – numbers and counters. After all, we're talking textbook exercise here:

> [T1] *Colloquium Stephani*, ed. Dickey, pp. 21b-22b: "ὅσα πρὸς τοὺς ἀρχομένους κατελέχθη αὐτοῖς, [21c] καὶ τὰ χρήζοντα καὶ ἀριθμούς, δακτύλους καὶ ψήφους, [21d] ταῦτα, ἐν ὅσῳ ἀποδίδομαι, οὗτοι ἔπραττον. [22a] οἱ λοιποὶ δὲ ἐξηγήσεσιν καὶ ἐπερωτήσεσιν ηὐκαίρουν, [22b] κατὰ δύο τάξεις, βραδύτεροι καὶ ταχύτεροι – quaecumque ad incipientes praebita sunt eis, et necessaria et numeros, digitos et calculos, haec, dum reddo, ei agebant. reliqui autem expositionibus et interrogationibus vacabant, per duas classes, tardiores et velociores [Dickey 2012, pp. 239-240 (slightly modified): whatever was explained to them as beginners, that is essential things as numbers, fingers and counting-stones, these things they were doing while I was attending to my work. But the rest <of the pupils> had free time for explanations and for <asking> questions, in two classes, the slower ones and the faster ones]".

Admittedly, bilingual conversation manuals hardly make for exciting reading, nor are they the stuff of great scholarship[10]. True to their nature, [T1]'s subject, vocabulary and narrative are so flat and so ordinary that they certainly were within everyone's reach. Of course, Boethius was not your average educated man; nor was he an ordinary Greek language student. Hence, one might take exception to

apud Romanos masculina, apud Graecos feminina [names which are masculine amongst Romans and feminine amongst Greeks]"; *Hermeneumata Pseudodositheana Leidensia*, I, p. 12, l. 322: 'ψῆφοι calculi'; *Hermeneumata Stephani (Glossae Stephanis)*, ed. Loewe / Goetz, p. 440b, ll. 55-59: "calculatio, ψῆφος, λόγος. [56] calculator ψηφιστής. [57] calculi, ψῆφοι. [58] calculus, ψῆφος. [59] calculor ψηφίζω"; *Hermeneumata Amploniana (Hygini)*, ed. Loewe / Goetz, 81a10-12: 'psefos calculus'. Informal, utilitarian, and/or educational, sub-literary texts in general are notoriously difficult to stemmatize, or even simply to date and locate with any certainty (and bilingual wordlists belong, if not to all, at least to one or two of the aforementioned categories). As far as these and germane collections are concerned, however, a tentative consensus has been reached, suggesting that, whatever their geographical origin, these materials were available for recycling and adaptation in the Latin-speaking West no later than the fourth century AD (cf. Dionisotti 1982, p. 123; Rapp 2004, pp. 1244-1245; Dickey 2012, p. 52; Zetzel 2018, p. 113 with relevant bibliography pp. 240-242).

10. One should not be too quick to condemn and dismiss, though – one can't help noticing that the account of the unknown pupil's daily routine both at home and at school (especially the "at home" part) bears an uncanny resemblance to Scarry 1969 account of Huckle Cat's "getting ready for school" morning drill (one of little Nahida's favourites): they both get up in the morning ("ἠγέρθην πρωΐ – surrexi mane" – "Huckle got up"), put their pants on ("ἤτησα ὑποδήματα καὶ περικνημῖδας – poposci calciamenta et ocreas" – "then Huckle got dressed. This is not the way to put on your pants, Huckle!"), wash their face ("προσηνέχθη ὕδωρ πρὸς τὴν ὄψιν εἰς ὀρνόλην – allata est aqua ad faciem in urceolum" – "he washed his face with soap and warm water"), scrub their teeth and gums ("ὀδόντας ἔτριψα καὶ οὖλα – dentes fricui et gingivas" – "he brushed his teeth") and leave for school under escort ("ἑτοιμασθεὶς οὖν εἰς πάντα, προῆλθον καλῇ κληδόνι, ἀκολουθοῦντός μοι παιδαγωγοῦ – paratus ergo in omnia, processi bono auspicio, sequente me paedagogo" – "Mother Cat walked with Huckle to the school bus stop"), etc. Apart from timeless habits, the text tells us very little about its meaning and purpose and nothing at all about its origin. However, as convincingly argued by its editor, Dickey 2012, p. 227 on stylistic grounds, the "colloquium Stephani" looks like a Western piece no later than the fourth century AD, quite possibly, a schoolbook excerpt.

treating him like one and object that – even if the existence of such linguistic aids in the Latin West as well as their circulation in Boethius' time and age, are more or less uncontroversial – we have no definite proof that he ever used them or that he ever felt the need to consult anything of the sort. A man of Boethius' upbringing, status and means simply did not need them to straighten out petty linguistic issues. Although we can confidently dismiss as mere fiction anecdotal reports about aid from Boethius' alleged associates – like, for example, the story that one Flavius Theodorus Dionisii, a distinguished trainee of Priscian's turned civil servant, "assisted Boethius with his translation of Aristotle's *Categories*" (or any other translation, for that matter)[11] – there's nothing wrong with the idea itself. Had he wished, Boethius would have been able to get the best linguistic advice that family, friendship, influence and money could buy. It simply defies imagination to think that a man of his wealth, cultural background and political connexions both in Rome and Constantinople would have had any problem finding out what precisely *ψῆφος* stood for – had he put his mind to it, that is[12].

5. "'Calculum ponere' *eigentlich* 'die Rechensteinchen aufs Rechenbrett setzen'"[13]

All of the above is merely academic, however. Boethius knew exactly what *ψῆφος* meant and what it was used for – abacus reckoning, what else? As it happens, we

11. Cf., e.g., Bjornlie 2013, p. 135 for a lively and up-to-date version of this particular fairy tale – with kindred scholarship and tentative evidence. For the record and future reference, the anonymous subscription – if genuine, as opposed to being a forgery by some unknown scribe (as Cameron 2011, p. 433-434 suspected) – namely, Paris, Bibliothèque nationale de France, Nouvelles acquisitions latines 1611, 51r: "contra codicem Renati viri spectabilis correxi, qui confectus ab eo est Theodoro antiquario qui nunc Palatinus est [I corrected <my copy> against a manuscript owned by the illustrious Renatus and copied by Theodorus the copyist who is now an official at court]" (cf. https://gallica.bnf.fr/ark:/12148/btv1b10037292c/f53) has little to do with Boethius' translation of Aristotle's *Categories* anyway, for it occurs as an afterthought to the colophon of one of his logical opuscula, that is Boethius, *De Hypotheticis Syllogismis* (for this and other subscriptions, see Zetzel 1981, p. 219-220).

12. The most likely scenario, I surmise, is Boethius' family by alliance. By the time Boethius got around to translating the *Sophistici Elenchi* – his collaboration with Symmachus was already tried and proven: all Boethius had to do was ask for Symmachus' expert opinion. Nothing out of the ordinary, there – that is, nothing Boethius had not already done in the past, most notably when he turned to his illustrious relative for advice about his first literary effort, a loose translation of Nicomachus of Gerasa's *Ἀριθμητικὴ εἰσαγωγή* (Boethius, *De Institutione Arithmetica*, 2-3). His foster father (and eventually father-in-law) – who, as Boethius himself put it, was "most proficient in both Greek and Latin" and a great patron of Byzantine scholars to boot – would certainly have indulged him and solved the riddle without breaking a sweat. If, for whatever reason, Boethius did not feel like asking Symmachus himself, he surely would not have had to look far in the company of trusted friends (the "honestissimorum coetus amicorum" he's reminded of in Boethius, *De Consolatione Philosophiae*, I, 4, 40, ed. Moreschini, p. 17, ll. 137-138) where, a few years earlier, he had learnt enough Greek to set his mind on translating word-by-word no less than all of Aristotle's works and of Plato's dialogues (Boethius, *Commentarii in Librum Aristotelis ΠΕΡΙ ΕΡΜΗΝΕΙΑΣ. Editio secunda*, ed. Meiser, pp. 79-80, ll. 9-9).

13. Gruber 2006, p. 188.

do not even need to look for bits of classical lore Boethius might have gleaned out of Roman golden and silver age literature – which, of course, one is going to dig out, eventually. In this particular instance, Juvenal – only to pick up a household name from a large pool of usual suspects[14] – will conveniently provide a solid precedent.

Juvenal is as good a classic as any and, if you ask me, a lot more fun than most. Boethius must have kept a copy of his satires in his prison's library[15] – the obvious choice to lift his spirit after things had definitely taken a turn for the worse:

14. Even if, for the time being, one leaves Cicero (Cicero, Marcus Tullius, *De Amicitia*, 58, ed. Powell, p. 345, ll. 18-26), Livy (Livius, Titus, *Ab Urbe Condita*, V, 4, ed. Ogilvie, p. 325, ll. 5-18) and a few others aside, Juvenal would still be in excellent company with the like of Seneca and Petronius who also used the counters metaphor as a vivid reminder of how decent people cope with grief and loss (the former) or put up a brave face when they are past consoling (the latter). Since Boethius – for reasons unknown – related more directly to Seneca's existential predicament (Boethius, *De Consolatione Philosophiae*, I, 3, ed. Moreschini, p. 10, ll. 28-37) and foreseeable outcome of falling out of royal favour (III, 5, p. 69, ll. 27-34), let's Nero's mentor speak for himself first – we'll go back to Petronius in a moment, for our own pleasure. Seneca, Lucius Annaeus, *Consolatio ad Polybium*, 9, 1, ed. Reynolds, p. 275, ll. 20-26: "illud quoque magno tibi erit levamento, si saepe te sic interrogaveris: utrumne meo nomine doleo an eius qui decessit? Si meo, perit indulgentiae iactatio et incipit dolor, hoc uno excusatus quod honestus est, cum ad utilitatem respicit, a pietate desciscere; nihil autem minus bono viro conuenit quam in fratris luctu **calculos ponere**. Si illius nomine doleo, etc. [Hine 2014, p. 88: you will also find it a great relief if you frequently ask yourself, am I grieving on my own account or on the deceased's account? If on my own, my display of devotion is meaningless; grief is only justified when it is honourable, so it begins to part company with love when it takes self-interest into consideration; and when it comes to mourning for a brother, nothing suits a good man less than being calculating. If I grieve on his account, etc.]". Sure enough, Seneca's advice hit a bit too close to home for Boethius-the-prisoner's comfort – after all no one, trained philosophers least of all, fancies being reminded that he's grieving for himself. That being said, the abacus counters image must have stuck with Boethius who, in [T3] below, will resort to the same metaphor in order to convey the idea that bereavement and accounting do not belong together. As one may expect from Petronius Arbiter, his reference to the abacus is a model of clarity and elegance: "nomen amicitiae si quatenus expedit, haeret; / calculus in tabula mobile ducit opus. / Dum fortuna manet, vultum servatis, amici; / cum cedidit, turpi vertitis ora fuga [if what we call friendship stays true to its meaning only for as long as one benefits from it, then it is like a counter doing volatile work at the board. While my fortune holds, you – my friends – you stick around; as soon as I am out of luck you shamelessly turn tail and run]" (Petronius, Gaius Arbiter, *Satyrica*, 80, ed. Schmeling, p. 236, ll. 1-4). While it is just possible that Petronius was consciously playing with two metaphors at once (the gaming board's, on the one hand, and the abacus', on the other), *quatenus expedit* fits better the accounting metaphor, which then runs along the same lines of Cicero's false friendship (referred to above) and its calculated balance between profits and losses as displayed by the counters on the counting board. Although she points out the ambiguity of the "calculus in tabula" analogy, Connors 1998, pp. 80-81 eloquently translates as we did in her footsteps "the calculating pebble does volatile work at the board". For the record, Habermehl 2006, p. 28 and Schmeling / Setaioli 2011, p. 339 rather favour the ludic version of the simile.

15. "The Highway Rat". As a matter of fact, one of Boethius' prose pieces is quite possibly – in fact, almost certainly – purposely reminiscent of the poet's tenth satire: "atqui divitiae possidentibus persaepe nocuerunt, cum pessimus quisque eoque alieni magis avidus quicquid usquam auri gemmarumque est se solum qui habeat dignissimum putat. [34] Tu igitur, qui nunc **contum gladiumque** sollicitus pertimescis, si vitae huius **callem vacuus viator intrasses, coram latrone cantares** [Watts 1999, p. 36: but wealth very often does harm its owners, for all the most criminal elements of the population who are thereby all the more covetous of other people's property are convinced

[T2] Iuvenalis, *Saturae*, IX, ed. Morton Braund, 38-41: "quod tamen ulterius monstrum quam mollis avarus ? 'haec tribui, deinde illa dedi, mox plura tulisti'. Computat et cevet. Ponatur calculus, adsint cum tabula pueri; numera sestertia quinque omnibus in rebus, numerentur deinde labores [Morton Braund 2004, pp. 353-355 slightly modified: yet what monstrosity is worse than a stingy pervert? 'I paid you this, then I gave you that, and later you got still more'. He computes it while wiggling his arse. All right, let's set out the counters, call in the lads with the reckoning board: count five thousand paid in total and then let's count up my exertions]".

Truly, no good deed ever goes unpunished; and Juvenal's ranting character is not the only one who – going through a rough patch – feels more than a little cheated for all his efforts and hard labour[16]. Boethius too must have felt he came up short despite his good will and moral integrity. More to the point, the idea of doing some existential reckoning might well have crossed his mind, for Philosophia set out to dismiss the whole notion of him playing accountant with personal profits and losses:

[T3] Boethius, *De Consolatione Philosophiae*, II, 3, ed. Moreschini, p. 35, ll. 35-38: "visne igitur cum Fortuna calculum ponere? Nunc te primum liventi oculo

that they alone are worthy to possess all the gold and precious stones there are. You are shuddering now at the thought of club and knife, but if you had set out on the path of this life with empty pockets, you would whistle your way past any highwayman]" (Boethius, *De Consolatione Philosophiae*, II, 5, 33-34, ed. Moreschini, p. 45, ll. 94-100) – "pauca licet portes argenti vascula puri nocte **iter ingressus**, **gladium contumque** timebis et mota ad lunam trepidabis harundinis umbra: **cantabit vacuus coram latrone viator** [Morton Braund 2004, pp. 367-369: though you're carrying only a few cups of plain silver when you set out on a journey at night, you'll be terrified of swords and sticks, and you'll panic at the twitch of a reed's shadow in the moonlight. A traveller who is empty-handed can sing in the mugger's face]" (Iuvenalis, *Saturae*, X, ed. Morton Braund, 19-22). A Juvenalian overtone is discernible in other passages, though less conspicuous – compare, for instance, Boethius, *De Consolatione Philosophiae*, IV, 5, 11-12, ed. Moreschini, p. 120, ll. 36-37 with Iuvenalis, *Saturae*, VI, 441-442. Juvenal's fourth-century revival is a matter of some controversy, depending upon how much stock one is willing to put in Servius (either as the main inspirator or one of several witnesses – along with Ammianus, Ausonius, Claudian, Prudentius, et alii – of the renewed general favour Juvenal was enjoying amongst well-read people): in a nutshell, are Servius' eighty-odd quotations from Juvenal mostly derivative (cf. e.g. Cameron 2011, pp. 452-453) or do they imply a marked interest in the great poet of the silver age (*ab una disce alios*: Monno 2009)? *Grammatici certant et adhuc sub iudice lis est* – even if it seems more plausible to assume that Juvenal was pretty much in the air du temps and needed no rescue when Servius' came of age. Be that as it may, by Boethius' era Juvenal's reputation had been firmly re-established for some time, and this is enough for the sake of our argument.

16. Unsurprisingly, Naevolus' postprandial ploughing and his longing for wealth (which – as Saller 1983 suggested – might just be a trifle more reasonable than generally assumed) and a better status than his current one (he describes himself as a *bipes asellus* – which, in his line of work, might be a trifle less self-deprecating than one may think at first) seem to have got everyone's undivided attention. As a result, the abacus is hardly mentioned at all (Courtney 1980 scarce remarks, pp. 431-432, are the – not so exceptional – exception). At any rate, *Satura* IX has got more than its well-deserved share of excellent scholarship: cf. e.g. Bellandi 1974; Cecchin 1982; Braund 1988, pp. 130-177; Notter 2008; etc.

praestrinxit. Si numerum modumque laetorum tristiumve consideres, adhuc te felicem negare non possis [and now you want to set out the counters <and square accounts> with Fortune? now that, for the first time, she has cast a malicious eye on you. If you were to sum up the number and fashion of things that brought you either joy or sorrow, you could not deny that you've been happy so far]".

The gist of Philosophy's rebuke is as powerful as it is plain: Boethius should count his blessings before he sits with Fortune at the abacus table and let the counters do the math. He's done very well for himself, up to that point – and the counters would certainly show that there's nothing wrong with his balance, so far.

Latin commentators had the truth of it, for they consistently endorsed the accounting image in its most ordinary and most familiar sense (as opposed to resorting to more subtle readings)[17]:

[T4] Guilelmus de Conchis, *Glosae super Boethium. Accessus ad Consolationem*, II, 3, ed. Nauta, p. 109, ll. 38-44: "visne igitur quandoquidem tot bona tibi contulit fortuna, cum ea calculum ponere id est computare? calculi sunt brevissimi lapides dicti a calcando. Sed quia in abaco sunt quidam caracteres ad modum aliorum calculorum qui ponuntur in abaco ad computationem faciendam, inolevit consuetudo ut calculum ponere diceretur pro computare [since indeed Fortune has granted you so many good things, 'now you want to' set out the counters with her, that is to square the accounts? 'calculi' are small stones which get their name from being tread upon. Since there are markings on the abacus which allow one to use a stone as if it had the same value as several others that are set on the counting board for the reckoning, it has become a habit to use 'to set out the counters' as a synonym for 'reckoning']".

[T5] Nicolaus Trevetus, *Expositio Super Boethio De Consolatione*, II, 3, 37, ed. Silk, p. 212, ll. 12-16: "visne igitur cum Fortuna calculum ponere id est computare ? calculus est parvus lapillus qui calcando non sentitur et quia talibus lapillis utebantur antique in computando pro uno solido ponendo lapillum unum pro duobus duos, ideo ponere calculum vel calculare idem est quod computare ['and now you want to set out the counters with Fortune?', namely to square accounts with her? a 'calculus' is a small stone which we do not feel under our feet

17. "As if four eyes were better than two". Practical and down-to-earth though it may be, the mediaeval understanding of Boethius "chance reckoning" is so far ahead of the modern competition that it hardly bears comparison with rival solutions, e.g. contemporary statistical drivels in the same vein as, say, Lüthy / Palmerino 2016. For all their supposed faults, mediaeval readers – the whole lot of them – had at least one redeeming quality: even the more biased ones read the text before having it say whatever their agenda called for. Lüthy / Palmerino 2016, on the contrary, simply do not have a clue about what the text is actually about – starting with who's talking to whom: "In Boethius' *Consolation* (II.3p), we have Fortune itself asking defiantly: 'do you wish to count out the score with Fortune?' (*visne igitur cum fortuna calculum ponere ?*). Through the mathematization of probability, we are attempting to do just that: 'reckon with fortune' etc." (Lüthy / Palmerino 2016, p. 17). Best of luck with that!

when we tread upon it. Since in ancient times such pebbles were used as counters – for instance, setting a pebble for a shilling, two pebbles for two shillings – it follows that 'to set out the pebbles' or 'to calculate' and 'to reckon' mean the same]".

For present purposes, William of Conches' and Nicholas Trivet's etymological concerns are of no great consequence. Suffice it to say that, as proven by recent developments, their views are as sound (or as far-fetched) as they come[18]. On the other hand, it is definitely worthwhile noting that both glosses – which, needless to say, belong to popular collections (both in their own right and in various revised forms)[19] – treat as a matter of course the association of *calculi* with ancient and traditional instruments and methods of calculation as well as (and the detail is far from being insignificant) most ordinary reckoning, that is accounting or computations involving money. Typically unafraid of stating the obvious, both commentators laid stress on two essential features, which spring to mind when Latin speakers (or Latin writers) encountered the word *calculus* or availed them-

18. As a matter of fact, traditional views on *calculus* etymology have been questioned and a new consensus has been gathering momentum for some time. Since Loicq 1960, the Latin words *calculus* and *calx* have steadily drifted apart (in particular, the former is not considered a diminutive of the latter any more). Moreover, Meid 2012, p. 150, note 9, has suggested an etymology along the lines of the pre-Indo-European root '*kar- / *kal-' 'stone' (most notably friable, calcareous rock eroded by water as pointed out by Alessio 1935-1936), whose reduplicated form '*kal-kal (= pile of stones)' occurs in 'calculus' (with a collective connotation which, by the way, has not been lost to specialists, cf. e.g. André 1978, p. 55). As a healthy memento of the old etymological rule of thumb ("vowels matter naught and consonants hardly at all"), Perono Cacciafoco 2015, p. 122 reminds us in an exquisite scientific english (no capital needed here) that "According to Villar, completing the Tovar discourse, in any case, it is questionable the opinion inherent in a possible pre-Indo-European origin of these roots".

19. Few late ancient texts have enjoyed throughout the Middle Ages (with the partial exception of the thirteenth century, that is) as much favour as Boethius' *Consolation of Philosophy*. A number of book-length studies bear witness to its mediaeval fortune: Courcelle 1967; Kaylor 1992; Hoenen / Nauta 1997; Glei / Kaminski / Lebsanft 2010. For readers in a hurry, King 2007 or Nauta 2009 should suffice. More to the point, on William of Conches' commentary, cf. Nauta 1999, pp. xv-cxxviii and on Nicholas Trivet's, cf. Brancato 2012, pp. 363-365. Considering that we know very little about its date of composition and the whereabouts of its author, we'll leave out, for the time being, the anonymous commentary sometimes ascribed to Thomas Aquinas (or to William Wheatley). All the same, its understanding of the issue at hand is as sound as William's and Nicholas': "nota, quod calculus in una significatione est parvus lapillus qui calcando non laedit. Et quia talibus lapillis utebantur antiqui in computando, ideo calculare vel calculum ponere ponitur pro computare vel pro rationem facere. Ergo dicit philosophia: vis ne cum fortuna calculum ponere? Quasi dicat: non debes: si computabis cum ea, ipsa inveniet te multo feliciorem quam miserum [you will notice that one of the meanings of 'calculus' refers to a stone that, when we tread on it, does not hurt <our feet>. Because in ancient times pebbles of the sort were used for reckoning, for that reason, the expression 'to calculate' or 'to set the counters' means 'to count' or 'to account for'. As if Philosophy were saying: you shouldn't. If you take up the counters with Fortune, she'll find out that you've had a lot more things to be happy about than to be sorry about]" (Ps. Thomas de Aquino, *In Boethii De Consolatione Philosophiae*, ed. Busa, p. 40b, ll. 27-34). Little has been published on the anonymous commentary (Courcelle 1967, pp. 322-323; King 2007, pp. 46-47; Lucia 2012) – a semi-published ANR by-product (Galonnier 2017) is the next best thing; and it would have been pretty good indeed, had it not suffered from a bad case of funded research deadline-frenzy by proxy.

selves of it, namely: (1) where we find *calculi* (on the counting board, with different values according to their different positions as possibly marked on the abacus itself) and (2) what we use them for (as symbols for what we count: first and foremost, coins).

6. Smoking gun

All of which brings us to our most compelling piece of evidence – internal evidence, that is. From the looks of it, Boethius did not care much for *ψῆφοι*. Indeed, on the few occasions he came across the word, it invariably got lost in translation, for Boethius came up every time with a different solution – other than simply calling a pebble a pebble, that is. For instance (and this is the only other example one will come by in his translation of the *Sophistici Elenchi*), Boethius must have thought that silver coins would catch the reader's imagination more than plain stones or tokens, for he translated *ψῆφος* as *denarius* in a well-known Aristotelian example of the fallacy of figure of speech:

> [T6] Arist., *De Sophisticis Elenchis*, 22, 178b11-13: "οἱ δὲ ὡς καὶ ὃ ἔχει ἔλαβεν· ἐδίδου γὰρ μίαν μόνον οὗτος ψῆφον·καὶ οὗτός γ' ἔχει, φασί, μίαν μόνον παρὰ τούτου ψῆφον [according to others, <the solution is> as in: what one possesses, he has received it. A man has given just one pebble to another, so – they say – this is what the latter has got, for he has received just one pebble from the former]".

> [T6B] Arist., *De Sophisticis Elenchis Translatio Boethii*, ed. Dod., p. 45, ll. 4-7: "quidam vero et ut quod habet accepit; dedit enim unum solum hic denarium; et hic habet, dicunt, unum solum ab hoc denarium; accepit enim ab hoc".

After all, it's no skin off anyone's nose whether someone gives someone else a rock or a chip – or more than one, for that matter. On the contrary, as soon as money changes hands, it's a different story altogether, even if we're talking small change here[20]. Be that as it may, [T6B] is as strong an indication as any that the tangible nature of *ψῆφοι* was all but lost to Boethius who was well aware that they could be handled (or mishandled, for that matter) as easily as

20. The same explanation (which is more of a suggestion) fits nicely a similar use of *digitus* as a translation for *ἀστράγαλος*: "εἰ ὅ τις ἔχων ὕστερον μὴ ἔχει, ἀπέβαλεν· ὁ γὰρ ἕνα μόνον ἀποβαλὼν ἀστράγαλον οὐχ ἕξει δέκα ἀστραγάλους [Hasper 2013: if someone possesses something and he does not possess it later on, he has lost it. In fact, someone who has lost just one knucklebone does not have ten knucklebones]" (Arist., *De Sophisticis Elenchis*, 22, 178a29-31; cf. 22, 179a21-22) – in Boethius' words: "si quod quis habens postea non habet amisit; nam unum solum amittens digitum non habebit decem digitos" (Arist., *De Sophisticis Elenchis Translatio Boethii*, ed. Dod., p. 44, ll. 13-15; cf. p. 46, ll. 24-25 where *denarius* translates *ἀστράγαλος*). At the end of the day, who cares if we are left with no dice to toss around... whereas it makes a heck of a difference whether you get to lose all your fingers or manage to keep most of them attached to your hands!

coins. In hindsight then, how could he have possibly got the *ψῆφοι* wrong? Let me spell it out for you: what is roughly the size of a coin and comes in handy when we have to work figures out? What else could it be, if not a reckoning-stone? My point exactly.

7. *Caveat*

While a good deal of the aforesaid is, if not indisputable, at least hardly controversial, is it enough to prove that Boethius dispensed with the word *calculus* on purpose and consciously omitted all reference to counters, counting boards and accounting still much alive, several centuries later, as attested by [T4], [T5] and cognate materials?

Probably not – on two counts.

First of all, even if one were to prove that Boethius could hardly ignore that *ψῆφος* in its most ordinary sense meant pebble or reckoning stone, this will get him only half way there. One still has to deal with the possibility that Boethius was aware of more than one meaning for the word *ψῆφος* – possibly "number" or "numeral"[21]. Accordingly, it is just possible that Boethius happened to learn some-

21. This is definitely a possibility, as suggested by slightly later Byzantine sources. Theophanes Confessor, for instance, recorded that, the same year the Umayyad Caliph expropriated the holy Cathedral of Damascus, he also replaced Greek with Arabic as the language of administration, except for the mention of numbers, that is: "καὶ ἐκώλυσε γράφεσθαι Ἑλληνιστὶ τοὺς δημοσίους τῶν λογοθεσίων κώδικας, ἀλλ' ἐν Ἀραβίοις αὐτὰ παρασημαίνεσθαι, χωρὶς τῶν ψήφων, ἐπειδὴ ἀδύνατον τῇ ἐκείνων γλώσσῃ μονάδα ἢ δυάδα ἢ τριάδα ἢ ὀκτὼ ἥμισυ [...] γράφεσθαι· διὸ καὶ ἕως σήμερόν εἰσι σὺν αὐτοῖς νοτάριοι Χριστιανοί [Mango / Scott 1997, p. 524: <al-Walid, that wretched man> also forbade that the registers of the public offices should be written in Greek; instead, they were to be expressed in Arabic, except for the numerals, because it is impossible in their language to write a unit or a pair or a group of three or eight and a half [...]. For this reason they have Christian notaries until this day]" (Theophanes Confessor, *Chronographia*, ed de Boor, p. 376, ll. 2-7 – textual and authorship-related issues are discussed in some detail in Jankowiak / Montinaro 2015, part I and II). In a different vein altogether but around the same time (give or take a few decades as tentatively suggested by Wheatley 1996 and Bugar 2016), Pseudo-Hyppolitus' eschatological ruminations involved numbers in the form of tattoos – ψῆφοι as it happened: "ἡ δὲ σφραγὶς αὐτοῦ ἐπὶ τοῦ μετώπου καὶ ἐπὶ τῆς δεξιᾶς χειρός ἐστι ψῆφος χξς. καὶ ὡς οἶμαι οὐδὲ ἀκριβῶς ἐπίσταμαι τοῦτο, κτλ. [the seal of the deceiver upon the forefront and the right hand is the number six hundred sixty-six. I kind of surmise that this is the case, but I do not know precisely, etc.]" (Ps. Hippolytus, *De consummatione mundi*, ed. Athanasopoulos, p. 28, ll. 16-17). On the other hand – for what it's worth – some six centuries after Boethius, Michael of Ephesus, despite being about as [arithmetically biased] as one can get, kept counters (*ψῆφοι*) and numbers (*ἀριθμοί*) within their respective semantic boundaries in his explanation of Aristotle's analogy. As a matter of fact, whilst digital dexterity and abacus expertise played no part in Michael of Ephesus' commentary either, Aristotle's mention of pebbles – as opposed to plain numbers – was, if not particularly meaningful, at least linguistically transparent: "χρᾶται δὲ τοῖς ἀριθμοῖς καὶ ταῖς ψήφοις πρὸς τὸ δεῖξαι τοὺς σοφιστὰς τῷ τὰ ὀνόματα μεταφέρειν ἐπ' αὐτὰ τὰ πράγματα παραλογιζομένους τοὺς ἀνεπιστήμονας. ὡς γὰρ ἐπὶ τῶν ψήφων οἱ μὴ δεινοὶ ἀλλ' ἐπιπόλαιοι καὶ ἰδιῶται τῷ τοὺς ἀριθμοὺς φέρειν ἐπὶ τὰ ἀριθμούμενα παρ' ἑαυτῶν καὶ παρὰ τῶν περὶ τὰς ψήφους καὶ τοὺς ἀριθμοὺς ἐπιστημόνων ἀπατῶνται καὶ διὰ τοῦτο

how that, figuratively, *ψῆφος* could possibly mean number and thought that – for some reason – *numerus* made better sense in the context of the prologue of the *Sophistici Elenchi*.

Second, one can only go so far with circumstantial evidence. What of Boethius' motives, if any? Without establishing probable cause to begin with, what's the point of discussing any further the facts of the case? What proof do we have that Boethius actually had Aristotle's analogy say what he wanted it to say, namely something along the lines of his own views on how language and computation – if they're connected at all – stand in relation to one another?

8. Pebble in the shoe

Count one is a bit of a moot point: whatever the right answer to the question turns out to be – the question itself has no direct bearing on how things went down in the Latin tradition insofar as the issue never came up for discussion. Boethius handed down his translation without much in the way of instructions for use. As a result, Latin commentators were no more privy to Boethius' mind and intention than we are and, for all practical purposes, it makes no difference whether or not he left the pebbles out of his translation by design or by accident. Moreover, it is not as if Mediaevals had any reason to suspect that something was missing in Boethius' translation, which made perfect sense as it stood. Unconcerned and, for the most part, unable to ascertain whether Boethius' Latin squared with Aristotle's Greek and to what extent, they trusted Boethius implicitly and took his translation at face value.

καὶ ὑπὸ τούτων παρακρούονται, τὸν αὐτὸν δή φησι τρόπον ἔχειν καὶ ἐπὶ τῶν λόγων τῶν ἐξ ὀνομάτων· οἱ γὰρ ἀξύνετοι καὶ ἀνεπιστήμονες τῆς τῶν ὀνομάτων δυνάμεως κακῶς διαλεγόμενοι παραλογίζονται καὶ αὐτοὶ διαλεγόμενοι καὶ ἄλλων ἀκούοντες [Aristotle introduces numbers and counters in order to explain how the sophists mystify those who lack knowledge by transposing words into the facts themselves. Just as those who are not skilled with counters, being careless and unaware of how numbers are related to the things they stand for, deceive themselves and are deceived by those who know their way around counters and numbers and, for the same reason, are misled by the latter, the same goes – Aristotle says – for arguments made out of words. In fact, the witless and those who pay no heed to the power of words make fools of themselves both when they take part into a discussion and when they listen to others]" (Ps. Alexander Aphrodisiensis (Michael Ephesius), *In Aristotelis Sophisticos Elenchos Commentarius*, ed. Wallies, p. 13, ll. 20-29). Even if Michael of Ephesus is no longer the unfathomable character he was before Browning 1962 and Ebbesen 1981, I, pp. 268-285 shed some much-needed light on his association with Princess Anna Comnena's philosophical circle and his style as an Aristotelian commentator, he still is an elusive figure – and he will be for the foreseeable future, as suggested by recent efforts to extract more information from the scanty historical data in our possession: cf. Golitsis 2018; Wilberding / Trompeter 2018; Trizio 2019 (which is both a lesson in sobriety against reading too much into known sources and a useful reminder that known manuscript material is always worth a second look).

9. Motive and opportunity

Count two is a different beast altogether. Whether or not Boethius' translation reflects first and foremost his own understanding of how calculations never fail while language seems to work only half of the time is a legitimate concern for the historian. As it happens, we might just have what we need to settle the issue: Sten Ebbesen put the pieces of this particular jigsaw together a while ago[22]. We'll have to take it from there and solve the problem accordingly. As observed time and again in Ebbesen's wake[23], Boethius' translation of the Aristotelian prologue of the *Sophistici elenchi* bears an uncanny resemblance to his celebrated account of the origin of logic – or, rather, let's put it the other way around, as we should: Boethius' celebrated account of the origin of logic bears an uncanny resemblance etc.

[Ur-Text^{B}] Arist., *De Sophisticis Elenchis Translatio Boethii*, ed. Dod., pp. 5-6, ll.14-13: "eodem autem modo et syllogismus et elenchus [15] hic quidem est, ille vero non est quidem, videtur autem **propter** [16] **imperitiam**; nam imperiti velut distantes longe speculantur. [17] Nam syllogismus quidem ex quibusdam positis est ut dicatur [18] diversum quid ex necessitate ab his quae posita sunt, elenchus [6.1] autem syllogismus cum contradictione conclusionis. Illi vero hoc [2] quidem non faciunt, videntur autem ob multas causas, quorum [3] unus locus aptissimus est et publicissimus per nomina. Nam [4] quoniam non est ipsas res ferentes disputare, sed nominibus pro [5] rebus utimur notis, **quod accidit in nominibus in rebus quoque** [6] **arbitramur accidere**, velut **in compotis ratiocinantibus**. Hoc [7] autem non est simile. Nam nomina quidem finita sunt et [8] orationum multitudo, res autem numero infinitae. Necesse est ergo [9] plura eandem orationem et nomen unum significare. Quemadmodum [10] igitur illic qui non sunt prompti **numeros ferre** a scientibus [11] expelluntur, eodem modo et in orationibus qui nominum [12] virtutis sunt ignari paralogizantur et ipsi disputantes et alios [13] audientes" (quoted – and translated – above).

[T7] Boethius, *In Isagogen Porphyrii Commentum. Editio secunda*, I, 2, ed. Brandt, p. 138-139, ll. 15-1: "ut in multis evenit Epicuro, qui atomis mundum

22. Ebbesen 1981, I, p. 253: "apparently the *Elenchi* passage, in which it is explained how the imperfect correspondence between words and things deceives people, was a famous one in late Antiquity, for Boethius, too, draws on it in his Second Commentary on the *Isagoge*. Lack of experience in the art of dialectic, he says <*In Isag.*, ed. 2a, I, 138 Brandt>, was responsible for the errors committed by Epicurus and his like who thought that facts about reality could be directly inferred from a consideration of expressions. They were wrong, Boethius continues, because the way words are related to things is not like the way numbers are. If you count correctly, using the fingers or an abacus, you can be sure that if the resulting number is one hundred the things that underlie the figure are a hundred. Not so with words: when you reason by means of words you may find something which is not matched in nature. This is clearly a paraphrase of 165a 6-17". How accurate Boethius' paraphrasis actually was is, of course, another puzzle (ours to solve, as it happens), which however cannot be unravelled unless we take our cue from Ebbesen.

23. Cf. e.g. Magee 1989, p. 122; Suto 2012, p. 47, note 22; Crialesi 2020, p. 113.

consistere putat et honestum voluptatem mentitur. Hoc autem idcirco huic <scil. Epicuro> atque aliis accidisse manifestum est, quoniam **per imperitiam** disputandi **quicquid ratiocinatione comprehenderant, hoc in res quoque ipsas evenire arbitrabantur**. Hic vero magnus est error; neque enim sese ut in numeris, ita etiam in ratiocinationibus habet. **In numeris** enim quicquid in digitis recte computantis evenerit, id sine dubio in res quoque ipsas necesse est evenire, ut si ex calculo centum esse contigerit, centum quoque res illi numero subiectas esse necesse est. Hoc vero non aeque in disputatione servatur; neque enim quicquid sermonum decursus invenerit, [139] id natura quoque fixum tenetur [<this kind of errors> occurs often in Epicurus, for he thought that atoms make up the universe and he falsely claimed that pleasure is a virtue. The reason why this happened to him as well as it happened to others is clear: because they all thought – as a result of their lack of experience in the art of argumentation – that whatever conclusion they reached by way of reasoning, the conclusion also applied to the things themselves. This is utterly mistaken. As a matter of fact, one thing is what happens when numbers are involved and another thing is what happens when arguments are. As a matter of fact, as far as numbers are concerned, whatever is the result one reaches when he reckons right on his fingers, there's no doubt that the same result must apply to the things themselves as well. For instance, if one hundred happens to be the result of one's reckoning, then the things matching those figures must also be one hundred. On the other hand, when we argue, things do not run as smoothly. As a matter of fact, it is not the case that whatever the path of an argument leads us to, it is also what we are to assume the natural order of things ends up with]".

Boethius' explanation of why we need logic in the first place ([T7]) is a remarkable piece of reverse engineering. It conveys the main idea of its Aristotelian blueprint ([Ur-Text^{A}]); moreover, it develops it according to the general pattern Aristotle laid out himself: poor dialectical skills and training lead to excessive trust in the ability of language to tell things as they are. Overconfidence in words is easily misplaced and, more often than not, it welcomes deception, error, misjudgement – you name it. That being said, even if [Ur-Text^{A}], [Ur-Text^{B}] and [T7] make the same general point (excessive reliance on words is a recipe for disaster), they marshal different facts in order to build their case. While Aristotle's argument in [Ur-Text^{A}] brings as close together as possible two distinct sets of symbols (abacus counters and words) insofar as they share the same liability (they are both prone to subtle but critical shifts in value and meaning), Boethius' translation ([Ur-Text^{B}]) and his repurposing of Aristotle's original design ([T7]) drive as far apart as possible two kinds of ratiocination (*ratiocinatio*, *ratiocinans*), namely calculation (*compotus*, *computans*) and debate (*disputare*, *disputans*), on account of the opposite ways numbers (*numeri*) and words as well as word-compounds (*nomina*, *orationes*) match the things we refer to when counting or debating. It then becomes immaterial to ask whether or not Boethi-

us' cautionary tale ([T7]) about arithmetical success[24], as opposed to discursive failure[25], sheds any light on Aristotle's analogy between computational and disputational hazards. In a nutshell, Aristotle was not so much interested in comparing why, on the one hand, everything adds up when we count right and why, on the other hand, things go awry when arguments misfire, as he was interested in comparing why (and how) both arithmetical reckoning by means of counters and verbal reckoning by means of words fail when we mishandle the symbols involved (counters and words, respectively). Which is, needless to say, a different story altogether. At any rate, it is not the story told by Boethius' translation ([Ur-Text^{B}]), together with his piece on the origin of logic ([T7]) – simple as that.

10. *Sententia latinorum* (*potius orthodoxorum*)

Again, Latin commentators had the truth of it – very early at that[26]. Two of the earliest extant witnesses provide the gist of Boethius' legacy and deliver its one-two punchline, namely "once you take the abacus counters out of the equation and make the analogy about numbers and calculations as such, it is the whole analogy that falls apart" – let's call them [ARITHMETIC BIAS] and [DISANALOGY BIAS] for short:

> [T8] *Anonymi Glosae in Aristotelis Sophisticos Elenchos*, ed. DE RIJK, p. 199, ll. 1-17: "quoniam non est ipsas res dicere ferentes, id est res non possunt significari in disputatione nisi per voces; sed utimur nominibus pro rebus notis, idest notandis, quod accidit in nominibus, id est coerentia vel discoerentia, in rebus arbitramur accidere ad similitudinem numerorum. HOC AUTEM NON EST SIMILE <165a10>, quia, ut dicit Boethius <*In Porphyri Isagogen*, II, pp. 138-139, ll. 14-1>, si ex calculo centum evenerint, centum oportet res esse subiectas et quicquid in digitos recte computantis evenerit, id procul dubio in natura rerum fixum tenetur, sed non quicquid in concursu sermonum evenerit. [...]. Demonstrata dissimilitudine inter voces et numeros statim demonstrat similitudinem inter eadem dicens <165a3-17>: QUE-

24. Inasmuch as whatever reckoning number stands in an immediate and unambiguous relation with the reckoned things whose number it is, one has to work hard to get off track when figures are involved.

25. Insofar as many a word stands in an ambiguous relation to the things it signifies, one has got to work hard to keep on track when dealing with linguistic expressions.

26. I.e. as early as the mid-twelfth century. DE RIJK 1962, I, p. 83 convincingly dated [T8], the anonymous Parisian glosses to around that time. It took Sten Ebbesen's ingenuity and erudition to bring together [Ur-Text^{B}] and [T7] in recent times. On the other hand, there's nothing out of the ordinary about a Parisian Glossator of around Peter Abelard's generation doing so and quoting Boethius commentary on Porphyry's *Eisagôgé* in connection with Aristotle's *Sophistici elenchi*: Boethius' explanation of the "ortus logicae disciplinae [origin of logic as a discipline]" was a popular topic indeed, as Hugues of Saint Victor (HUGO DE SANCTO VICTORE, *Didascalicon*, ed. BUTTIMER, pp. 19-20, ll. 4-27) and John of Salisbury (IOANNES SARISBERIENIS, *Metalogicon*, II, 2, ed. HALL / KEATS-ROHAN, p. 58, ll. 10-21) bear witness.

MADMODUM IGITUR ILLIC id est in numeris, illi homines qui non sunt prompti ferre numeros, id est qui nesciunt numerare, expelluntur a scientibus, scilicet a ratione numerorum, eodem modo et in orationibus, id est quod sophistae expelluntur a sapientibus. Vel sic: quemadmodum illi numeri qui non sunt prompti ferre, idest qui non possunt ferre numeros, id est proprietatem propositi numeri, ut quinarius non potest ferre proprietatem binarii, expelluntur, idest reiciuntur a scientibus, eodem modo in orationibus expelluntur a sapientibus illi termini et orationes quae non possunt ferre proprietatem syllogismorum [since it is not possible to discuss things by carrying them around, that is to say since things cannot be argued about unless we use words in their stead, we avail ourselves of names as symbols in order to refer to the things we want to refer to. Therefore, what words bring about, namely whether what we speak about results in a correlation or lack thereof, we assume that the same follows concerning the things themselves as well, just like we do when numbers are involved. 'Still it is not the same', for – as Boethius explains – if one hundred happen to be the result of one's reckoning, then the things matching that figure must also be one hundred and, accordingly, whatever is the result one reaches when he reckons right on his fingers, there's no doubt that we are to assume that the same also follows in the natural order of things. Now, this is not what happens when we lay down the path of an argument. [...]. Once Aristotle has established the difference between numbers and words, he turns to their similarity and establishes it without delay: just as with numbers, those who are good at working out figures show those who are not, namely those who do not know how to reckon, that they are out of their depth when they handle figures, likewise those who know how to argue show that the sophists are out of their depth when they discuss. Or else: just as some numbers are unfit and cannot uphold other numbers, that is they do not display the property of a given number – for instance, five does not have the properties of an even number – and are cast aside, that is are rejected by those who are knowledgeable in these matters, in the same way some words and sentences cannot display the properties nor meet the requirements of a proper deduction and are cast aside by those who know how syllogisms work]".

[T9] *Anonymi Aurelianensis I Commentarium in Sophisticos Elenchos*, ed. EBBESEN, pp. 27-28, ll.14-13 and p. 29, ll. 1-12: "NAM QUONIAM EST <165a6>, id est contingit, disputare non ferentes, id est non proponentes ipsas res de quibus disputatur, sed utimur nominibus disputando pro rebus; [114b] cetera praedicto modo legantur. Nota ideo Aristotelem se communicasse ita male arbitrantibus ut arrogantiam vitet et se hac arte indigere ut alios notet. Quoniam, ut Boethius dicit in secundo commento super Porphyrium <*in Porphyrii Isagogen* II, I, 2, p. 138>, antequam ars ista esset tradita et remedium ad ipsam, singuli arbitrabantur in rebus accidere idem quod in nominibus, et ideo fere omnes fallebantur putantes quod dicitur de nomine dicendum esse de re, et ut nomen dicitur de nomine, sic res dicatur de re. VELUT IN COMPOTIS <165a9>, vel 'IN NUMERIS' secundum aliam translationem, quasi diceret: quoniam nominibus utimur gratia rerum, videtur [28] nobis accidere in rebus quicquid in nominibus, decepti similitudine numerorum, quibus similiter gratia rerum utimur, nam et in rebus quidem accidit fere quicquid in numeris, ut si numerus binarius et res duae, si numeri pares vel impares et res si-

militer, et tot res quot numeri, quod non accidit in orationibus; et hoc est 'VELUT', id est: sic arbitramur accidere in rebus ut in nominibus velut accidit ratiocinantibus in compotis id est in rationibus numerandi, ut scilicet illi inveniant fere eodem modo accidere in numeris et rebus. Compotus est numerandi ratio. HOC AUTEM NON EST SIMILE <165a10>, quod scilicet se **habeat in orationibus et rebus quemadmodum in numeris et in rebus**, etc. [...]. QUEMADMODUM <165a13>. Inconvenienter videtur inferre, cum dixerit non similiter esse in numeris et orationibus, id est **non similiter habere se numeros et orationes ad res**. Sed si quis bene inspiciat poterit hic notare locum a maiori hoc modo: **dixerat enim hoc non esse simile eo quod difficillimum sit falli in numeris**, cum in rebus eodem accidat modo, facillimum autem in orationibus, quare non est mirum si accidat falli in orationibus, cum etiam et in numeris accidat, et hoc est QUEMADMODUM IGITUR ILLI QUI NON SUNT PROMPTI FERRE NUMEROS, id est qui non sunt expediti in arte numerandi et in computatione numerorum, ut si quaeratur ab eis quanta sit summa istorum numerorum, scilicet XX.L.D.III.V, et nesciant reddere summam nisi detur tempus meditandi ['in fact, since it is', that is to say since we happen to argue without carrying around or putting on display the very things we talk about, but we use words instead of things in our discussions, the rest is to be understood in the aforementioned way. Now, do notice that Aristotle expressed himself thus lest those who are quick to misjudge accuse him of arrogance; take also notice that Aristotle wrote as if he lacked such an art himself, just like the others. Since, as Boethius explains in his second commentary on Porphyry, before the technique sophists resort to was handed down along with its antidote, people deemed that what happens with words also is the case for the things the words stand for. As a result, almost everyone was mistaken because they thought that whatever words allow to say of one another, the same applies to the things themselves and that, for the same reason, just as a given word is said of another, the things themselves are also said of one another. 'Just as with calculations' or, according to another translation, 'with numbers' as if Aristotle was saying: since we use words as substitutes for things, we are under the impression that things behave in the same way words do and we are deceived because of what happens with numbers, which we also use in a similar way as substitutes for things. As a matter of fact, things and numbers behave in almost exactly the same way. For instance, when a number is two or a multiple of two, then the things are two as well. Again, when numbers are either even or odd, then the things they stand for are too. Whilst there are exactly as many things as their number says there are, this is not what happens when we speak about things. And this is what 'just as' means here, namely we think that what happens with words also happens with things as this is the case for those who calculate with their reckoning, that is with numerical ratios, insofar as those who calculate find out that what happens with numbers is exactly identical to what happens with things. Reckoning here means numerical ratio. 'Still it is not the same', that is to say **linguistic expressions and things are not in the same relation as numbers and things**, etc. [...]. 'Just as', that is we are mistaken when we infer from words how things are in the same way we do it with numbers, for Aristotle has just said that the same does not apply to numbers and linguistic expressions alike, that is to say numbers and linguistic expressions do not stand in the same relation to things. If we read carefully, we'll dis-

> cern here an argument *a fortiori* as follows: Aristotle maintains that words and numbers are not alike insofar as it is **extremely hard to be wrong when numbers are concerned**, the reason being that what happens with numbers is identical to what happens with things; whereas it is very easy to be wrong when linguistic expressions are involved. Accordingly, there is nothing surprising when one makes a mistake involving linguistic expressions, for this can also happen when numbers are involved. And this is what Aristotle refers to when he says: 'just as in the case of those who aren't any good with figures', namely those who are not deft at counting numbers and reckoning figures, as when they are asked to say how much is twenty plus fifty plus five hundred plus three plus five and are unable to answer unless they are given some time to work the sum out]".

As [T8] and [T9] show, Boethius' translation ([UR-TEXT^{B}]) and Boethius' cue ([T7]), when taken together, are a mixed bag at best. In the former ([UR-TEXT^{B}]) there is no mention of computational symbols (pebbles) as analogical counterparts to linguistic ones (words both taken by themselves and compounded together) – most certainly, calculations (*compoti*), let alone numbers (*numeri*), are not symbols or, at any rate, they are not symbols in the sense stone counters and words are. In the latter ([T7]) the results we end up with when we reckon and the conclusions we reach when we argue are pitted against each other. Latin commentators followed Boethius on both counts: that is, they usually went along with the [ARITHMETIC BIAS] Boethius hardwired into them, namely that computations and numbers – Boethius' substitutes for Aristotle's counters – are what the Aristotelian analogy is all about, it being understood that therefore it is not an analogy at all ([DISANALOGY BIAS]). As a matter of fact, on Boethius – and the Boethian tradition's – terms, Aristotle's simile explains, to an extent, why we cannot expect linguistic expressions to behave in the same way numbers do, insofar as – precisely – the calculations involved are plain arithmetic ones, which can't go wrong as such. However, this does not help us much understand why both arguments and calculations rely on symbols and, more to the point, why – under certain conditions, similar conditions to be sure – linguistic and computational symbols do more harm than good (this is the way an analogy is supposed to work, isn't it?).

Later commentators built on both Boethian foundations with their usual ingenuity and exegetical finesse[27]. Two more highlights will provide a sense of how [ARITHMETIC BIAS] and [DISANALOGY BIAS] bolstered each other and became the standard story:

27. EBBESEN 1993 supplies extensive information about available editions and extant manuscripts of Latin commentaries. An updated list will include (detailed references are provided in the first section of the bibliography below): *Anonymi Cantabrigiensis Commentarium in Aristotelis Sophisticos Elenchos*; *Anonymi Marciani Commentarium in Sophisticos Elenchos Aristotelis*; *Anonymi Pragensis Quaestiones super* ARIST., *De Sophisticis Elenchis*; *Anonymi Mazarinei Quaestiones Super Librum Elenchorum*; IOANNES DUNS SCOTUS, *Quaestiones super Librum Elenchorum Aristotelis*; GUALTERUS BURLAEUS, *Quaestiones super Sophisticos Elenchos*.

[T10] *Anonymi Cantabrigiensis Commentarium in Aristotelis Sophisticos Elenchos*, ed. Ebbesen, pp. 68-70: "NAM QUONIAM NON EST <165a6> i.e. non contingit semper 'NOS FERENTES IPSAS RES DISPUTARE', i.e. nos in disputatione ipsas res deferre posse. Ferre dicitur rem in disputationem qui rem illam de qua fit sermo potest demonstrare ad oculum. Hoc autem non possumus semper facere, quoniam quandoque res absens est, quandoque incorporea, quandoque de aliquo indeterminato fit sermo; 'SED TUNC UTIMUR PRO REBUS NOMINIBUS NOTIS' <165a7>, i.e. significativis rerum. Vel notis i.e. cognitis et usitatis. Et quoniam, inquam, oportet [69] nos uti nominibus pro rebus, quando hoc fit, 'ARBITRAMUR QUOD ACCIDIT IN NOMINIBUS ACCIDERE IN REBUS' <165a9>, unitatem scilicet in significato secundum nominis unitatem attendendo et semper diversitatem significationum ex varietate nominum considerando. Unde decipimur putantes vere esse syllogizatum in aequivoco. Arbitramur, inquam, similiter esse in rebus et in nominibus, 'VELUT RATIOCINANTIBUS IN COMPOTIS' <165a9>, i.e. in computationibus – in illis, inquam, videtur **similiter esse in numeris et numeratis, et merito**, nam progressionem rerum numeratarum sequitur progressio numeri et econverso; paritatem numeri comitatur paritas rerum numeratarum, et si numerus finitus est, res numeratae finitae sunt. Omnis equidem proportio in numeris considerata in numeratis rebus sibi similiter respondet. Arbitrantur, inquam, quidam sic esse in significantibus et significatis ut computantibus in numeris et numeratis; 'HOC AUTEM NON EST SIMILE' <165a10>, nam cum numeri certam comprehensionem certarum <rerum> comitetur comprehensio, nec sic est in nominibus et rebus, 'NAM NOMINA QUIDEM <FINITA> SUNT ET ORATIONUM MULTITUDO FINITA EST, RES AUTEM INFINITAE SUNT NUMERO' <165a10>. [...] [70] [...] QUEMADMODUM <165a13>. Ex praedictis patet **aliter esse in numeris quam in sig<nifi>cantibus, unde horum ad invicem manifestatur differentia**. Ne ergo nullo modo convenientia videantur, ostendi<t> in quo simile accidat in his et illis. Dicit ergo quod sicut in numeris qui nesciunt multiplicationes et divisiones numerorum expelluntur a peritis, sic et qui significationes vocabulorum nesciunt a scientibus abiciuntur. Et hoc est: in praedictis differunt nomina a numeris, ERGO i.e. sed QUEMADMODUM ILLIC i.e. in numeris QUI NON SUNT PROMPTI FERRE NUMEROS i.e. qui non sunt periti circa numerorum proprietates (ferre dicitur ille numeros qui novit scientiam multiplicandi <et> partiendi) EXPELLUNTUR A SCIENTIBUS et pro imperitis reputantur, sic et in orationibus se habet quod illi sc. QUI SUNT IGNARI VIRTUTIS NOMINUM i.e. significationis nominum paralogizantur ['for since it is not etc.', namely insofar as it is not always the case that we discuss things by bringing them into our conversations, that is insofar as it is not always the case that we can bring the things themselves into our discussions. By bringing something itself into a conversation Aristotle refers to what happens when we can put under someone else's eyes the very thing the discussion is about. We can't do that every time, because sometimes we talk about missing things, sometimes about immaterial things and sometimes about things in general. 'We then use words as symbols instead of things', that is words which refer to things or, if we understand the word 'notis' otherwise, words which we know and use every day. And because, as I say, we need to use words instead of things, when this occurs 'we deem that what happens in the case of words, happens in the case of things as well', namely we expect that one and the same word always has the same

meaning, whilst different words have different meanings. As a result, we deceive ourselves when we think that a proper deduction has occurred while in reality equivocity has prevented it from happening. We assume, I say, that the same goes for words and for the things words stand for, just as those who reckon think when they go about their computations, that is when they do their calculations. In computations, I say, **the same goes for numbers and the things numbers stand for – and rightly so**: as a matter of fact, incremental series of numbered things follow the series of numbers and the other way around. Equal amounts of things match their number, and if the number is a finite one, then the things the number stands for are finite too. Indeed, the same relation that obtains between numbers is also to be found amongst the things numbers stand for. According to some, the words we use in order to say the things we say stand in the same relation as the numbers we come up with when we reckon stand in relation to the things whose numbers they are. But 'this is not the same' – as a matter of fact, getting a number right goes hand in hand with sorting out how many things exactly the number stands for, whereas this is not the case when words are involved: 'in fact, words as well as sentences are finite in number, whereas things are infinite in number'. [...]. 'Just as' – from what we have just said, it is clear that **numbers and words are at variance so that their difference stands out**. In order to rule out that they are related in any way, Aristotle shows what it is that happens in similar fashion when we work out numbers and when we resort to words. To that effect, Aristotle says that just as those who, working out numbers, do not know how to multiply and how to divide are outsmarted by those who are good at it, the same happens to those who know little about the power of words and are no match for those who are knowledgeable in this regard. And this is: words and numbers differ the way we said, that is 'then just as in the case of numbers', namely 'in the case of those who are not proficient at crunching numbers', that is those who are not familiar with the properties of numbers (handling numbers means here knowing how to do multiplications and how to perform divisions) are overmatched by those who have such knowledge and are looked down upon as ignorant, the same goes for those who are engaged in a conversation: they are deceived insofar as they have little knowledge of the power of words, namely what it is that words stand for]".

[T11] Robertus Grosseteste, *Commentarium in Sophisticos Elenchos*, ms. Oxford, Merton College, 280, f. 4rb: "quod autem decipi possumus per nomen ostendit 'quod accidit in nominibus in rebus etiam arbitramur accidere etc.' <165a8-9> et causam quare sic arbitramur dicit eo quod non afferimus res in disputatione, sed pro rebus utimur nominibus notis. Illud idem confirmat per simile quoddam ut quod videtur esse simile quod accidit computantibus: apud eos numerus significatorum respondet semper numero significantium et ideo non decipiuntur. Sed apud disputantes non est ita, quod innuit cum dicit 'hoc autem non simile' <165a10> et causam quare non est simile subiungit et est ratio talis: tam nomina quam orationes sunt finita, res vero numero infinitae, plures igitur sunt res quam nomina. Si ergo significantur omnes res per nomina necesse est idem nomen et eandem orationem plura significare. Quoad hoc <non> advertentes, sed unam rem per unum significari credentes decipiuntur. Et ponit iterum similitudi-

> nem inter disputantes et computantes. Nam sicut computantes vituperant et expellunt computare nescientes, sic disputantes vituperant et confundunt virtutem nominum ignorantes [Aristotle shows that words can be deceiving by saying 'we assume that what happens with words also is the case for the things the words stand for'. Aristotle also says that the reason why we believe so is that we do not bring the things themselves we discuss into our conversations; rather, we use words as symbols instead. Aristotle drives the point home by way of an analogy, for this is somewhat similar to what happens to those who work out figures. As far as those who reckon are concerned, the number of signified things always matches the number of signifiers – this is why those who deal with numbers are not deceived. This does not apply to those who deal with arguments, as Aristotle suggests by saying: 'but this is not the same'; and Aristotle adduces as proof the fact that words as well as sentences are finite in number, whereas things are infinite. Accordingly, there are more things than there are words. As a result, if we are to refer to all things by way of names, then the same name and the same sentence must refer to more than one thing. But those who are not alert to such fact and believe that one name stands for one thing will be deceived. Aristotle resorts again to the analogy between those who reckon and those who argue: just as those who can work figures out disparage and humiliate those who can't, those who can argue disparage and humiliate those who do not understand the power of words]".

If we leave aside peculiar or idiosyncratic features – such as [T8]'s optimism about sorting out syllogistic compatibility and incompatibility at the sophists' expense – the general picture is clear enough: numbers and words have precious little in common. What ultimately sets them apart is that we can confidently take the former at face value, but not the latter. Numbers stand in a direct, straightforward and perfectly univocal relationship with the things whose number they are: when figures add up and calculations come together, reckoning numbers and reckoned things stand in a perfectly one-to-one relation with one another. As Boethius and Latin commentators in his wake put it, if – by our reckoning – there are one hundred what's-their-names out there, we won't find out, later, that there were in fact ninety-nine of them or one-hundred-one for that matter (if we got the maths right to begin with, that is). Words and word-compounds are nothing like that: they fail to achieve the same kind of transparency insofar as their relationship to the things they refer to is neither direct nor straightforward, let alone univocal. As a result, while numbers are relatively foolproof and we can put as much trust in them as we can possibly muster, words are likely to cause all sorts of trouble and we are well advised to proceed with utmost caution when matching them with the things they refer to.

11. *Sententia modernorum* (*potius orthodoxorum*)

Little, if anything, has changed to this day. [ARITHMETIC BIAS] still provides the foundation of contemporary understandings of Aristotle's computational analogy as an analogy in name only [DISANALOGY BIAS]. A few distinguished examples – one from each group of scholars worth mentioning in this connection – will show how the standard story has built momentum (or, rather, never lost it) and has become very popular with historians of Ancient and Mediaeval Philosophy alike as well as with bona fide philosophers who have joined the consensus over the years:

> [T12] DORION 1995, p. 206: "(*ad* 165a3) the case of the names we use instead of things is not exactly similar or even analogous to the case of the pebbles we use when we reckon. Because, for a reason Aristotle will introduce immediately afterwards, between words and things there is not the one-to-one relationship obtaining between counters and the unities constitutive of numbers".

> [T13] KRETZMANN 1967, pp. 362-363: "ambiguity, Aristotle maintained, is theoretically unavoidable, [363] for since 'names and the sum-total of formulas [*λόγοι*] are finite while things are infinite in number... the same formula and a single name must necessarily signify a number of things'. This will, however, give us no trouble unless 'we think that what happens in the case of the names also happens in the case of the things, as people who are counting think of their counters', which are in a one-to-one correspondence with the things counted (*Sophistical Refutations*, 165a5)".

> [T14] FOUCAULT 1971, pp. 43-44: "let us leave to one side the extension that must be given to this text. One thing that is clear here is the location of the sophistical effect. It is made possible by the fact that it is not things themselves which are manipulated in the discourse, but their verbal symbols. Precisely, their name. But if this symbolization makes the sophism possible, it does not explain it. The sophism does not take place in the dimension in which words are signs. It takes place in a certain difference between names and things, between the symbolic elements and the elements symbolized. In what does this difference consist? It does not consist in that by which words produce an effect of meaning, whereas things do not. No more does it consist in the difference between *phusis* and *nomos*, between the natural character of things and the conventional character of words. It consists in the fact that there is a finite number of names and an infinite number of things, that there is a relative scarcity of words; that we cannot establish a bi-univocal relation between words and things. In short, the relation between words and what they designate is not isomorphic to the relation that enables one to count".

12. *Italiani brava gente*

If there's such a thing as an intellectual geography of Aristotelian scholarship, Italian staunch support of the majority view ([ARITHMETIC BIAS] + [DISANALOGY BIAS]) would certainly make for an interesting case in point. Exceptions are exceedingly few and far between (two overall that I know of, namely FAIT 1996 and – as a distant second – GAZZIERO 2021b). Moreover, they are unlikely to turn the tide any time soon – there's strength in numbers and Italian numbers are solid and overwhelmingly against a change away from the mainstream interpretation:

> [T15] PAGLIARO 1962, pp. 45-46 (= DI CESARE 1981a, pp. 22-24 and DI CESARE 1981b, pp. 16-20 – down to the word): "Aristotle introduces a sharp distinction between the language of numbers, on the one hand, and the language of spoken words, on the other hand. [...]. Plainly, what sets apart those who speak and those who reckon with pebbles (it being understood that their kinship is limited to the fact that neither deal directly with the things themselves) is the fact that, when we count, symbols' extensional relationship to things is straightforward – in fact, univocal insofar as one pebble refers, say, to one book, two pebbles refer to two books and so on and so forth. On the other hand, language operates with signs whose reference has a wider scope. As a matter of fact, their reference to concrete objects results in a joint determination, both connotative and extensional: for instance, the word 'book' refers to a variety of books which differ not only in shape and content, but also in number, be it one book, two, three or all of them for that matter (we say, for instance, 'the book contributes to the dissemination of culture'). Fallacies arise from within the scope of meaning so understood as a concept".

> [T16] BELARDI 1975, pp. 141-142 (= BELARDI 1976, p. 83): "*psêphoi* and *pragmata* stand in a one-to-one (1:1) relationship on account of there being so many pebbles, the *calculi*, as there are things to refer to – their relationship is therefore a numerical representation, namely a reckoning. *Onomata* and *pragmata*, on the other hand, stand in a different relationship altogether, that is a one-to-many relationship (1:n, where 'n' is a placeholder for a whole number whatsoever). As a matter of fact, even if linguistic signs count as one, for instance the word 'man' is one sign, they each stand for an unlimited number of things, men-things in this particular instance – their relationship is therefore a symbolic representation, namely a word. Accordingly, the relationship between *psêphoi* and *pragmata* rests on a numerical identity between pebbles and things. On the contrary, the relationship between *onomata* and *pragmata* can be hardly quantified – it is indeterminate or, rather, indeterminable on account of the infinite scope of things names apply to. As a matter of fact, each name can refer to whichever actual or possible individual out of the infinite number of individuals of the same kind the name stands for by virtue of an abstract generic notion which applies to them all".

> [T17] COSERIU 1979, p. 436 (= COSERIU 1981, pp. 10-11): "Aristotle compares names and counters precisely to show that they do not work in the same way and

that the relationship between name and thing is sui generis. Obviously, Aristotle's point here is not so much that things and names are not the same, as it is that the relationship between them bears no analogy to the relationship between counters and things. Counters stand in a one-to-one relationship to the things they refer to [...]. Their relationship is direct: counters simply stand for things. Counters have no 'meaning'. Their only function is to represent things or to refer to them, directly. Names are different. A name does not refer directly to any given thing. What it stands for is one single meaning through which it refers to a multitude of things (essentially, it refers to whatever falls under its meaning, namely everything that is what the name means or displays the features the name refers to). This is precisely why 'those who are not cognizant of the power of words' (*οἱ τῶν ὀνομάτων τῆς δυνάμεως ἄπειροι*) run into all sorts of trouble".

[T18] Gusmani 2004, p. 155 (≈ Gusmani 1986, p. 538 and Gusmani 1993, p. 111): "Aristotle means to contrast, on the one hand, the way abacus counters work (these are symbols just as words are, but they stand in a 1:1 relationship to the things whose numbers they are) and, on the other hand, the way linguistic items work (with the possible exception of proper names, which are not relevant here, these are supposed to refer to n things of the same kind)".

[T19] Chiesa 1991, pp. 227-229 – *reportatio*: Aristotle's 'arithmetic comparison' sets out to restore the truth about language by debunking the alleged term-to-term correlation between words and things – a fallacious, self-serving assumption which is tantamount to treating language as a vote count of sorts where the number of ballots (*ψῆφοι*) actually stands in a one-to-one relationship with the number of votes cast in the booth.

[T20] Lo Piparo 2003, pp. 184-186: "what Aristotle tells us is, precisely, that words, and not pebbles, are symbols. [...]. As Aristotle observes, the way words-which-are-symbols and the way pebbles-which-are-not-symbols refer [185] to facts differ from one another. To keep track of – say – the sheep one buys or sells, one simply has to match a pebble to each sheep. On the other hand, in order to convey one's intent to buy sheep rather than cows, one needs both words – in this particular instance, 'sheep' and 'cow' – which allows him to assign, individually and separately, all possible sheep and all possible cows. That is to say, words like 'sheep' and 'cow' function as operators by means of which each element of a virtually infinite set of sheep is identified as such (that is, as a sheep) and each elements of a virtually infinite set of cows is also identified as such (that is, as a cow). The correlation in this case is no longer a 'one-to-one' relationship (a pebble ⇔ a sheep) but a 'one-to-many' (the word 'sheep' → many sheep). [...] [186] If words referred to things the way pebbles, counters or tokens do, then rhetoric, literary works, false syllogisms, reductions to the absurd, metaphors would simply be impossible. But words are nothing like pebbles".

[T21] Sorio 2009, p. 301: "by comparing names (*ὀνόματα*) and pebbles (*ψῆφοι*), Aristotle highlights an important difference between the two: for we cannot bring

the things themselves (*πράγματα*) into our debates – in fact, in our discussions at large (of course, Aristotle's remarks apply first and foremost to dialectic, but are not restricted to the dialectical sphere alone) – but we use names as symbols (*σύμβολα*), that is as substitutes for things, one can be mislead, as Aristotle suggests in the same breath, into thinking that names and counters stand in the same relationship to things. As a matter of fact, when we work numbers out, the relationship is a one-to-one straightforward numerical correlation: five pebbles, for instance, stand exactly for five coins. It therefore seems that, according to Aristotle, pebbles are not *σύμβολα* or, at any rate, they are not symbols in the same way names are. It also appears that the relationship between counters and *πράγματα* involved in the *λογίζεσθαι* cannot be analogous to the relationship between names and things involved in the σημαίνειν".

[T22] Gusmani / Quadrio 2018, p. 58: "in *Soph. el.* 165a16, Aristotle provides a tentative theory of the symbolic character of linguistic signs. Here *δύναμις* refers to the *ὀνόματα*'s 'capacity' to refer to several *πράγματα* of the same class, as opposed to the 'capacity' of the pebbles of the abacus which are also symbols, but stand in a one-to-one relationship with the numbered objects. As a matter of fact, as far as their reference goes, linguistic expressions are inherently polyvalent, which allows them to express, by means of a limited number of signs, an unlimited number of aspects of the extralinguistic reality".

[T23] Crialesi 2020, p. 112: "according to Boethius, the cardinality of the set of reckoned numbers will always be identical to that of the set of real things. That is to say, natural or cardinal numbers are in a bijective function with things. It is not inappropriate to identify this conception as Aristotelian, if only we consider that Boethius derives the idea of the correctness of arithmetical reckoning, and thus of the capacity of numbers to signify the reality, from an Aristotelian text" – what text? Crialesi 2020, p. 113: "the roots of this conception of numerical calculation displayed in the Second Commentary on the *Isagoge* are detectable in Aristotle's *Sophistical Refutations*, which Boethius himself translated into Latin".

Granted that naming and reckoning are nothing alike (all things considered, we no more add, subtract, divide or multiply words than we speak in numbers), one might then start to wonder whether Aristotle's analogy is not 'mistaken' after all (Schreiber 2003, p. 12 made the ludicrous claim, in so many words) – or is a different understanding of Aristotle's analogy possible? By now, it should be clear that our answer is 'yes' – provided, of course, we give up either [arithmetic bias] or [disanalogy bias]. Better still we might bring the whole house of cards down and drop both assumptions. Latin commentators were not ones for half-measures and, in this respect, we can definitely take a page or two from their book[28].

28. This is, of course, a half-truth at best. That being said, there's complicated and too complicated. So we'll keep it relatively simple for the sake of the current argument, which purports to show how

13. *Sententia latinorum* (*minus orthodoxorum*)

In this connection, two Latin commentators definitely stand out, in a good way: Anonymus Bavaricus and William of Ockham. While neither seemed to push a particular agenda of their own (on the face of it, they were simply more right than they thought) both circumvented the two biases that are the hallmarks of the standard story. As a matter of fact, not only did they get past the idea that there's nothing more at stake in [Ur-Text] than smooth arithmetic routines and the arithmetic skills involved in adding, subtracting, etc. ([arithmetic bias]), but they also broke free from the concomitant notion that, for this reason, the way we usually steer clear of problems when we work out numbers provides a foil for highlighting the predicaments we get ourselves into when we misapply words, as opposed to shedding any significant light on why language fails us in the first place ([disanalogy bias])[29]:

good mediaeval commentators were at their best. As befits an homage to Jean Celeyrette, Gazziero forthcoming will deal with the whole array of alternative solutions within the arithmetical framework: non-standard or subtractive notations of Roman numerals (Nicholaus Parisiensis, *Notulae super Librum Elenchorum*, ms. Praha, Knihovna Metropolitni Kapituli, L.76, f. 56rb), algorisms (Aegidius Romanus, *Expositio super Libros Elenchorum*, ed. Venetiis 1496, f. 6ra), rhythms and metres (Aegidius Romanus, *Expositio super Libros Elenchorum*, ed. Venetiis 1496, f. 6ra), rithmomachy (*Anonymi Aurelianensis I Commentarium in Sophisticos Elenchos*, ed. Ebbesen, p. 29; and Albertus Magnus, *Expositio Sophisticorum Elenchorum*, ed. Borgnet, p. 529b) – you name it. Here's one highlight out of several the subject has to offer: whilst mainstream and essentially in line with their arithmetical background, Nicholas of Paris' and Giles of Rome's suggestion that the positional character of numerical notation is to be blamed was a huge step in the right direction. Even if their observations can hardly apply to Aristotle, they both got that much right: the whole point of Aristotle's analogy is that those who count can get it wrong too. In so many words: "sicut ille qui nescit computare propter hoc quod figura una in numeris secundum diversos situs non unum sed multa significat, 'expelluntur', id est decipitur 'a scientibus', sic ille qui non cognoscit virtutes vocabulorum a scientibus decipitur [just as someone who does not know much about working figures out, insofar as <e.g. he overlooks the fact that> one and the same digit has not the same but different values when its position changes, is entrapped, namely is deceived by those who have such knowledge, in the same way he who ignores the power of words is deceived by those who have such knowledge]" (Nicholaus Parisiensis, *Notulae super Librum Elenchorum*, ms. Praha, Knihovna Metropolitni Kapituli, L.76, f. 56rb); "sicut apparet in algorismos, ubi una figura aliter et aliter situata alium et alium numerum importat, quia si primo loco ponitur repraesentat se ipsam et secundo loco decies se ipsam, sicut ergo in talibus posset esse deceptio accipiendo unum numerum pro alio, sic et in orationibus, propter nominum multiplicitatem, accidit esse deceptio [as algorisms make it plain insofar as the same digit is worth a different number as soon as it changes its place – for instance, in one place it is worth its numerical value, whereas in another place it is ten times worth what it was. Accordingly, therefore, just as one can get a number wrong and confuse it with another number, in the same way one can be deceived in a conversation insofar as words may refer to more than one thing]" (Aegidius Romanus, *Expositio super Libros Elenchorum*, ed. Venetiis 1496, f. 6ra). Truth be told, Robert Kilwardby made a similar claim but did not provide much in the way of explaining why numeral symbolism can be misleading too (cf. Robertus Kilwardby (?), *Commentarium in Aristotelis Sophisticos Elenchos*, in **C**, f. 278rb; **P**, f. 2vb).

29. The truth, nothing but the truth and yet not the whole truth (again). One might be tempted to mention Anonymus monacensis as a third unsung hero of the abacus saga and to reconstruct along the same lines his understanding of Aristotle's analogy, but we won't – here. Sure enough, the

[T24] *Anonymi Bavarici Commentarium in Aristotelis Sophisticos Elenchos*, ms. München, Bayerische Staatsbibliothek, Clm. 8002, f. 2rb-2va: "tunc cum dicit VELUT etc. <1, 165a9> declarat illud per simile et dicit [2va] quod simile huic sit in compotis ratiocinantibus. Illi enim aliquando loco unius librae ponunt unum lapillum et pro alia illum et pro alia alium et sic cum tres sunt lapilli credunt tres esse denarios vel tres esse solidos. Et similiter si in loco viginti librarum ponatur unus lapis pro tanto alius et tunc ad numerum lapidum sumatur numerus librarum non esset bonum; eodem modo ex ista parte quia nominibus notis pro rebus utimur ideo quod accidit in nominibus accidere arbitramur in rebus. Deinde cum dicit HOC AUTEM NON EST SIMILE <1, 165a10> ponit causam defectus aequivocationis et dicit quod defectus est multitudo significatorum. Istam sententiam ponit sic HOC AUTEM NON EST SIMILE <1, 165a10> et istud debet retorquere ad illud quod primo fuit dictum de causa apparentiae, quod scilicet causa est in ista fallacia quando nos ita credimus accidere in rebus et in nominibus quod sicut vox est una et res sit una. Hoc autem non est simile quoniam aliquando vox est una et tamen sunt multa eius significata, quae multitudo significatorum decipit nos [by saying 'just like, etc.' Aristotle introduces an analogy and states that what happens to those who count when they go about their calculations is alike to what happens here. Indeed, those who reckon sometimes assume that one pebble is worth one pound, another pebble is worth another pound and a third pebble is worth one more pound. Accordingly, since there are three pebbles, they therefore infer that there are as many schillings or pence. On the other hand, if we are to assume that one pebble is worth twenty pounds and another pebble is worth as much, then figures will not add up if we keep comparing the number of pebbles, on the one hand, and the number of pounds, on the other hand. The same goes for the other half of the analogy, for we

anonymous commentator must have thought that those who reckon do not fare much better and may turn out to be every bit as mistaken as those who put their trust in words. More to the point, he nailed it when he surmised that Aristotle's analogy has to do with the way we handle stone counters, wood sticks or whatever else we use to count, reckon or calculate with (*Anonymi Monacensis Commentarium in Sophisticos Elenchos*, **M**, f. 4ra; **A**, f. 7ra: "lapilli, ligna, vel aliquid alterum mediante quo computant vel numerantes in compotis vel computantes"). That being said, he was not so eager to make the point that computational symbols are as shifting and ambiguous as linguistic ones – which is the main reason Aristotle brought them together in the first place. Accordingly, maybe one should not read too much into his most intriguing suggestion – namely: "ERGO QUEMADMODUM ILLI etc. <165a13> qui non sunt prompti, id est docti FERRE NUMEROS, id est qui nesciunt computare prompte expelluntur a scientibus computare **prompte et velociter**. EODEM MODO ET IN ORATIONIBUS <165a15>, id est a parte orationum, illi QUI SUNT IGNARI VIRTUTIS NOMINUM id est ignorantes significationes nominum et quicquid possit apprehendi et intelligi per illa PARALOGIZANTUR id est decipiuntur ['just as in the case of those who' are not proficient, i.e. are not expert at processing numbers or do not know how to reckon promptly, are no match for those who know how to reckon swiftly and quickly; 'the same applies to discussions', i.e. on the side of discussion. 'Those who are not familiar with the power of words', i.e. those who are not cognizant of the meanings of words and everything one can learn from the power of words are misled by fallacious reasonings, that is are deceived]". Anonymus monacensis' legitimate concern with the swiftness and promptness in calculations (*computare prompte et velociter*) might, after all, have less to do with the manual dexterity some display in moving the counters around, which is as close as one can possibly get to the truth of Aristotle's analogy, and more to do with how quick one is able to go through numbers, which is pretty standard lore (cf. e.g. Anonymus Aurelianensis' [T9], most notably p. 29, ll. 11-12).

use words as substitutes for things and we therefore believe that what is the case for words is also the case for the things words stand for. Next, by saying 'still it is not the same', Aristotle dwells on the cause which brings about the fallacy of equivocation and he states that its flaw consists in the multiplicity of things an ambiguous term refers to. Aristotle makes this point by saying 'still it is not the same' and one has to understand the claim by referring it back to what Aristotle has previously said about what grants the fallacy of equivocation its deceptive allure, that is to say the fact that we believe that the same goes for both the words and the things they stand for, so that we are led to believe that one and the same word refers to one and the same thing. But this is not the case insofar as sometimes the same word refers to more than one thing and its polysemy deceives us]".

[T25] Guilelmus de Ockham, *Expositio super Libros Elenchorum*, I, 3, ed. del Punta, p. 11, ll. 86-89: "ponit exemplum de ratiocinantibus in computis, quia in illis apparet aequivocatio; nam in illis aliquando unus lapillus significat unum denarium aliquando duodecim, aliquando unam libram aliquando sexdecim [Aristotle resorts to the example of accountants performing calculations, for ambiguity <also> occurs in calculations. As a matter of fact, while reckoning, one and the same pebble sometimes means one penny sometimes means twelve pence, sometimes one pound sometimes sixteen pounds]".

[T24] and [T25] are a testament to Latin commentators' matter-of-fact ingenuity and no-nonsense, down-to-earth approach[30]. Indeed, there's much we can learn from both Anonymus Bavaricus and William of Ockham's unorthodox views on what is going on in [Ur-Text].

14. Lesson n° 1: "what is a pebble analogy about, if not pebbles?"

The first lesson we can draw is the most obvious – and yet it has proved elusive time and again. [T24] and [T25] make it plain that, contrary to what [arithmetic biased] commentators would have us believe, there is more to Aristotle's pebble analogy than plain numbers and smooth arithmetical calculations. In fact,

30. How Anonymus Bavaricus and William of Ockham managed to get all the abacus facts straight (despite being at a considerable disadvantage, that is) is, of course, a bit of a mystery and a story worth telling in its own right. As Gazziero forthcoming will show, it involves industrious minds who designed, built and modified counting boards (abacus inventors and abacus experts) as well as unscrupulous end-users who took advantage of some of their features (merchants, accountants, book-keepers and the like). For the time being, we'll have to rely on the fact that mediaeval commentators actually put two and two together and made the connection between Aristotle's analogy and the abacus. We will also have to rely on the scanty but rock solid evidence we already provided. In this particular instance, William of Conches tells us pretty much everything we need to know for the sake of our argument – most notably, [T4] confirms that the mediaeval abacus was a positional device where one and the same counter could be moved around and change its value accordingly.

by discarding or belittling the pebbles in the pebble analogy one is most likely to miss Aristotle's point altogether. In [Ur-Text] pebbles are of interest by themselves and their function is certainly neither to remind us that counter-assisted calculations follow the exact same rules as purely arithmetic ones nor to remind us that some of the people that toss the tokens around are more proficient than others when it comes to processing numbers as such or performing calculations at large. Rather, pebbles are there to warn us that those who rely on counters in their computational transactions and those who rely on words in their verbal interactions share the same predicament, namely: words, in the course of the same argument, and counters, in the course of the same calculation, do not always have the same value and – as if that wasn't bad enough – this is not something people with bad intentions advertise up front. As a result, neither words nor counters are entirely safe to play with, precisely because neither counters nor words – as opposed to plain numbers, say – operate at a level of transparency that would make them virtually indistinguishable from the things they stand for.

15. Lesson n° 2: "pounds, shillings and pence"

The second lesson we can learn from [T24] and [T25] is also of the obvious kind and yet, like the previous one, it has also been largely overlooked. Unlike most commentators, Anonymus Bavaricus and William of Ockham made ample allowance for computational concerns other than the purely arithmetical. More to the point (and more importantly) they both referred to specialized reckoning involving coins and monetary non-decimal conventions as opposed to focusing on numbers and arithmetical operations as such[31]. Understanding Aristotle's abacus analogy along the lines of practical computational routines – such as public accounting, private bookkeeping, business transactions and the like – might well be the best way of making sense of [Ur-Text]. On the one hand, it squares nicely with a vast array of ancient literary and epigraphic sources where the abacus is most commonly – in fact, almost exclusively – associated with counting money[32],

31. Without reading too much into it and without going into too much detail, Ockham's shift between one penny and twelve pence is telling. Whether consciously or unconsciously, it reflects the 1:12 standard conversion rate (12 pence = 1 shilling) between denominations (*denarii* and *solidi*) Mediaevals were familiar with (the same ratio is mentioned in e.g. *Anonymi Fallaciae Londinenses*, ed. de Rijk, p. 662, ll. 22-29). On Ockham's monetary environment, cf. the recent survey (1150-1350) in Kelleher 2018 (together with the extensive bibliography it provides).

32. Gazziero 2021b presents evidence from fifteen epigraphic collections and discusses some twelve staple texts which strongly support the conclusion that all known features of the ancient abacus had one thing in common: they were all meant to accommodate the needs and comfort of traders, auditors, bankers and other money peddlers whose interest in numbers did not go beyond counting coins, exchanging currencies, charging interest rates and, of course, preying on each other when selling and buying goods.

and, on the other hand, it is remarkably consistent with Aristotle's language and expression, right down to [Ur-Text]'s vocabulary itself[33].

16. Lesson n° 3: "failure means failure"

A third lesson we can take from [T24] and [T25] is that [Ur-Text] is a cautionary tale without bright side or silver lining: everybody and everything fails – those who reckon and what they reckon with no less than those who argue and what they argue with. Simply put, failure is the whole point here, failure to spot subtle and yet momentous changes in the worth of counters and in the meaning of words which plague discussions and calculations alike.

17. Epilegomena

Once we give up the idea that numbers as such took centre stage in Aristotle's counter analogy, we can set it back upon its feet by shifting its focus from trying to showcase why dealing with numbers is so successful whereas dealing with words is so troublesome to trying to explain why pebble reckoning and verbal sparring are both accident-prone – prone to the same accidents, that is. For this is, arguably, the main reason why Aristotle brought pebbles and words together to start with: being symbols whose value can change with us having a hard time keeping track or even noticing, pebbles and words are every bit as tricky. More to the point, they both require that we pay constant attention to what is (worth) what and that we watch out for those who will take advantage if we don't keep up. And this is, arguably, the most valuable lesson which [Ur-Text] – understood along the same lines Anonymus Bavaricus and Ockham did – can teach us about Aristotle's views on language, its involvement in arguments and how they play out: when it comes to squaring accounts – be it by means of arithmetical or verbal reckoning – there are those who play by the rules and those who don't. As it happens, just being good at numbers or being arithmetically proficient is not enough to keep con men and traffickers at bay. Knowing one's way around counting boards and digital dexterity at pushing the pebbles around or at least the ability to understand and follow their movements on the abacus is just as important. Likewise,

33. Ancient sources (as gathered and commented upon in Gazziero 2021b) also strongly support the conclusion that when *λογίζομαι* (as well as related words: *λογισμοί*, *λογιζόμενοι*, etc.) and *ψῆφοι* showed up in the same sentence as in [Ur-Text] 165a9-10, people were counting money on their own (Theophrastus, *Characteres*, XXIV, 12, ed. Diggle, p. 134, ll. 15-17) or someone else was counting money for them (Athenaeus Naucratita, *Deipnosophistae*, III, 117e3-118a13). Even *παραχρούω* in [Ur-Text] 165a15 had a nice, cheating-money-out-of-people ring to it (Arist. (quod fertur), *Mechanica*, 849b34-38).

going about one's conversational business with a decent grasp of the general principles of verbal communication and basic argumentation is not enough to stop fallacy-mongers in their tracks. Knowing one's way around linguistic pitfalls and tricks with words is at least as important. Admittedly, there's nothing particularly profound nor particularly exciting about all that; but, as a wise man once said: "better to be bored and safe than outgunned and outmanoeuvred at every turn" (or words to that effect).

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Abstract: The prologue of the *Sophistici elenchi* is as close an Aristotelian text gets to dealing with language as a subject matter in its own right, only in reverse. Language and its features bear consideration to the extent that they account for some major predicaments discursive reasoning is prone to, both as a separate and as a common endeavour. That being said, the linguistic pitfalls that trick us into thinking that whatever is the case for words and word-compounds is also the case for the things and facts linguistic items stand for reveal as much about good linguistic habits and sound argumentation as they themselves are revealed by the principles and rules our argumentation goes by. In this connection, Aristotle resorted to a curious (or not so curious) analogy between words and counters which played a major role in explaining why language is such a powerful source of illusion and deception. As it happens, Aristotle accounting analogy is a case in point for showcasing the level of sophistication mediaeval Latin literature on fallacies achieved as early as the first half of the twelfth century. As a matter of fact, Western commentators managed to build compelling cases both in favour of and against the understanding that was to become and still is the standard story – which, of course, speaks volumes about their exegetical proficiency and technical expertise. On the one hand, trusting implicitly Boethius' translation and well aware of his views on disputational hazards as opposed to computational reliability, they usually understood Aristotle's comparison as if it was an analogy in name only. On the other hand, despite Boethius' translation put them at a considerable disadvantage, Latin commentators were able to construe Aristotle's analogy as bringing together two sets of symbolic variables (words and counters) that are neither entirely free nor entirely bound – which expose them to subtle but critical shifts in value and meaning.

Keywords: Aristotle; Boethius; William of Ockham; Language; Arithmetic; Logic; Argumentation; Fallacies; Translation.

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Theology, Fallacious Reasoning and Heresy on the Borders of the Twelfth and Thirteenth Centuries: Some Remarks on the *Fallaciae in theologia* and Amalricians*

Treatises on fallacies – i.e. logical traps, directly inspired by Aristotle's *Sophistical Refutations* – constitute a new genre that emerged in the second half of the twelfth century. Several treatises on fallacies have come down to us. First, the *Fallaciae Londinenses*, preserved in a single manuscript, were edited by Lambertus Maria de Rijk. These fallacies closely follow the Aristotelian *Sophistical Refutations* and are not written specifically for theology. Then, the *Fallaciae* and *Loci Theologici* of Guillelmus de Montibus which, according to Yukio Iwakuma, who edited them, should be dated after 1186 and before 1200[1]. The treatises of Guillelmus de Montibus are "explicitly addressed to theologians" and their "aim is didactic", as Luisa Valente writes[2]. The *Fallaciae in theologia* or *Fallaciae in sacra pagina* (*inc*. "Columnae basis triplicis innititur fides nostra"), according to the different title given in the manuscripts, still unpublished, is an anonymous text on the errors that can be committed in theological discourse. It was discovered by Jean Leclercq[3], who edited the prologue, where we have an explicit mention of heretics and their sophisms (*cavillationes*) which must be rejected – the reference to heretics will be discussed below. This one text in particular and the theological fallacies as a genre have been studied by Franco Giusberti[4] and Luisa Valente[5].

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1. Iwakuma 1993, p. 4. The *Fallaciae Magistri Willelmi* were first edited by L.M. de Rijk in volume two of his *Logica modernorum*.

2. Valente 1999, p. 221. See also Rosier-Catach 1988.

3. Leclercq 1945.

4. Cf. Giusberti 1982.

5. Cf. Valente 2008 where the author puts the role of fallacies into perspective by showing the two-way influence between logic and theology.

1. Manuscripts

Jean Leclercq knew of only three manuscripts of the *Fallaciae in theologia*: Florence (no. 1 in the list below), Paris, Mazarine (no. 3), and Paris, français 19951 (no. 4)[6]. Franco Giusberti reports seven[7], and we can add manuscripts from Leipzig (no. 2), Paris, latin 14417 (no. 5)[8], and Princeton (no. 6). Thus, from seven we have now reached ten manuscripts; this is a very considerable number for an anonymous scholastic text from the late twelfth/early thirteenth century:

1) Florence, Biblioteca Medicea Laurenziana, Plut.20 dex.13, ff. 123ra-125ra
2) Leipzig, Universitätsbibliothek, MS 835, ff. 64rb-69rb
3) Paris, Bibliothèque Mazarine, 891, ff. 127a-130c
4) Paris, Bibliothèque nationale de France, français 19951, ff. 33r-62r
5) Paris, Bibliothèque nationale de France, latin 14417, ff. 312rb-313va
6) Princeton, University Library, MS 189, ff. 1r-14v
7) Rome, Biblioteca Vallicelliana, B 82, ff. 220r-226v
8) Toulouse, Archives départementales de la Haute-Garonne, 5 (D 56), ff. 254vb-257vb
9) Venice, Biblioteca dei PP. Redentoristi (S. Maria della Consolazione, detta "della Fava"), 4 (lat. CLXXIII), ff. 69va-71vb
10) Zagreb, Nacionalna i sveucilišna knjižnica, MR 97

In the thirteenth-century Leipzig manuscript (no. 2) the *Fallaciae* is entitled *Tractatus de fallaciis in sacra scriptura occurrentibus*. It is sandwiched between Alain of Lille's *Contra haereticos* (ff. 26ra-64rb) and the *Summa contra hereticos* by Pseudo-Iacobus de Capellis (ff. 69rb-104ra), an anonymous anti-Cathar text dated 1230s[9].

The thirteenth-century Parisian manuscript (no. 5) is made up of two separate volumes from the library of Saint Victor. The second volume is of interest here because it contains the *Fallaciae in theologia* with the title *Ars refellendi hereticos*. It also includes works by authors such as Richard of St. Victor, Stephen Langton, Praepositinus of Cremona, Hugh of St. Victor, and the questions entitled after their incipit 'Quare'. All these indications lead us back to the teaching of theology in Paris at the turn of the twelfth/thirteenth centuries[10].

6. Leclercq 1945, p. 44.

7. Giusberti 1982, p. 97. Giusberti reports that Francesco Del Punta told him he had found another manuscript in Wrocław. Unfortunately, I have not been able to locate it.

8. This manuscript is already listed in the online index "In principio", by Brepols (http://apps.brepolis.net/BrepolisPortal/default.aspx).

9. Cf. Ps. Iacobus de Capellis, *Summa contra hereticos*, ed. P. Romagnoli, p. 42.

10. Here is a summary of the contents of the volume according to the Paris National Library's catalogue (https://archivesetmanuscrits.bnf.fr/ark:/12148/cc75171d/cd0e532): Stephanus Langton, *Reportatio super Historiam scholasticam Petri Comestoris* (ff. 129-158); *Glosa in Isaiam, Danielem et Prophetas Minores* (ff. 159-241); Praepositinus, *Summa super Psalterium* (ff. 242-308); Hugo de Sancto Victore, *Didascalicon* (excerpta) (309); Richardus de Sancto Victore, *Liber Exceptionum* (excerpta) (ff. 309-312); Questiones de ecclesiasticis officiis intitulate "Quare" (ff. 313-315).

No. 6 in the list is an Italian manuscript from the fifteenth century (between 1460 and 1469), of unknown provenance. It contains both medieval texts and writings by humanist authors such as Leonardo Bruni, Gasparino Barzizza and Giorgio Valagussa[11]. The *Fallaciae* is untitled[12].

The Vallicelliana manuscript (no. 7) is composite. The third codicological unit (ff. 220-254) that interests us here dates from the fourteenth century, according to the online catalogue 'Manus'[13]. Our treatise on the *Fallaciae*, which appears at the beginning of this third part, is acephalous: "[...] si non unus Deus solus est...". A title has been added by a hand different from that of the scribe: *Tractatus theologicus de fallaciis*. The *Fallaciae* is followed by the *Hierarchia Alani* (*Tractatus de angelica yerarchya* in the Vallicelliana manuscript), ff. 226v-231r, edited by Marie-Thérèse d'Alverny. This is followed by Alain of Lille's *Regulae theologicae*, ff. 231v-254v. It should be noted that the Florence manuscript (no. 1 in the list) also contains both the *Hierachia Alani* and the *Fallaciae in theologia*[14].

No. 8 is a French manuscript from the fourteenth or fifteenth century: the *Fallaciae in theologia* follows Nicole Oresme's *De communicatione idiomatum* with no break in continuity[15]. This fact should be noted, because in manuscript no. 4 as well, the *Fallaciae* has been bound in a single volume with a fifteenth-century copy of Nicole Oresme's *Livre des divinations*.

It is probably too early to draw conclusions. In any case, the *Fallaciae in theologia* can be found in the manuscripts that I have been able to consult in the company of works by Parisian theologians of the twelfth or early thirteenth century (in three cases with Alain of Lille); or in two cases with works by Nicole Oresme. Their presence in a fifteenth-century Italian manuscript shows that the *Fallaciae* had a wide circulation. However, the older manuscripts, those of the thirteenth century, point to a French or even Parisian milieu.

11. High resolution images of the ms. available on the Princeton Library Special Collections – Manuscripts website: https://catalog.princeton.edu/catalog/11547901. The best description of the manuscript can be found on the website "Les enluminures" (https://www.textmanuscripts.com/medieval/leonardo-bruni-barzizza-valagussa-60431&p=6): "[Miscellany of Humanistic Texts], including Leonardo Bruni, *Ad Petrum Paulum Histrum Dialogus*; Gasparino Barzizza, *Tractatus de Compositione*; Giorgio Valagussa, *Elegantiae Ciceroniae,* ff. 69v-74; Cicero, *De inventione, Book I,* v. 1-6, ff. 75-88v; [Anonymous], *Commentary on the grammatical aspects of the* Decretals: incipit Prologue, '[I]stud proemium in quatuor divido partes. In prima quarum salutatio ponitur...'; rubrics, *De summa tri[nitate] et fid[e] ca[tolica]; De constituti omnibus; De rescriptis; De constitudine; De postulatione; De electione*; incipit, 'Fideli. Errore grecorum quod volebant...', explicit, '... episcopus per alios faciat episcopos expediri'. This is a commentary on the grammatical aspects of the *Decretals*, compiled by masters of the University of Bologna. It is not recorded in Bursill-Hall 1981 and is likely unpublished".

12. The first folio is available on the website "Les Enluminures".

13. https://manus.iccu.sbn.it/opac_SchedaScheda.php?ID=16498

14. The latter knew the Vallicelliana and Florence manuscripts. See d'Alverny 1965, pp. 219-221.

15. The manuscript contains Jacobus de Altavilla, *Commentarium in Sententias*; Nicole Oresme, *De Communicatione idiomatum*; *De Fallaciis in theologia*.

2. Author

Another clue to the origin of the *Fallaciae in theologia* can be found in the text itself – this is an example that the author puts forward when examining the *fallacia compositionis et divisionis*:

> "I send a servant to Tours, proposing to him two different routes: one via the city of Chartres where he has many enemies, and the other via Orléans where I have acquaintances and friends"[16].

It is obvious that the author who speaks in the first person is in Paris, because it is from Paris that one can reach Tours either via Orléans or via Chartres; these are still the two possible routes even today. A few lines later the two alternative cities are proposed again. This passage has not been noticed by other specialists – with the exception of Marie-Thérèse d'Alverny[17]. *Turonis*, *Carnotum*, *Aurelianum* read perfectly in the Leipzig manuscript, although I have not collated all the manuscripts. The example is inserted in the context of the discourse on predestination and free will. Convinced by a whole series of arguments, the servant chooses to travel via Orléans and not via Chartres, but in the course of his journey he comes across criminals and is killed. Texts produced in a school context often contain concrete examples that speak to the listeners and such examples are often indicative of the identity of the teacher or, as in this case, of the place where he teaches.

3. The *Fallaciae in theologia* and the Amalrician Heresy

Fallacious reasoning in theology is often mentioned in attacks on heretics. This aspect is admittedly known to scholars, but has not been thoroughly investigated. As indicated above, the fight against heretics and their sophisms is expressly mentioned in the prologue of *Fallaciae in theologia*, once edited by Jean Leclercq[18]. However, the text has never been studied from this perspective. For this rea-

16. *Fallaciae in theologia*, ms. Paris, BnF, français 19951, f. 52r (transcription Francesco Del Punta): "Item, ego destino servum Turonis proponens ei duplicem viam: unam per Carnotum ubi multos habet inimicos, aliam per Aurelianum ubi notos habeo et amicos". I was able to consult Francesco del Punta's transcription of the *Fallaciae* thanks to the generosity of Luisa Valente (Università degli Studi di Roma, La Sapienza) and Leone Gazziero (CNRS, UMR 8163 "Savoirs, textes, Langages"). I would like to thank them for this.

17. d'Alverny 1965, pp. 220-221: "Ce ms. contient ff. 123-125, – she is describing the Florence manuscript (no. 1) – un traité *De fallaciis in sacra pagina*, rédigé en France d'après des allusions aux villes de Tours, Chartres et Orléans". However, Marie-Thérèse d'Alverny does not suggest that the author is in Paris.

18. Leclercq 1945, p. 45; ms. Paris, BnF, français 19951, f. 33r (transcription Francesco Del Punta): "Sicut autem triplici funiculo subsistit et roboratur fides nostra, sic triplici laborat incommodo. Nam eam persequitur materialis gladius, manifestus haereticus et hostis domesticus. Contra duo ul-

son, the present study will investigate the *Fallaciae in theologia* in the context of the Amalrician heresy. Amaury de Bène (Almaric of Bena) was excommunicated *post mortem* by a Provincial Synod presided over by Peter of Corbeil, the Archbishop of Sens, in Paris in 1209 or 1210. Ten of his followers were degraded and handed over to the secular arm, i.e. to the officers of King Philip Augustus, who condemned them to be burned, while four others were sentenced to life imprisonment. No text by Amaury of Bène or his disciples has been preserved. Their doctrine is only known from the chroniclers and from his detractors, such as Garnier of Rochefort, author of the *Contra Amaurianos*. The latter text was discovered and studied by Barthélemy Hauréau[19]. It was first published by Clemens Baeumker in 1926, who proposed to attribute it to the Cistercian Garnier of Rochefort, abbot of Clairvaux, and bishop of Langres for some years; it was republished by Paolo Lucentini in 2010. It is clear that the *Contra Amaurianos* is an exceptional witness, written before the arrest of the Amalricians, and more importantly before the Provincial Synod of 1209 or 1210, as there is no reference to the condemnation of Amaury and his followers. Thirteen theses are attributed to the Amalricians in the *Contra Amaurianos*. Thesis no. 9 is undoubtedly the best known of all: *Deus est omnia in omnibus*. As Clemens Baeumker once indicated, to illustrate this thesis Garnier of Rochefort re-uses a part of a chapter from his *Isagogae theophaniarum symbolicae*, still unpublished, which is preserved in ms. Troyes, Médiathèque du Grand Troyes, 455. This is very important, because Garnier of Rochefort now attributes to the Amalricians statements that in the *Isagogae* he had attributed to the Manichaeans. Paolo Lucentini, for his part, has shown that Garnier of Rochefort had in fact used the *Summa "Quoniam homines"* of Alain of Lille in both works[20]. The *Contra Amaurianos* should also be compared with the *Fallaciae in theologia*.

Three fallacies are mentioned in chapter 9 of the *Contra Amaurianos*, which illustrates thesis no. 9 of the Amalricians: *Deus est omnia in omnibus*. Garnier of Rochefort combats the Amalricians as follows:

> "Moreover, taking the authority of the Apostle who says: 'God will be all in all' [*I Cor* 15:28], they say that God is all in all. They proceed in this way: God will be all in all. But all that will be is, since change does not affect God. So, God is all in all. But what is more absurd than that God is stone in stone, Godin in Godin? Let Godin therefore be worshipped, not only by dulia but by latria, since he is God. Moreover, let the mole or the bat be worshipped, since God in the mole is a mole, and in

tima incommoda, hoc est contra cavillationes manifesti haeretici et domestici hostis insidias, modum et artem repellendi earum versutiam suscepimus in tractatum. Cavillatur autem multipliciter".

19. Hauréau 1880, pp. 85-90.

20. Lucentini 2005. In the same paper, Paolo Lucentini links chapter 1 of the *Contra Amaurianos* with the *Summa "Qui producit ventos"* by Praepositinus of Cremona, and with the *Epistola de incarnatione verbi* by Anselm of Canterbury.

> the bat a bat. The poor [*Amalricians*] do not understand why this is said. [...]. But the argument can be refuted as follows: 'God will be – he says – all in all'. And all that will be, is. So, God is all in all. Falsification: 'The charity of God will be in all who are to be saved. And all that will be charity, is the charity of God. Therefore, the charity of God is in all who are to be saved'. And this is false, since many are to be saved who are not yet born. But to this they object: All that is in God is God[21]. But all are in God, since 'that which has come to be was life in Him' [*John 1:3-4*]. Therefore, God is all. They are therefore mistaken, for they do not understand the Scriptures, nor do they pay attention to the reason for what is said. [...]. Falsification: 'The power of God is the same as the charity of God. But the power of God is in the stone. Therefore, the charity of God is in the stone'. Or: 'The father is distinct from the Son in fatherhood. But the divine essence is the Father. Therefore, in the fatherhood [lege *in the divine essence*] he is distinct from the Son'"[22].

In the third falsification *(fallacia)* one must correct *paternitate* to *divina essentia*, otherwise the conclusion is identical to the major premise of the syllogism. In the apparatus of the sources, Paolo Lucentini brought the first "fallacy" closer to Praepositinus of Cremona's *Summa* "*Qui producit ventos*": "Instantia: Dei essentia nihil aliud est quam caritas. Et divina essentia est in lapide. Ergo caritas est in lapide"[23].

The anonymous author of the *Fallaciae in theologia* says in the prologue that "cauillatur autem multipliciter". He therefore examines the different types of

21. On this saying quoted by the masters *in sacra pagina* of the twelfth century, see Valente 2000. This saying is very often attributed to Augustine, but it is not found as such in his works. It is quoted by Abelard and in the writings of his school, by the Porretans, by Alain of Lille and others: the uses and interpretations that have been made of it are multiple. Luisa Valente brings Garnier of Rochefort closer to Alain of Lille and Simon of Tournai – a master of theology in Paris in the second half of the twelfth century, who died in 1201, author of the *Institutiones in sacram paginam*. For Garnier of Rochefort, as Luisa Valente put it, this sentence means that "the theological propositions express no inherence but identity" (Valente 2000, p. 733).

22. Garnerius de Rupeforti, *Contra Amaurianos*, IX, ed. Lucentini, pp. 26-30: "Item. Occasione illius auctoritatis, quam inducit Apostolus dicens: *Deus erit omnia in omnibus*, dicunt quod Deus est omnia in omnibus. Sic enim procedunt: *devs erit omnia in omnibvs*. sed qvicqvid erit, est, qvia mvtatio non cadit in devm. ergo devs est omnia in omnibvs. Sed quid est absurdius quam quod Deus est lapis in lapide, Godinus in Godino? Adoretur ergo Godinus, non solum dulia sed latria, quia Deus est. Immo et talpa uel uespertilio adoretur, quia Deus in talpa talpa est et in uespertilione uespertilio. Non intelligunt miseri quid propter quid dicatur. [...]. Sic autem potest argumentum refelli: 'Deus erit – inquit – omnia in omnibus. Et quicquid erit, est. Ergo Deus est omnia in omnibus'. Fallacia: 'Caritas Dei erit in omnibus saluandis. Et quicquid erit caritas, est caritas Dei. Ergo caritas Dei est in omnibus saluandis'. Et hoc falsum est, quia multi saluandi sunt qui necdum nati sunt. Sed ad hoc instant: qvicqvid in deo est, devs est. sed in deo svnt omnia, qvia *qvod factvm est in ipso vita erat*. ergo devs est omnia. Ideo errant, quia non intelligunt Scripturas, nec quid propter quid dicatur attendunt. [...]. Fallacia: 'Idem est potentia Dei et caritas Dei. Sed potentia Dei est in lapide. Ergo caritas Dei est in lapide'. Vel sic: 'Pater paternitate distinguitur a Filio. Sed essentia diuina est Pater. Ergo paternitate [*lege* diuina essentia] distinguitur a Filio'".

23. Praepositinus de Cremona, *Summa* "*Qui producit ventos*", I, 6, 12, ed. Angelini, p. 238, ll. 50-52.

fallacies, according to the list of *Sophistical Refutations*, as well as the other *fallaciae* texts of his time. As observed by Jean Leclercq, the author will neither contemplate "sophisms *ex accentu*, which do not occur in connection with Holy Scripture, nor *petitio principii* and the paralogism *propter non causam*. But with regard to all the others, he accumulates examples, in order to train the mind of the theologian to discern easily the apparent contrarieties and to reconcile them"[24]. Luisa Valente follows Jean Leclercq's lead: "the *Fallaciae in theologia* is intended to prepare students of *sacra pagina* for the practice of refutation"[25]; and she points out that these themes are dealt with in theological summas which, unfortunately, are still mostly unpublished: "We can often find the same questions in the theological summas of the time [...]. In the end, one could say that this work constitutes a collection of theological questions ordered, not according to the systematic subdivision of the subjects treated, a structure usual in the summas, but according to the systematic subdivision of the instruments used to resolve the questions, a subdivision modelled on the scheme of the *Fallaciae*"[26]. It must be emphasized that the author selects from the *Sophistical Refutations* those sophisms that are useful for theology. We are thus in the presence of a case of Christian Latin appropriation of Aristotle's philosophy. Whether Aristotle's *Sophistical Refutations* had a decisive impact on the development of theology in the West is a complex matter, and one that is not unanimously agreed upon by scholars. Moreover, following a suggestion by Franco Giusberti, Luisa Valente has brought the *Fallaciae in theologia* closer to Peter the Chanter's *De tropis loquendis*[27]. She believes that these two texts, eccentric in relation to the theological production of the time, come from the same intellectual circle. However, the *De tropis loquendi* is more concerned with exegesis and preaching, and it takes up only two types of fallacies from the *Sophistical Refutations*[28], while the *Fallaciae in theologia* is more concerned with speculative theology and disputation. For my part, I place more emphasis on the fact that the *Fallaciae in theologia* faithfully reproduce the pattern of the *Sophistical Refutations* and on their practical aspect, which serves to counteract the fallacious reasoning of heretics and bad dialecticians. This text is a weapon for unmasking heretics and refuting their sometimes-tantalizing demonstrations. If we have to situate its writing in Paris, we can imagine that it is the work of a member of "Peter the Chanter's circle", to echo the expression with which Baldwin[29] referred to the theologians who gravitated around Peter. One may also add that, more recently, Jean-

24. Leclercq 1945, p. 45.
25. Valente 1999, p. 222.
26. Valente 1999, p. 223.
27. Valente 1997, pp. 54-55.
28. Giusberti 1982, pp. 92-93.
29. Baldwin 1970

Pierre Rothschild[30] noted that the *De tropis loquendi* was used, or even taken up verbatim in Garnier of Rochefort's *De contrarietatibus in Sacra Scriptura*.

Fallacia equivocationis is the first type of fallacy analyzed in the *Fallaciae in theologia*. Since it is the most relevant to the present topic – i.e. the fallacious reasoning in the statements made by Amalricians according to Garnier of Rochefort's *Contra Amaurianos* – it is also the only type of fallacy I will be discussing in this article.

In his translation of the *Sophistical Refutations*, Boethius renders the word *ὁμωνυμία* as *equivocatio*. The anonymous author of the *Fallaciae in theologia* provides a definition of *equivocatio*: "Equivocation is a different meaning of the same word. The fallacy of equivocation is the error that arises from the different meanings of the same word"[31]. The author then distinguishes several types of fallacy of equivocation, eight types to be exact. The following examples fall into two types, namely "ex varia transumptione"[32] and "ex vario officio":

> "Now the changing transfer of the word occurs in this example: The Holy Spirit is in this stone, or everywhere, but the Holy Spirit is charity, so charity is in this stone or everywhere. Now the meaning of this name 'charity' changes; for although it is said in the proper sense of the virtue, it is transferred to the Holy Spirit. Now, in the phrase 'charity is in this stone', the virtue is meant, or this preposition 'in' indicates the inherence of the form in the subject, not its essence. But in the phrase 'the holy spirit is in this stone', what is meant is that it is in it by essence, not by inherence. There is thus a double equivocation: the first comes from the transfer of the noun, the second from the changing role of the preposition"[33].

In the *fallacia ex vario officio* the equivocation is caused by the different roles played in a proposition by the different parts of speech. It is the preposition "in" that is examined in the following extract:

> "The sentence is misleading because of the changing role in it [*of the preposition 'in'*], as follows: 'everything that is in God is God', but the punishment of this one is

30. Rothschild 2013.

31. *Fallaciae in theologia*, ms. Paris, BnF, français 19951, f. 33v (transcription Francesco Del Punta): "Equivocatio est dissimilis eiusdem vocis acceptio; fallacia equivocationis est deceptio proveniens ex dissimili eiusdem vocis acceptione".

32. Giusberti 1982, p. 97 writes "ex vocabuli transumptione" but I follow Del Punta's transcription "ex varia transumptione".

33. *Fallaciae in theologia*, ms. Paris, BnF, français 19951, f. 41v (transcription Francesco Del Punta): "*Ex varia transumptione* [...]. Incidit autem varia transumptio in hoc exemplo: 'Spiritus Sanctus est in hoc lapide vel ubique, sed caritas est Spiritus Sanctus; ergo caritas est in hoc lapide vel ubique'. Variatur autem significatio huius nominis 'caritas', nam cum proprie dicatur de virtute a se transumitur ad Spiritum Sanctum. Dicto autem 'caritas est in hoc lapide', supponitur virtus, vel hec prepositio 'in' notat inherentiam non essentiam forme ad subiectum. Sed dicto 'Spiritus Sanctus est in hoc lapide', intelligitur quod est in eo per essentiam non per inherentiam. Est ergo hic duplex equivocatio: una surgens ex transumptione nominis, alia ex vario officio prepositionis".

> in God, so the same punishment is God. Here we are misled by the fact that the role of the preposition changes. In fact, in the sentence 'all that is in God, is God', this preposition 'in' indicates the essence, since all that is in God by essence, is God Himself. But when it is said 'the punishment of this one is in him', this must be understood to mean that it is in God according to foreknowledge. In the same way, when it is said, 'All that is in the Father is the Father, but the Son is in the Father, therefore the Son is the Father': in the first proposition the essence is designated, in the second the separation of persons; for it is said that the Son is in the Father according to generation. In the same way, this example which we have adduced above can explain another case: 'The Holy Spirit is in everything, but the Holy Spirit is charity, therefore charity is in everything', which is false. Indeed, in the conclusion [*of the syllogism*] this preposition 'in' indicates the inherence of the form in the subject; yet, in the major premise it indicates the essence, i.e. 'he is everywhere'. [...]. One is misled in both cases by the fact that the ablative can indicate the formal cause or the efficient cause"[34].

The anonymous author explains the various pitfalls behind the prepositions: 'in', 'ex', 'a', 'apud'. It is all these subtle distinctions that help to unmask false reasoning. The *Fallaciae in theologia* clearly illustrate all the logical traps and their application to theology. It is perhaps an attempt to create a foundation for theology as a science by laying down precise logical rules that should be observed in theological discourse.

In conclusion, the *Fallaciae in theologia* seems to be useful for understanding Garnier of Rochefort's *Contra Amaurianos*. However, all these discussions can be better understood if they are situated in the theological debate of the second half of the twelfth century. Simon of Tournai also, like Alain of Lille, argues for the impropriety of using terms that properly apply to creatures to speak of God, in other words he defends the equivocity of theological discourse: the terms are applied to God, only in an improper sense. There is thus a *translatio*, that is to say a transfer or a *transumptio*, as the author of the *Fallaciae in theologia* says. Praepositinus of Cremona, on the other hand, was an advocate of the univocity of language; the same terms apply according to the same meaning to creatures and

34. *Fallaciae in theologia*, ms. Paris, BnF, français 19951, f. 44r (transcription Francesco Del Punta): "*Ex vario officio dictio* his fallitur hoc modo: 'quidquid est in Deo Deus est, sed damnatio istius est in Deo; ergo ipsa Deus est'. Hic fallitur ex eo quod prepositionis variatur officium. Dicto enim 'quidquid est in Deo Deus est', hec prepositio 'in' notat essentiam, quia quidquid est in Deo per essentiam est Deus ipse. Sed cum dicitur 'damnatio istius est in eo', ea ratione †dictum intelligere† quod est in Deo secundum prescientiam. Simile est cum dicitur 'quidquid est in Patre, Pater est, sed Filius est in Patre; ergo Filius Pater est': in prima propositione notatur essentia, in secunda disiunctio personarum; dicitur enim Filius esse in Patre per generationem. Ad idem spectat hoc exemplum, quod supra posuimus ad aliud ostendendum: 'Spiritus Sanctus est in qualibet re', sed Spiritus Sanctus est caritas; ergo caritas est in qualibet res, quod falsum est. Nam in conclusione notat hec prepositio 'in' inherentiam forme ad subiectum; in maiori autem propositione notatur essentia iuxta quam modum, scilicet 'est ubique' [...] [f. 46v] Fallitur iterum ex eo quod ablativus potest notare causam formalem vel causa efficientem".

to God[35]. It is most probably this theory of univocity that Amaury of Bène had followed and interpreted in a crude way, judging from what Garnier of Rochefort suggests. At the meantime, several studies converge to show that Garnier of Rochefort had used works by Parisian masters to expose and to contest the theses of the Amalricians. Further research will be necessary on unpublished texts, in particular on the summas of theology from the end of the twelfth and beginning of the thirteenth centuries, in order to shed light on this period, which is ultimately very poorly known, marked by the beginnings of the University of Paris.

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Abstract: The *Fallaciae in theologia* is an anonymous, still unpublished text from the late twelfth or early thirteenth century on the errors that can be made in theological discourse. Ten manuscripts are known, dated from the thirteenth to fifteenth centuries. The author is probably a Parisian master, perhaps from the circle of Peter the Chanter. The *fallacia equivocationis*, as illustrated in the *Fallaciae in theologia*, is related to the fallacies committed by the heretic disciples of Amaury of Bène, according to Garnier of Rochefort in the *Contra Amaurianos*. In general, heretics are often accused of using fallacious reasoning during the Middle Ages.

Keywords: Theology; Amalrician heresy; *fallaciae*; Parisian twelfth-century schools; Garnier of Rochefort.

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Byzantine Treatments of Fallacy: The Reception of Aristotle's Account*

The relevance of argumentation theories in Byzantium does not need much proof – a short look into the Byzantine theological debates and dialogical literature suffices to make the point that argumentation and its related fields (particularly logic, dialectic and rhetoric) played an important role in the Byzantine culture. Although some aspects of the dialectical perspectives of Byzantine debates have been already studied[1], there is still much work to be done both on the theoretical level of examining the argumentation theories as such and on the practical level of studying the application of such theories in the actual Byzantine dialectical and rhetorical practice. This paper aims at examining a part of the elements related to argumentation, that is, false reasoning. Even so, I shall not manage to deal with all accounts and instances of false reasoning in Byzantium, but I shall necessarily restrict my inquiry even further by focusing on the Byzantine reception of Aristotle's accounts of fallacies. I shall not deal with material found in theological texts or rhetorical treatises, but I shall focus on the philosophical discussions of the matter, in particular as expressed in Byzantine commentaries dedicated to Aristotle's treatises.

Given the fact that Byzantine philosophy still remains in many respects an uncharted field, a study of the Byzantine treatments of fallacy needs to start with (I) an overview of the material, that is, of the primary sources of our inquiry. After this overview, I will focus on the Byzantine discussions regarding the sources of false reasoning. By this I mean (II) the material and formal sources of fallacies, and (III) the means of deception. The first point refers to a well-known and widely used distinction between the matter and form of an argument, whereas the second point refers to the practice of producing fallacies, for instance by

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1. For instance, Cameron / Gaul 2017; Kaldellis / Sinniosoglou 2017.

means of hiding the relevant information. Related to this point are also references to the detection of fallacies.

1. Overview of the Material

The first place to look when examining Aristotle's account on fallacies is his treatise on the *Sophistical Refutations* and luckily the Byzantines also thought so and thus composed several commentaries on this text. The Greek commentary tradition of the *Sophistical Refutations* (*Soph. el.*) has been already examined in depth by Sten Ebbesen, who in his exemplary work on this tradition presented the interrelation between the various commentaries as well as their sources[2]. I will rely in what follows on his analysis, but, as will be shown in the next sections, I aim to give a more positive evaluation of particular aspects of the commentators' interpretation than the one Ebbesen offers, who argues that there is nothing original in the Byzantine treatment of fallacies (as well as of other topics)[3]. Therefore, I will not deal here with the relation and dependence of one commentary on the other, since Ebbesen has established this already. Instead, I will focus on the texts that he showed to be particularly influential and present some interesting philosophical aspects of their account. Among the most independent and complete commentaries (in the sense of commenting on the most of the Aristotelian text and not consisting on a few single scholia) the most influential is the commentary written by Michael of Ephesos (twelfth century). His work, which Ebbesen shows to consist in scholia that Michael collected from earlier material and edited into a unified commentary, survives in two versions, the later and more elaborate of which is printed in the *Commentaria in Aristotelem Graeca* II.3. An earlier collection of *scholia* ('Commentary II' in Ebbesen) as well as an eleventh-century compendium of logic have been identified as sources of Michael's commentaries. The compendium, known as the Anonymous Heiberg, contains a summary of the Aristotelian logical treatises (except for the *Topics* and *Posterior Analytics*) including an account of fallacies. An anonymous commentary from the twelfth century ('Commentary III' in Ebbesen) is also a useful source of information, since it is not only original in its interpretation, but also influenced subsequent commentators like Leo Magentinos (late twelfth - early thirteenth century). The latter author, known for his commentaries on Aristotle's *Organon*, composed a commentary on the *Sophistical Refutation* for which he draws, according to Ebbesen, on both Commentary III and, on Michael's late commentary. Two additional texts from the thirteenth century are also useful for our purposes: first, Nice-

2. Ebbesen 1981, see especially chapter 5. My short presentation of the Byzantine commentaries in this paragraph is based on Ebbesen's analysis in this chapter.

3. Ebbesen 1981, I, p. 60 *et passim*.

phoros Blemmydes' compendium of logic and physics form c. 1260 includes an account of fallacies; second, a paraphrase of the *Sophistical Refutations* attributed to Sophonias (thirteenth - fourteenth century) and edited in *Commentaria in Aristotelem Graeca* XXIII.4. This paraphrase was to be the source for Chortasmenos' paraphrase of the *Sophistical Refutations*. John Chortasmenos (fourteenth - fifteenth century) composed *scholia* also on other treatises of the *Organon*, including the *Topics*, as well as introductory notes on Aristotle's logic (*Prolegomena to Logic*) and rhetoric (*Prolegomena to rhetoric*), both of which comprise material relevant to fallacies.

In fact, the rhetorical treatment of fallacies can be traced back to Aristotle, who in his treatise on *Rhetoric* dedicated a chapter to fallacious enthymemes (II 24). The two extant commentaries on the *Rhetoric*, both written in the twelfth century, one by an anonymous author and another by some Stephanos, present an analysis of this chapter which, as will be shown below, exhibits many similarities to other Byzantine texts on fallacies, but they also present in a unique way issues pertaining to Aristotelian syllogistic and the means of deception.

Finally, information about the Byzantine treatment of fallacies can be found in scholia on Aristotle's *Topics*[4]. For instance, a fourteenth century manuscript (Hier.Patr. 150) includes both scholia that have been attributed to Michael of Ephesos and a long extant commentary, which is also found in further manuscripts[5]. Leo Magentinos, who was mentioned above as the author of a commentary on the *Sophistical Refutations*, also wrote a commentary on the *Topics* accompanied by a prooemium that resembles his prooemium on the commentary on the *Sophistical Refutations* regarding his analysis of the role of dialectic[6]. John Chortasmenos, besides the introductory account on logic mentioned above, is also the author of an introduction to dialectic and of scholia on the *Topics*. Finally, a few independent treatises on dialectic written in the Byzantine period also comment on Aristotle's *Topics* and entail discussions on fallacies, for instance John Italos' (eleventh century) *Treatise on Dialectic* and *Inquiry on the matter of syllogisms and their construction*.

As we will see, all these texts offer material relevant to the study of fallacies in Byzantium. We will focus on particular aspects of this material, as sketched out in the introduction.

4. For a detailed presentation of the commentary tradition on the *Topics* see Kotzabassi 1999.
5. ms. Par.gr. 1845; ms. Vat.Regin. 116; ms. Par.Coisl.327 (in excerpts).
6. See Ebbesen 1981, II, pp. 280 ff.

2. Form and Matter of a Syllogism

The most elemental question that needs to be asked is what it means for an argument to be fallacious. Aristotle, both at the beginning of the *Topics* and in the *Sophistical Refutations* distinguishes between two types of contentious or sophistical arguments: those that syllogize from apparently acceptable premises and those that appear to syllogize from acceptable or apparently acceptable premises (*Top*. I 1, 100a24-101a4; *Soph. el.*, 164a20-165a4). The former, he states, are syllogisms whereas the latter are not syllogisms, since they are non-deductive, meaning that the conclusions do not follow from the premises[7]. Fallacious or sophistical arguments include then (1) syllogisms formed from apparently correct/acceptable content, and (2) apparent syllogisms.

Besides this distinction regarding the sources of fallacy, the *Sophistical Refutations* draw on a further distinction, namely the one between fallacies dependent on the linguistic expression (*in dictione*) and fallacies independent of the linguistic expression (*extra dictionem*). A problem arises due to the fact that Aristotle does not explain the relation between the two distinctions. Are the thirteen linguistic or non-linguistic fallacies discussed in this treatise fallacious due to their not leading to a conclusion through the used premises or due to the content of the premises? At a later point in the treatise (*Soph. el.*, 168a18 sqq.), he argues that the distinction between fallacies *in dictione – extra dictionem* might be unnecessary, since all fallacies can be said to come about due to the ignorance of the definition of refutation and syllogism. If this is so, then Aristotle probably took the source of all thirteen fallacies to be rather formal, since they characterised by failing to be syllogisms. The same problematic applies to the list of fallacies in the *Rhetoric*, where Aristotle explicitly speaks of apparent enthymemes and states that they are non-syllogistic in the sense that they appear to deduce without really deducing without even referring to cases in which apparent enthymemes are apparent in the sense that they proceed from apparent true/acceptable/probable premises while still being syllogistic.

These points are dealt with by the Byzantines, who seem to make two relevant presuppositions. The first regards the analysis of arguments in terms of form and matter. The second, related to the first one, regards the relation between sophistical and other types of arguments. Starting from the former, the Byzantines adopted a late antique distinction between logical form and logical matter of an

7. It has therefore been argued that the fact that Aristotle calls both types of arguments contentious or sophistical means that we cannot simply identify fallacious arguments with invalid arguments, since the apparent acceptable arguments would still be 'valid'. However, it should be noted that we cannot speak of valid and invalid syllogisms in the sense of the formal Aristotelian syllogistic, when examining the account of the *Topics* and *Sophistical Refutations*, which do not seem to entail traces of Aristotle's figure-syllogistic. See King 2013, p. 193.

argument, that is, between the figure of the syllogism used and the various material that can fit into this figure. This division was introduced already by Alexander of Aphrodisias, who called the three syllogistic figures forms which can be filled with different sorts of matter[8]. Alexander referred to the different sort of matter as the premises appropriate to the different types of syllogism: even though the syllogism remains the same with respect to its form, if the premises are true, then the syllogism is demonstrative; if they are acceptable, then it is dialectical; if they are only apparently acceptable, the syllogism is sophistical (*In Top*. 2, 15-26). Although this would imply that sophistical syllogisms are only materially defective, elsewhere, he refers to eristic or sophistical syllogisms as being also formally defective (*In Top*. 21, 13-19). Similarly, other late antique authors also adopted the distinction between matter and form and refer to sophistical syllogism as being defective either in form or in matter[9].

Contrary to this view of the sources of sophistical syllogisms as being both formal and material, some Byzantine thinkers, while adopting Alexander's division into form and matter, take only those syllogisms that are materially defective to be sophistical[10]. As we shall see, they argue that sophistical syllogisms differ from dialectical and demonstrative ones only with respect to their matter, while sharing the same form. In this view and contrary to Aristotle's first distinction presented above, sophistical syllogisms are only those that syllogize from apparently acceptable or apparently true premises, but are still syllogisms. The non-deductive ones, namely those defective in form, will then not fall under the type of sophistical arguments.

The earliest text to express this view is Anonymous Heiberg, who explicitly states that sophistical syllogisms are not deceptive due to their logical form,

8. Alexander Aphrodisiensis, *In Aristotelis Analyticorum Priorum librum I commentarium*, ed. Wallies, p. 6, ll. 16-21 and p. 52, ll. 19-25; Alexander Aphrodisiensis, *In Aristotelis Topicorum libros octo commentaria*, ed. Wallies, p. 2, ll. 9-15. See Lee 1984, pp. 38-39.

9. See for instance Ebbesen 1981, I, p. 94 where he notes that Philoponus made some controversial claims regarding sophistical syllogisms, sometimes stating that they are all materially defective, sometime that they are formally defective.

10. In focusing on the material defects of syllogism and thus analysing the fallacies of the *Soph. el.* as being primarily materially false they deviate from earlier authors such as Alexander or Philoponus. In this, therefore, I disagree with Ebbesen 1981, I, p. 96. I should note, however, that there are also Byzantine authors who adopt Alexander's distinction, but repeat the problematic issues of the Aristotelian text without explicating in which sense sophistical arguments are fallacious or how we are to understand the treatment of particular fallacies of the *SR* and *Rhetoric*. For instance, Commentator III on the *Sophistical Refutations* comments on Aristotle's initial remark about the distinction between true and apparent syllogisms (164a23-24) and states that it corresponds to the distinction between matter and form without explicating further how this distinction applies to the analysis of the thirteen fallacies. Leo Magentinos on the other hand, when commenting on the same passage, states that it refers to the distinction between demonstrative/dialectical and sophistical syllogisms, but without indicating whether this means that the terms 'true' or 'apparent' are meant formally (namely deductive/non-deductive) or materially (namely for demonstration true and primary, for dialectic truly acceptable, for sophistic apparent).

but due to their matter by making use of premises that appear to be true[11]. This view has certain implications for the interpretation of particular fallacies. In fact, Anonymous Heiberg's statement about the material source of the sophistical arguments is followed by his summary of the thirteen fallacies presented in the *Sophistical Refutations*, indicating that he takes these fallacies to be defective in their matter and not in their form. This view, although it contradicts Aristotle's statement that sophistical arguments may be formally defective, is an interesting solution to an ongoing debate regarding Aristotle's description of particular fallacies. As mentioned above, after the initial statement of the *Sophistical Refutations* of the two types of fallacies (the non-syllogistic and the apparently endoxastic ones), this distinction does not appear in the presentation of the thirteen fallacies and Aristotle does not specify in what sense he takes the fallacies to be defective, but only later does he reevaluate his analysis and state that all fallacies can be traced back to the ignorance of the definition of refutation and syllogism (*Soph. el.*, 168a18 sqq.), thus implying that they are all formal fallacies. However, this creates problems in our understanding of particular fallacies[12], problems that can be solved by Anonymous Heiberg's solution of taking all fallacies as materially defective.

The view of Anonymous Heiberg is also shared by other Byzantine authors. Although not explicitly referring to the distinction between matter and form, Commentator II on the *Sophistical Refutations* interprets the fallacies of the treatise primarily as being false independently of the soundness of the syllogism. His interpretation of the few references to fallacies being non-syllogistic is indicative of this strategy: in some cases, he directs the reader to the *Analytics'* figure syllogistic (*In Soph. el.*,167b34-35), whereas in another he states that the conclusion does not follow from the premises, but takes this to mean that the premises have concluded something different from what was presented as the conclusion, in which case the paradoxical conclusion has not even been deduced (*In Soph. el.*, 168a21). In this latter passage, he neither refers to the 'form' of the syllogism nor explains how this syllogism is unsound. His interpretation rather reminds us of Aristotle's description of the linguistic fallacy in the *Rhetoric*, which is said to come about in dialectic when something is presented as the conclusion of the argument without having been deduced (1400a1-13)[13]. On the other hand, in other passages, he argues that the syllogistic as presented by

11. *Anonymi Logica et Quadriuium cum Scholiis Antiquis* (= *Anonymus Heiberg*), 49, ed. Heiberg, p. 39, ll. 14-20: "ὥσπερ δὲ τῶ σίτῳ συναναφύεται τὰ ζιζάνια, καὶ τοὶς ῥόδοις παραπεπή-γασιν ἄκανθαι, καὶ τοῖς σωτηρίοις τῶν φαρμάκων συντρέχει τὰ δηλητήρια, οὕτω καὶ τοῖς συλλογισμοῖς οἱ παραλογισμοὶ συμπεφύκασιν οὐ τῶ τρόπῳ τῶν συλλογισμῶν τὸ πταιστὸν ἐπαγόμενοι, τῇ δὲ ὕλῃ καὶ ταῖς σεσοφισμέναις προτάσεσιν ἀληθείας χρῶμα περικειμέναις πρὸς ἐξαπάτην τοὺς ἀμαθεῖς παρασύροντες".

12. Accordingly, he has been accused of wrongly presenting non-formal fallacies as being formal. See King 2013, pp. 197-198; Kirwan 1979.

13. Cf. *In Soph. el.*, 174b8-9; 171a6-7, where the commentator argues that by 'syllogism' Aristotle

Aristotle in the *Prior Analytics* underlies the theory of the *Sophistical Refutations,* since the former treatise 'includes' (*periechei*) the principles of the latter (*In Soph. el.*, 177a16-18).

John Italos' treatise *Inquiry on the matter of syllogisms and their construction* also adopts Anonymous Heiberg's scheme and introduces the distinction between matter and form of a syllogism: by form, he states, he means the syllogistic figures, whereas by matter he means the truth or falsity of the propositions that construe the syllogism (p. 1, ll. 1-10). Interestingly, his exposition of the syllogistic forms took place in an earlier treatise *On Dialectic*, where, as we will see below, he also argued that dialectic, demonstration and sophistic differ only with respect to the truth-value of their premises, that is, their matter (p. 2, ll. 3-11). The examination of the matter of syllogisms is the subject of the former treatise, in which Italos undertakes to inquire into matter both generally and particularly with an analysis of syllogisms (p. 1, ll. 11-13). What the treatise then includes is, first, a division of matter into demonstrative, dialectical and sophistical (Ch. 4), and second, an analysis of the *topoi* used for the construction of syllogisms (Ch. 5 sqq.). What is interesting in this inquiry is, first, the very fact that the analysis of the forms and matter of syllogism is the subject of different inquiries. This is justified, of course, by Italos' view that the inquiry into form is more general, since form applies to all kinds of syllogism. Second, the *topoi* are recognized as mechanisms for the construction of all syllogisms but with respect to their matter – the inquiry into the *topoi* is namely described as a part of the more general description of matter of syllogism. Hence, the *topoi*, defined as starting points of argumentation (p. 4, ll. 7-9), provide the matter of all types of syllogism (demonstrative, dialectical and sophistical). In his treatise on *Rhetoric*, Italos adds rhetorical syllogisms to this scheme and argues that they also make use of the distinctions already made (p. 37, ll. 21-23)[14]. Here, he repeats his view regarding *topoi* being starting points for the construction of all syllogisms and adds that the difference between rhetorical syllogisms, on the one hand, and dialectical and sophistical syllogisms, on the other, is that rhetorical problems use only accidents as starting points, whereas, dialectical and sophistical syllogisms might also start from a genus, a definition or a proper attribute (pp. 43-44, ll. 16-5), i.e. the four predicables of the *Topics*. The fact that rhetorical syllogisms are said to be constructed from accidents probably derives from Aristotle's statement that rhetoric makes use of signs and

means simply the conclusion, and examines how the falsity can lie either in the premises (when the question entails a contradiction) or in the conclusion (Cf. *In Soph. el.*, 177a12).

14. Anonymous Heiberg (Ch. 64) also includes rhetorical as well as poetical syllogisms in his presentation of the sorts of matter that can fit into the forms of the syllogistic figures. Also in his view, the difference between the various types lies in their truth-value. Demonstrative are always true, dialectical are usually true, rhetorical can be equally true or false, sophistical are usually false and poetical are always false. This is an interesting description, especially because it implies that there are sophistical arguments that are true, that is, that accidentally have true conclusions.

probabilities, which in the *Rhetoric* are further analyzed in terms of their relation to what they signify[15].

Italos' exposition of a criterion for differentiating rhetorical from dialectical/sophistical syllogisms leads to the question of the relation between the other types of syllogisms. Since the form is always the same, the criterion of differentiation between the various types of syllogisms is the matter. I mentioned above Alexander's presentation of the difference between demonstrative, dialectical and sophistical arguments. Italos also refers to this issue: demonstrative syllogisms are from primary and true premises and have necessary subject matter; sophistical syllogisms are from particular and false premises constructed from imaginative things, and have a contingent subject matter, whereas dialectic can deal both with the necessary and the contingent. A similar thought had been expressed before by Philoponus (*In. An.Pr*. pp. 1-4, ll. 1-25), who also presented the difference between the three types on the basis of the rational capacity responsible for the construction of each and argued that demonstrative syllogisms are always true (getting their principles from the intellect), sophistical always false (getting their principles from *phantasia*), and dialectical being both true and false (depending on opinion)[16]. However, what is new in Italos' analysis is his presentation of dialectic as being the middle between the two extremes: dialectic shares in both extremes and for this reason it is a capacity (*dynamis*) to both construe and refute an argument[17]. Unlike Philoponus' description of sophistical syllogisms as aiming at refuting every view that is considered as true, Italos claims that dialectic aims at refuting the arguments of the sophists as well as at constructing the true arguments, while being able to both refute and construct what is in between (for instance, what falls under the realm of ethics) (*Dial*., pp. 2-3, ll. 3-9). Besides the implications this view has for the role of dialectic, it is also important for grasping Italos' understand-

15. See Arist., *An.Pr.*, 70a10 and Arist., *Rhet*., 1357a32-33. For the connection between signs and accidents also in the commentaries on Aristotle's *Rhetoric* see Vogiatzi 2019, pp. 130-134.

16. For an analysis of Philoponus' account see Ebbesen 1981, I, pp. 91-94. Although Philoponus might be a source of the Byzantine reference to the psychological sources of false reasoning, it is important to note that such references to the sources of reasoning within the soul are very common in the Byzantine literature, especially in the *Prolegomena* to logic and rhetoric. Of the texts examined so far, Anonymous Heiberg (Ch. 64) offers a similar account as Philoponus, which is later also adopted by John Chortasmenos in his *Prolegomena* to logic. See Hunger 1969, pp. 210-214. However, the Byzantine *Prolegomena* to rhetorical treatises offer similar accounts extensively arguing for the part of the soul responsible for reasoning or art (See Rabe 1931). Therefore, I take it, we can consider such references as something characteristic of Byzantine *Prolegomena*-literature, even though it can be found in earlier late antique texts as that of Philoponus.

17. Ioannes Italus, *Dial*., p. 2, ll. 3-9: "Τὴν διαλεκτικὴν οἱ παλαιοὶ ἀποδεικτικῆς μέσην οὖσαν καὶ τέχνης σοφι-στικῆς εἰκότως αὐτὴν δύναμιν ἐκάλεσαν ὡς τῶν ἄκρων μετέχουσαν καὶ τὸ αὐτὸ δυνάμενην κατασκευάζειν ἅμα καὶ ἀνασκευάζειν. Λέγω δὲ ἄκρα τὴν ἀποδεικτικὴν καὶ σοφιστικήν, ὡς τὴν μὲν ἐκ πρώτων καὶ ἀληθῶν οὖσαν, τὴν δὲ ἔκ τινων φανταστῶν τοὺς λόγους ποιουμένην, οὓς δὴ ἐπὶ μέρους τε καὶ ψευδεῖς εἶναι συμβέβηκεν".

ing of the role of sophistical arguments. As mentioned above, sophistical syllogisms, being particular (*epi merous*) and false (*pseudeis*), are said to be constructed out of certain imagined things (*ek tinôn phantastôn*). The reference to imagination is most probably related to Philoponus' connection of sophistical syllogism to *phantasia*. What is new in Italos is the reference to the *logoi* of the sophistical syllogisms as being both based on certain imagined principles and particular. This is repeated in various contexts: in a later passage of the treatise *On Dialectic*, John argues that categorical syllogisms may be either demonstrative and scientific or dialectic and endoxastic or sophistic and dealing with ultimate particulars (*tôn eschatôn*) (p. 4, ll. 18-19). Similarly, in his treatise on the *Rhetoric*, he states that sophistic differs from both dialectic and rhetoric in that the latter are universal, while sophistic is particular (p. 44, ll. 6-8). How are we to understand these statements? There seem to be two options: the particularity refers either to the quantity of the premises or to the kind of views the sophist employs. By the first, I mean that the premises are particular as opposed to universal, for instance 'Pittacus is good; Pittacus is wise; Therefore, all good are wise', a syllogism that is false by deducing universally from particular premises. However, this cannot be what Italos has in mind. First, this would contradict his often-expressed view that the sophistical arguments are primarily materially defective. Second, there are many sophistical arguments that do not deduce from particular premises and we have seen above how he explicitly took the *topoi* to be starting points also of sophistical syllogisms[18]. What remains is the second option, namely that sophistical syllogisms start from the ultimate particulars in the sense that it is a particular view – the view of the opponent – that the argument aims at refuting. This may also be connected with Aristotle's statement in the *Rhetoric* (1356b28-35) that neither rhetoric nor dialectic examine what is persuasive to one person, but what is persuasive to *such* a person. In this sense, rhetoric and dialectic (as well as other arts) examine what is universally applicable. We would expect that for Aristotle sophistic (which follows the principles of dialectic) is also such an art, but apparently John has a different view. According to the references to the particularity of sophistic mentioned above, it seems that in his view sophistical arguments are not universal in the way dialectical and rhetorical ones are and they are not endoxastic in the way of dialectical syllogisms, but are from ultimate particulars. This, I take it, means that sophistical syllogisms do not take as starting point what a certain type of people might accept (namely the *endoxa* from which dialectic starts), but what particular occasions require, namely the particular views/arguments that the opponent will accept in this particular setting. The issue of

18. Especially in cases where *topoi* are used as universal premises of a syllogism it is impossible that both premises are particular.

acceptability of the premises is very important for the study of fallacies, especially in the understanding of fallacies as materially false. As mentioned above, Aristotle's description in the *Topics* and the *Sophistical Refutations* of what we called material defect is based on the acceptability or apparent acceptability of the premises[19]. If then my understanding of Italos' description of sophistical syllogisms is right, then sophistical syllogisms are not simply those that are false, but those that are (falsely) accepted by the particular person at whom the argument is directed. This is a point that also underlies Michael of Ephesos interpretation of the goals of contentious reasoning. As he points out, the goals of the sophists (refutation, falsity, paradox, solecism and garrulity) are all apparent, meaning that they do not need to be true, but it suffices if they are accepted as such by the interlocutor (*In Soph. el.*, 20.10-13, pp. 74-75, ll. 20-12). This view is interesting with respect to the authors' understanding of what a fallacy is: it is not simply a bad or false argument, but we can speak of fallacy only in cases when the argument has the appearance of a true argument and is accepted as such. An invalid argument that does not look true will not be a fallacy, because it will not convince[20].

A confirmation of this view can be found in John Italos' treatise on the *Rhetoric*, which starts by comparing rhetoric and dialectic. As he states, the two disciplines differ with respect to their addressee and subject matter as well as with respect to the way they produce false reasoning: rhetoric is usually wrong due to making use of affirmative and particular premises, whereas dialectic is wrong due to the proposition it uses and the disposition (*diathesei*) of the respondent (p. 33, ll. 7-9). Taking this statement together with what we said above, it seems that a rhetorician might produce false reasoning by making frequent use of particular premises in the first sense of particularity mentioned above, whereas the dialectician will produce a fallacy, when his argument, besides being false, does not meet the particular view accepted by his opponent[21]. An issue that requires clarification is that rhetorical syllogisms are said to be often formally defective, namely due to their deducing (universally) from two affirmative and particular premises, which would be a syllogism of the third figure. However, the source of the confusion might lie both with Italos' effort to include rhetorical syllogisms in his previous description of the other types of syllogisms, which differ from one an-

19. On the issue of acceptability of the premises see also next section, where I present the interpretation of the Anonymous commentator on the *Rhetoric* of the acceptability of premises of rhetorical syllogisms. Interesting is that he uses the point of what is accepted by the audience for explaining what can be left out for the sake of brevity.

20. I'd like to thank Prof. Peter Adamson for pointing out to me that I need to emphasize this point.

21. Based on what Aristotle says about sophistic differing from dialectic with respect to the *prohairesis* (Arist., *Rhet*., 1355b15-21) and also based on how John Italos argued that dialectic shares on both apodeictic and sophistic, I take that when he talks here of the dialectician producing false reasoning, he might as well mean the sophist.

other only with respect to matter while sharing the same form, and with Aristotle's description of enthymemes and apparent enthymemes in the *Rhetoric*. As we will see below, both of these points are also addressed by the Byzantine commentators on Aristotle's *Rhetoric*.

A point that needs to be examined is in what respect syllogisms are materially defective. I have referred so far to defective premises with respect to matter as being false. However, as I mentioned already, Aristotle speaks of this type of sophistical syllogisms as having apparently endoxastic premises. In other words, fallacy is produced not only when the premises appear to be true while being false, but also when they appear to be acceptable despite not being so. Although some Byzantine authors took 'apparently acceptable' simply to mean false[22], the meaning of 'apparently acceptable' has caused trouble for others: what would be an *endoxon* that appears to be acceptable but is not really?[23] Already Alexander offered an interpretation of the meaning of 'apparent *endoxa*' by giving examples such as 'what you have not lost, you have', from which one deduces 'you have not lost horns; therefore, you have horns' or 'every manly/brave man (*andreios*) has bravery', from which one deduces 'the tunic is manly (*andreios*); therefore, the tunic has bravery (*andreia*)' (*In Top*., p. 19, l. 28; p. 23, l. 19). In both cases the *endoxon* is apparent because the multiplicity of the meanings of the statements escapes the notice of the interlocutor. The last example is repeated by Michael of Ephesos with the explanation that we have to do with a material fallacy, since the syllogism is formally sound by being deduced in the first figure having one universal and one particular premise (*In Soph. el.*, p. 4, ll. 7-20). Just as Alexander, Michael notes that sophistical arguments (being the same as contentious arguments)[24] have two forms: they are either defective in matter by coming about from such apparent *endoxa* or they are defective in form, in which case they are not even syllogisms. Although this seems to be a step away from the authors that we examined above who took sophistical syllogisms to be only materially false, Michael later returns to this view taking a step away from Alexander: sophistical and contentious syllogisms are properly speaking those that are defective in matter, whereas those syllogisms that are formally defective (without excluding that they are also materially defective) are called paralogisms (*In Soph. el.*, p. 7, ll. 4-13; p. 8, l. 7 sqq.). As I said above for John Italos and Anonymous Heiberg, this view has implications for Michael's interpreta-

22. See Leo Magentinos, *In Top. I* (Ebbesen 1981, I, p. 95); Commentator II on the *Sophistical Refutations, In Soph. el.*, 171b36-37 (Ebbesen 1981, II, p. 86-88) states that both dialectical and sophistical syllogisms use accepted *endoxa*, the difference being that in dialectic the accepted *endoxa* are truly *endoxa*, whereas in sophistic they are not.

23. See King 2013 for an overview of the problematic issue.

24. See Ebbesen 1981, I, p. 96 on the Byzantine interpretation of the first passage of the *Soph. el.* and the identification of sophistical with contentious syllogisms.

tion of the particular fallacies of the *Sophistical Refutations*. As he argues, it is only for clarity that Aristotle refers in the latter treatise to formal aspects of syllogisms, since syllogism generally speaking (*haplôs syllogismos*) is the genus of the particular types of syllogism and genera need to be examined or recapitulated before their species (*In Soph. el.*, p. 7, ll. 15-22). In fact, in his interpretation of the sixth chapter of the *Sophistical Refutations* dealing with the reduction of all fallacies to the fallacy of *ignoratio elenchi*, Michael states that only the fallacies from taking the non-cause as cause, of begging the question and of the consequence are fallacies in the sense of not deducing an appropriate conclusion from the premises (*In Soph. el.*, p. 55, ll. 27-29). Michael even seems to criticize Aristotle's move in analyzing all fallacies in terms of their correspondence to the definition of syllogism and refutation, when he says '[Aristotle] says how they are to be reduced, or rather he himself reduces them' (*In Soph. el.*, p. 55, ll. 16-17). This does not mean that Michael denies that the other fallacies can also be formally defective and, in fact, in his analysis of the reduction of each fallacy to the ignorance of refutation, he often refers to the way in which the examples given by Aristotle are also formally fallacious. The point of criticism seems rather to lie with his understanding of the primary source of fallacy: we speak of a fallacy, for instance, from equivocation, consequence or accident, primarily because of their use of premises that appear to be true or acceptable without being so.

This also applies to the previous authors we examined: the fact that these authors take sophistical syllogisms to be defective in matter does not mean that they exclude that they might also be formally defective. The point of interest lies in what they take sophisms to be and in what respect they take sophistical syllogisms to be primarily fallacious. It seems then that the authors we examined so far take fallacies to depend primarily on the falsity of the premises, namely on the false or apparently acceptable content of the premises, from which the conclusion might be deduced even in a valid syllogism (but also in unsound syllogisms). Moreover, the point of interest lies on the authors' motivation in offering such a 'material' reading. In particular, this 'material' reading can be justified by the commentators' general approach towards the *Organon*: in the group of texts that we have examined so far, the *Organon* is said to include the treatises that deal with all types of syllogisms that fall under the figure-syllogistic of the *Analytics*. This would justify the inclusion not only of demonstrative and dialectical arguments, but also of rhetorical and sophistical ones – only if sophistical arguments are those that are materially defective, since invalid arguments cannot be sophistical syllogisms because they are not syllogisms at all. This is not the case, for instance, for the Byzantine commentators on the *Rhetoric*, who interpret fallacies primarily with respect to their formal defects. This second group of texts seems to also make the effort to unify Aristotle's various accounts, but, as we will now see, not by excluding the non-syllogistic parts of logic.

Turning now to the rhetorical fallacies, in his *Rhetoric*, Aristotle devotes his attention to what he calls the body of persuasion, namely enthymemes. He then examines the elements of enthymemes, that is, signs and probabilities as well as the rhetorical *topoi*, both the common and the specific to particular subject matters. His inquiry includes an examination of the apparent enthymemes, which, he says, correspond to the apparent syllogisms of dialectic (1356b1-2). In a later passage setting forth the plan of the following investigation, Aristotle states that he will examine the apparent enthymemes which 'are not enthymemes, since they are not syllogisms' (1397a3-4; cf. 1400b34-36). Similarly, when referring to sign arguments he states that tokens (*tekmêria*) are the only irrefutable signs, whereas the other signs are refutable by being non-syllogistic (1357b13-14; 1401b9-14; 1403a3-11). Although the *Rhetoric* does not make use of the figure syllogistic of the *Analytics*, in the latter treatise the 'non-syllogistic' language is replaced by reference to the syllogistic figures and it is argued that invalid sign arguments occur in the second and third rather than in the first figure (*Pr.An*. II 27, 70a12 sqq.). It seems then that Aristotle takes the rhetorical fallacies to be formal in the sense of only appearing to be syllogisms. This can be confirmed by his analysis of particular fallacies, for instance the fallacy from linguistic expression, which is said to come about when a statement is presented as if it is deduced without really being so (1401a1-7). Aristotle does not speak in the *Rhetoric* of enthymemes that are deduced from what appears to be probable or a sign without really being so or from what appears to be acceptable by the audience.

Although both commentators on the *Rhetoric* follow Aristotle and interpret fallacious arguments primarily as formal fallacies, there are aspects in both Stephanus' and the Anonymous' commentary that show that they make an effort to investigate the material defects of the fallacies as well. In particular, in his discussion of sign arguments, Stephanus states that a syllogism can be fallacious either in matter or in form or in both[25]. We seem then to return to the presentation of the distinction between formal and material fallacies found in Alexander: apparent enthymemes (just like sophistical arguments in Alexander) can be either formally or materially defective or both formally and materially. Stephanus explains what this means: the formal fallacy refers to the deduction in the second and third syllogistic figures (*In Rhet*., p. 265, ll. 19-24), whereas the material fallacy refers to the premises being false while appearing true (*In Rhet*., p. 266, l. 7). This last point is important in grasping the exact source of material rhetorical fallacy: the enthymeme is in this sense not formally fallacious, but the fallacy is due to the premis-

25. Interestingly, Stephanus explicitly connects the rhetorical account of fallacies to the account of the *Sophistical Refutations* and argues that the distinction between fallacies *in dictione – extra dictionem* applies also to the rhetorical fallacies. This means that, in his view, the rhetorical account of the fallacy from the form of the expression has the same content as the dialectical one. See Vogiatzi 2019, pp. 89 ff.

es having a certain appearance. Therefore, for a fallacy to come about it does not suffice that an argument is unsound (invalid), but it has to appear true in order for it to convince. In addition to this explicit reference to the material source of fallacy, Stephanos offers an example not found in Aristotle's text that can illustrate how a sign might be apparent. In particular, he argues that signs have a certain temporal relation to the signified thing: when they are simultaneous, then they are proper signs of the thing they signify, whereas if they precede or antecede the thing they might give the impression of being signs of the thing but falsely so (*In Rhet*., p. 266, ll. 20-26). It is possible that with this comment Stephanos expresses exactly the way in which enthymemes might be apparent by relying on apparent signs while still being enthymemes.

The Anonymous commentary also contains an account of rhetorical arguments as being fallacious due to the apparent acceptability of the premises. As we will see in detail in the next section, the Anonymous author argues that, since rhetorical syllogisms are more concise than other syllogisms, the orator has to omit some information that is already known, while the audience 'agrees' that what is left out is known. In this sense, a fallacy comes about due to the apparent acceptability of a premise that was omitted by the speaker (*In Rhet*., pp. 1-2, ll. 13-30).

However, as I mentioned above, the two Byzantine commentators on the *Rhetoric* interpret the rhetorical account of fallacies primarily as presenting formal fallacies. The reason for this might be twofold: first, especially in the analysis of sign arguments the parallel account of the *Prior Analytics* leads to the interpretation of sign arguments in view of the figure syllogistic, and as I have argued elsewhere[26], they take the analysis of sign arguments to be exemplary for the analysis of other fallacies as well, especially of the fallacies of consequent and accident; second, their effort to justify the *Rhetoric*'s position within the *Organon* leads them to a more analytical or 'logical' interpretation of many aspects of Aristotle's rhetorical account, including emotions and the ethical *topoi*.

This reference to the 'logical' interpretation of the rhetorical account leads us to another point of connection among the Byzantine authors. We have seen above how some Byzantine authors argued that all types of syllogisms, including demonstrative, dialectical, sophistical and rhetorical syllogisms differ from each other with respect to their matter, while still having the same form. This is an important point for many of the Byzantine authors who base their interpretation exactly on the fact that Aristotle's logical theory is unified in resting upon the figure-syllogistic. Anonymous Heiberg (Ch. 64) calls the syllogistic figures the instrument (*organon*), to which the various types of matter are subjected (*hypoballomenên hylên*), hence justifying the inclusion not only of *Rhetoric*, but also of the *Poetics* within the *Organon*. Interestingly, many Byzantine manuscripts

26. Vogiatzi 2019, pp. 114-134.

of these texts include diagrammatic depictions of the syllogisms, thus literary illustrating the 'form' of the syllogisms. Such depictions can be found besides the commentaries on the *Analytics*, in Stephanos' commentary on the *Rhetoric*, and the long Anonymous commentary on the *Topics*.

In conclusion, I take it that the Byzantine authors' adoption of the distinction between form and matter of the argument was central to their interpretation. The distinction was not only integrally incorporated into the interpretation of various aspects of the treatises, but it was particularly used as a mechanism to unify the Aristotelian logical treatises. It is, therefore, an unfair estimation to consider the division into matter/form as irrelevant to the interpretation of the *Sophistical Refutations* and of the other treatises discussed above, simply because they do not presuppose the syllogistic figures.[27] Although it is true that Aristotle's *Sophistical Refutation* (just as the *Topics* and the *Rhetoric*) do not presuppose the Aristotelian figure syllogistic, the Byzantine use of the distinction between matter and form is interesting precisely because it shows that the authors took the account of fallacies in Aristotle to share something with the account of the other types of syllogisms and thus justified Aristotle's treatment of it. The very fact that many authors took 'sophistic' to refer to the material defects of the premises confirms this: they all belong to the same discipline, that is logic. Especially in the non-rhetorical accounts, the fact that the 'material' reading prevailed is a strong way of suggesting that the discussions of fallacies are part of logic proper and that the *Organon* is unified and has the syllogistic as its criterion of unification. The rhetorical accounts aim at showing the unity of the *Organon* as well, also by justifying the treatment of invalid arguments: It is not only because the treatment of fallacies helps us to avoid fallacies that we should examine Aristotle's accounts, but it is also because a unified syllogistic account must entail an account of false reasoning.

3. Means of Deception

We have seen how the Byzantine commentators on Aristotelian treatises dealing with fallacy presented the sources of fallacious reasoning and how they integrated the distinction between form and matter into their interpretation of fallacies. However, there is a further question that arises concerning both the material and formal fallacies, namely why are we deceived or why we do not perceive the fallacies. In other words, what tricks are used so that a fallacy deceives one into giving their consent to the argument? Aristotle refers in the *Sophistical Refutations* and in the *Topics* to the fact that only an ignorant audience will be deceived by some kinds of argument, thus indicating that an expert will be more alert to fallacies or

27. Ebbesen 1981, I, p. 97.

will be able to detect them in time. Similarly, in the *Rhetoric* we read that the audience is simple-minded and cannot follow a long train of thought (1357a7-14), which might be the reason why a fallacy brought about in a rhetorical speech can easily remain unnoticed and deceive the audience. It is due to the audience's ignorance or incapability of detecting defects in reasoning that a fallacy is successful. The issue of the audience's or interlocutor's deception by fallacies is also dealt with by Byzantine commentators, who often adopt Aristotle's statement that it is the ignorant who will be primarily fooled by such arguments. However, they also refer more explicitly to the tricks used in deceiving such an audience.

One trick is to hide the fallacy in premises that remain unexpressed, another is the creation of confusion by expressing many additional points between the premises of the argument. Both tricks are presented as particularly relevant for the rhetorical practice due to the brevity of rhetorical syllogisms. The issue of brevity is discussed by Aristotle in the *Rhetoric* where it is stated that enthymemes must be precise and brief so that they can be easily followed by the simple-minded audience (1357a14-21). However, the meaning of this statement has been interpreted both by ancient and modern readers in accordance with their overall assessment of Aristotle's theory of enthymemes. On the one hand, those who take the Aristotelian rhetorical theory to be pre-syllogistic, argue that Aristotle means simply that rhetorical arguments have to be concise in the sense of expressing only what is necessary without repeating what is already known. On the other hand, those arguing that Aristotle's account of rhetorical syllogisms is compatible with and presupposes his syllogistic take the statement to mean that rhetorical syllogisms, unlike the other types of syllogisms, express only one of the premises and imply the other[28]. As we might expect given what we have seen so far, this latter train of thought is also followed by the Byzantine commentators on the *Rhetoric* and analyzed in terms of Aristotle's syllogistic, which, as we said above, played an important role in the commentators' interpretation of most aspects of the treatise. Hence, the commentators argue that, since enthymemes have to be briefer than other syllogisms, this brevity can be achieved through the suppression of one of the premises. The Anonymous commentator explicates this further by arguing that the suppression of one premise is in a way an "agreement" between the speaker and the audience that what is known does not need to be stated (*In Rhet*., p. 2, ll. 10-30). This makes him discuss two ways of making use of a fallacy: first, a fallacy can come about when the suppressed premise is not accepted by the audience or would not have been accepted had it been expressed. In this case, the fallacy lies in the fact that the above-mentioned agreement does not take place – the hidden

28. For the latter view see Madden 1952, p. 373 and Grimaldi 1972, pp. 87-88. For criticism of this view see Rapp 2016, p. 187. For a more detailed discussion of the issue as well as of the Byzantine commentators' interpretation of the rhetorical fallacies see Vogiatzi 2019, pp. 40 sqq. and pp. 80 sqq.

or unexpressed premise is taken as "agreed" although it is not so. Second, a fallacy can be covered up when the speaker adds many sub-arguments between the premises and, hence, takes advantage of the fact that the audience cannot follow the long train of thought. In this case, the fallacy is hard to spot due to the speaker taking advantage of the audience's defects.

Regarding the latter point, the Anonymous argues that the audience's incapability to follow long trains of thought and to anticipate the argument (*ouk eisi proeilêmmenoi*) makes them able to only follow one step of the argument at a time. This means, in his view, that they are incapable of detecting a fallacy that is based on presuppositions made earlier in the argument. The audience – due to its being simple-minded – cannot predict that the speaker's use, for instance, of a homonymous term, will lead to the audience's later acceptance of a false statement (*In Rhet*., pp. 1-2, ll. 13-6). Interestingly, the commentator seems to suggest that rhetorical arguments have to be concise and brief, *in order for* the audience to be able to detect such fallacies. In fact, the whole passage discussing this issue seems to be advice aimed at the orator to make use of brief and clear arguments that are either self-evident or accepted by the hearer, so that the speech will be persuasive. But fallacies can remain unnoticed in the rhetorical context when the speaker intentionally adds many secondary arguments in between the main steps of argumentation.

Related to this is also the case when the orator presents his statement as a conclusion of an argument, although nothing has been deduced. Aristotle speaks of this case in his account of the fallacy from verbal expression, but presents it as pertaining mainly to dialectic (1400a1-13). Interestingly, the Anonymous account of rhetorical pre-syllogisms can add to our understanding of his views of means of deception. In particular, when commenting on Aristotle's statement that rhetorical syllogisms consist either of premises that have been proven before or of premises that require proof (1357a7-17), the Anonymous adds that an example of the former can be seen when the orator summarizes the conclusions of previously expressed arguments by saying, for instance, such as "I have proved a, b and c" (*In Rhet*., p. 2, ll. 10-30). However, as we read in his account of the fallacy from verbal expression, although the requirement of clarity necessary due to the audience's inability to follow long arguments can justify the use of such conclusion-like or summary-like statements, this kind of statements can be the source of deception, if various unconnected propositions are brought together in order to give the impression that they belong to the same argument (*In Rhet*., p. 148, ll. 15-20). We can imagine that this trick can be particularly successful, when the orator presents many arguments at once, which he then summarizes, but also adds points that have not, in fact, been shown at earlier stages of the argument. The audience of the rhetorical speech now might be falsely led to believe that something was proven by only paying attention to the inferential terms in the orator's speech.

Regarding the trick of hiding of the fallacious premise, the Anonymous also expresses an interesting view. As I mentioned above, a view adopted by both commentators on the *Rhetoric* (as well as by some modern interpreters) is that rhetorical syllogisms are brief in the sense of not expressing both premises, but only one. As Anonymous has argued, although this is legitimate, when the audience has 'agreed' on this (*In Rhet*., p. 2, ll. 10-30), the lack of one premise can be the source of deception when the missing premise is false (*In Rhet*., p. 148, ll. 3-11). In particular, the commentator argues that it is the major universal premise that is omitted and that, when added, can be shown to be contrary to the minor premise. What is interesting in this view, is that it expresses both the source of fallacy and the source of deception: the contrariety between the two premises or the false content of the major premise is the source of fallacy, whereas the fact that this premise is unexpressed is the reason why the fallacy escapes the notice of the audience. The fact that speakers make use of unstated terms in order to deceive the interlocutor is also mentioned by Michael of Ephesos, who however refers to the omission of a term that turns the syllogism defective (*In Soph. el.*, p. 59, ll. 7-8). A more explicit statement of this view, strongly reminiscent of the Anonymous on the *Rhetoric*, can be found also in Commentator II on the *Sophistical Refutations*, who argues that a syllogism can become brief when one of the premises is omitted, which might be defective (*In Soph. el.*, 182b20-21).

Stephanos in his commentary on the *Rhetoric* also interprets brevity in terms of the omission of the major premise. He argues that the omission of this premise not only makes the enthymeme compact and brief, but it also plays a role in persuading the audience who need to 'invent' it in order to be fully persuaded – or (if it turns out to be false) to reject it (*In Rhet*., p. 303, ll. 13-34). Besides the historical connotations of the reference to invention, this view is interesting because it shows the procedure of deception by an unspoken premise as well as the detection of the fallacy: the speaker omits one premise that is supposed to be known; the hearers are supposed to invent or provide the missing premise, which, if true, will be accepted or, if 'opposite', will be rejected. The feature of opposition is namely taken as the source of fallacy (the major premise is false and, therefore, in opposition to the true minor premise), while the omission of the major premise hides the antithesis, which becomes evident only when the premise is provided by the hearer[29].

In addition to deception by hiding the fallacy in the missing premise, the Anonymous commentator offers an alternative reading of the way the audience can be misled into giving their consent to a false argument. In particular, when further discussing the fallacy from the verbal expression, he refers to the 'slight distinction' between forms of propositions that escape the notice of the audi-

29. On the feature of antithesis and the similarity between Anonymous' and Stephanus interpretation see Vogiatzi 2019, pp. 89 ff.

ence. One such slight distinction that is used primarily in dialectical or sophistical contexts is the distinction between indefinite and universal propositions. By using an indefinite proposition such as 'men...', the speaker leads the audience to believe that what is said is true universally, as if the statement was 'all men...'. The Anonymous seems to take as a starting point Aristotle's account of indefinite propositions in the *Analytics*, where it is stated that they should be considered as equivalent to particular ones (24a16-23; 26a29-30; 29a27-29). A similar point is also made by Michael of Ephesus in his commentary on the *Sophistical Refutations* (p. 67, ll. 17-21), where he argues that the similarity in form (*homoioschêmosunê*) of a particular and universal statement, such as between 'this man' or 'men', can be the source of deception[30]. Elsewhere he refers again to similarities in expression as the source of deception and argues that slight associations and similarities (*apo mikras koinônias kai homoiotêtos*) may result in one's being deceived (*In Soph. el.*, p. 12, ll. 21-22).

4. Conclusions

As mentioned in the introduction, the examination of fallacies in the Byzantine thought is a vast enterprise that cannot be sufficiently fulfilled within such a short study. For this reason, I chose to examine some issues that seem to recur in various Byzantine texts. These recurring themes, in particular the analysis of fallacies primarily in terms of material defects and the explanation of the sources of deception, indicate that the Byzantines were able (1) to identify both some of the most problematic questions of the topic, in particular the question regarding the sources of fallacious reasoning as well as the question regarding the relation between the two distinctions made by Aristotle, and the lack of any earlier examination of other questions, such as the question regarding the sources of deception, and (2) to offer compatible solutions. These points in turn have two interesting consequences. First, they indicate a mature philosophical thought and not non-critical adoption of earlier views. I have hopefully shown in my analysis how the authors adopted and developed earlier accounts as well as how they introduced new ones. Second, the fact that the solutions offered are often similar or compatible, shows that we can speak of a "Byzantine philosophical tradition" in the sense that the authors not only had the same background, but also the same goals. In the case of the study of fallacies, the goals seem to be the presentation of a unified logical theory, the inclusion within this theory of dialectic, rhetoric, sophistic and poetic, as well as the complete examination of all aspects related to this theory.

30. Cf. Comm. II on *Soph. el.*, 169a33.

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Abstract: The Byzantine commentary tradition on Aristotle's treatments of false reasoning – most notably in the *Sophistici elenchi* and the *Rhetoric* – is by no means uncharted territory. Still, it warrants further investigation on account of its poor standing amongst historians of Byzantine logic. The paper focuses on the conceptual tools Byzantine commentators brought to bear in their efforts to identify the sources of false reasoning and the ways fallacious arguments actually mislead and deceive – such as the distinction between logical form and logical matter, as well as the principles of specious appearance and the means of deception.

Keywords: Logical Form and Logical Matter; Anonymus Heiberg; Michael of Ephesus; John Italos.

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Outside the Logic of Necessity: Deontic Puzzles and 'Breaking' Compound Causal Properties in Islamic Legal Theory and Dialectic

1. Introduction

This study will examine an important group of illegitimate moves involving causal properties as identified by Medieval Muslim jurists in the intertwined domains of legal theory (*uṣūl al-fiqh*) and dialectic (*jadal*). More precisely, we will focus on the discourse surrounding the dialectical objection called *kasr*, or 'breaking', which deliberates certain proper and improper paths to challenging and defending the causal components of a correlational argument (*qiyās*) in which the *ratio legis* (*ʿilla*) of the root-case's ruling (*ḥukm*) is a compound of two or more properties. The developmental history of this dialectical objection is complicated; long and heated controversies centred on which modes of *kasr* (and responses to *kasr*) were fallacious and which were not. There were even those who rejected *kasr* at the outset, some with arguments paralleling classical and medieval Latinate claims that one cannot refute an argument whose premises have a meaning *in sensu compositio* by the blunt separation of its parts.

In the present study, we will restrict our analyses to the relevant discussions of two eleventh century CE theorists: the renowned Shāfiʿī jurist Abū Isḥāq al-Shīrāzī (d. 1083 CE) and his equally prominent, one-time pupil, the Mālikī jurist Abū al-Walīd al-Bājī (d. 1081 CE), both of whom elaborated two main pathways to 'breaking' an opponent's compound *ʿilla*, either by (1) replacing one of its properties with something similar, or (2) removing one of its non-efficient properties. The rules and responses prescribed in the sources we examined for each type make it clear that they correspond to two primary conjunctive modes for an *ʿilla*'s valid composition: either (type 1) *meaning dependence* of one property (or more) upon the other (with all of them together being claimed to occasion the ruling), or (type 2) *independent conjunctive composition*, in which one or more properties might be efficient, but not the remainders (and from which it follows that if *A*, *B*, ..., occasion the ruling, then either *A* occasions the ruling or *B* occasions it, or...).

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Moreover, we will briefly confront the additional, but fallacious, modes of *kasr* denounced by al-Shīrāzī and al-Bājī with forms of certain deontic paradoxes and puzzles we may group under the rubric *logical extrapolation fallacy*. These play a dominant role in contemporary deontic logic, although they have roots in the medieval Latin tradition (or, arguably, are already present in Megarian and Stoic logic). More generally, fallacies of *logical extrapolation* should include the illegitimate reduction of some pattern of argumentation to a pre-existing logical system (be it modal or otherwise).

Our primary claim is that, whereas logical extrapolation produces fallacies or paradoxes by unsafely applying inference rules of standard alethic and/or logical necessity to the deontic realm, the fallacies generated by invalid modes of *kasr* in Islamic legal theory (wherein both logical rules and semantic rules for reasoning with deontic modalities are expressed dialectically) constitute a genuine source for reflecting on what patterns of reasoning should be endorsed for determining causality in Law – and, perhaps, more generally, also for establishing causality in certain natural (as opposed to normative) epistemological contexts.

2. The Logical Extrapolation Fallacy

Contemporary approaches to Standard Deontic Logic (SDL) have followed G.H. von Wright's idea of applying the developed framework of modal logic to the deontic realm[1]. According to this approach – further extended with the model theoretical semantics of Jaakko Hintikka and Saul Kripke[2] – obligation was conceived as a special reading of the necessity operator of basic modal propositional logic. According to this view deontic reasoning can be reduced to a special kind of modal logic.

Possible-world interpretations of deontic necessity struggled, however, with a wealth of philosophical and logical puzzles that threatened the framework right from the very start – already by the sixties, and at the propositional level[3]. Of course, deontic necessity is still a kind of necessity. But standard possible-world semantics – without deep modifications – is not the instrument required, vis-à-vis the prescriptive dimension on performances of actions, to grasp the meaning of those norms governed by the assumption of liberty – or, more precisely: of legal or ethical *liability*.

Paul McNamara and Risto Hilpinen provide, in their contribution to the *Handbook on Deontic Logic and Normative Systems*, a systematic, historical overview of

1. See von Wright 1951 and von Wright 1963.

2. Hintikka 1957; Hintikka 1962; Hintikka 1968; Kripke 1963.

3. For recent overviews of these challenges see Hilpinen / McNamara 2013; Navarro / Rodriguez 2014.

such problematic, reductionist projects. One of the puzzles they discuss is the following reconstruction of Stephen Langton's (1150-1228) paradox[4]. It results from reducing (Aristotelian) modal necessity to deontic necessity, and constitutes an excellent example for our reflection on the genealogy of fallacious arguments within the deontic realm[5].

Necessarily, if this man visits his sick father, then the father is sick.

But it does not follow that

If this man ought to visit his sick father, then his father ought to be sick.

These and similar counterexamples are rooted in the following form of reductionism for which Leibniz has been praised as much as blamed:

N($A \supset B$) (whereby '**N**' stands for necessity)

(**O**$A \supset$ **O**B) (inferred via modus ponens from **N**($A \supset B$), followed by introduction of implication and substituting **N** with '**O**', obligatory)

Notice that this example of the sick father is not specific to the modal necessity operator but extends to inferences governed by logical necessity as well. This also holds for many other well-known deontic puzzles, including: Ross 1941's *Burning-Letter* paradox; Chisholm 1963's puzzle on *Conditional Obligations*; and Prior 1958's *Good Samaritan* paradox, in which latter there follows from: *It is obligatory that Jones help Smith who is being mugged*, that: *It is obligatory that Smith is being mugged* (assuming the premise is rendered as a conjunction).

More generally, the problem may be seen as an illicit extrapolation from the modal logic of necessity to the set of rules governing deontic reasoning in the moral and/or legal realms. This signifies a special kind of fallacious argumentation: identifying illegitimate moves which consist in extrapolating a set of rules for logical reasoning to a pattern of reasoning within some specific field of knowledge, and drawing a paradoxical or puzzling consequence that patently contravenes the conceptual architecture of that field. As mentioned, we will call this the *fallacy of logical extrapolation*, though it is not new. Ample Megarian and Stoic debates on the fruitfulness of material implication for temporal and modal contexts long ago involved discussions of such puzzles. The garden of logical extrapolation fallacies is quite rich.

This is particularly important in the context of Islamic Dialectical theory, wherein rules for legal reasoning were not constituted by importing logical rules from somewhere else and subjoining them to rules of dialectic, but the dialectical framework itself originated dialectical rules for legal-logical reasoning. In the

4. Hilpinen / McNamara 2013, p. 8.
5. Our rendering of the example is based on Knuuttila / Hallamaa 1995, p. 77.

general dialectical theory of Shams al-Dīn al-Samarqandī (d. 1322), the *ādāb al-baḥth wa-l-munāẓara*, which streamlined and universalized a predecessor juristic dialectic, logical rules were (as in the predecessor dialectic) consciously and rigorously formulated as dialogical rules[6]. This explains why we will not, in principle, find fallacies of extrapolation therein. However, there is no dearth of cases for the identification and study of illegitimate moves in legal and deontic reasoning more broadly.

3. Islamic Deontic Imperatives: Outside Logical Extrapolation

3.1. *Ibn Ḥazm's Deontic Imperatives*

In previous joint studies by S. Rahman, W. E. Young, and F. Zidani[7], the authors argue that Ibn Ḥazm of Córdoba's (994-1064) *Facilitating the Understanding of the Rules of Logic and Introduction Thereto, with Common Expressions and Juristic Examples,* wherein, among other things, he thoroughly investigates deontic notions and their modal counterparts, assures him a place among the fathers of the logic of norms. Moreover, in these studies we show that, in the context of what we called *Islamic heteronomous imperatives*, 'puzzles' of extrapolation are not puzzles at all. The point is that this approach does not require one to block the use of any logical rules as standard solutions do, but allows logical inference rules to cohabit with deontic rules – if, that is, sufficient care is given to the meaning constitution of deontic assertions generated by the logic that governs the normative realm. Insights from the Islamic tradition that facilitate our reconstruction include the following:

– Prescriptions are understood as prescriptions to do rather than prescriptions that take us from one state of affairs to another: *Tun Sollen* rather than *Sein Sollen*.

– Actualizations (performances) of prescriptions are subjects of predication. In other words, performances of actions are bearers of qualifications such as *law-abiding* (rewarded) or *law-breaking* (sanctioned)[8]. The distribution of reward and sanction yields the classification of deontic modalities into *obligatory, forbidden, recommended permissible, reprehended permissible,* and *evenly permissible*.

– Norms presuppose freedom of choice or moral and legal liability. In other words, each type of prescription (obligatory, forbidden, etc.) assumes as hypoth-

6. See Miller 2020, pp. 103-123.

7. Most of this current article's sections 3.1 and 3.2 has been reproduced from the following: Rahman / Granström / Farjami 2019; Rahman / Young / Zidani 2021 and Rahman / Young / Zidani 2022.

8. By way of clarification, 'sanction' in this study is meant only in its sense of 'penalty', and 'sanctioned' as 'penalized'.

esis that the corresponding type of action can be deliberately carried out or not carried out.

3.2. *Ibn Hazm's Deontic Imperatives*

In the general realm of human actions, Muslim jurists identified five deontic qualifications[9]. Ibn Ḥazm defines them as follows[10]:

1. ***wājib, farḍ, lāzim***. Obligatory action is the one which: If we do it we are rewarded. If we do not do it we are sanctioned.

2. ***ḥarām, maḥẓūr***. Forbidden action is the one which: If we do it we are sanctioned. If we do not do it we are rewarded.

3. ***mubāḥ mustaḥabb***. Recommended permissible action is the one which: If we do it we are rewarded. If we do not do it we are neither sanctioned nor rewarded.

4. ***mubāḥ makrūh***. Reprehended permissible action is the one which: If we do not do it we are rewarded. If we do it we are neither sanctioned nor rewarded.

5. ***mubāḥ mustawin***. Evenly permissible action is the one which: If we do it we are neither sanctioned nor rewarded. If we do not do it we are neither sanctioned nor rewarded.

In the aforementioned studies we proposed a logical analysis based on Ranta 1994's Type Theoretical Grammar and Martin-Löf 1984's Constructive Type Theory (CTT). This fully interpreted framework allows a logical analysis of imperatives that makes explicit the conceptual links at work here. More precisely it allows one to express for each deontic qualification both (1) the dependence of rewarding or sanctioning on the presupposition that there was a real choice, and (2) the fact that these performances actualize the type of action specific to the imperative at stake (obligatory, forbidden, etc.) Thus, if we are describing an *obligatory action*, we need to express the following:

Obligatory action:

(i) *If the individual **g** made the choice to perform an action of type A (i.e., if there is a performance by **g** that actualizes the **left side** of the disjunction) then some (legal) procedure **b**, determined by the Law, attributes a form of **reward** specific to this type of performance.*

(ii) *If the individual **g** made the choice to omit performing an action of type A (i.e., if there is a performance by **g** that actualizes the **right side** of the disjunction)*

9. Contractual qualifications (valid, invalid, null and void) were also developed, but will not be discussed here.

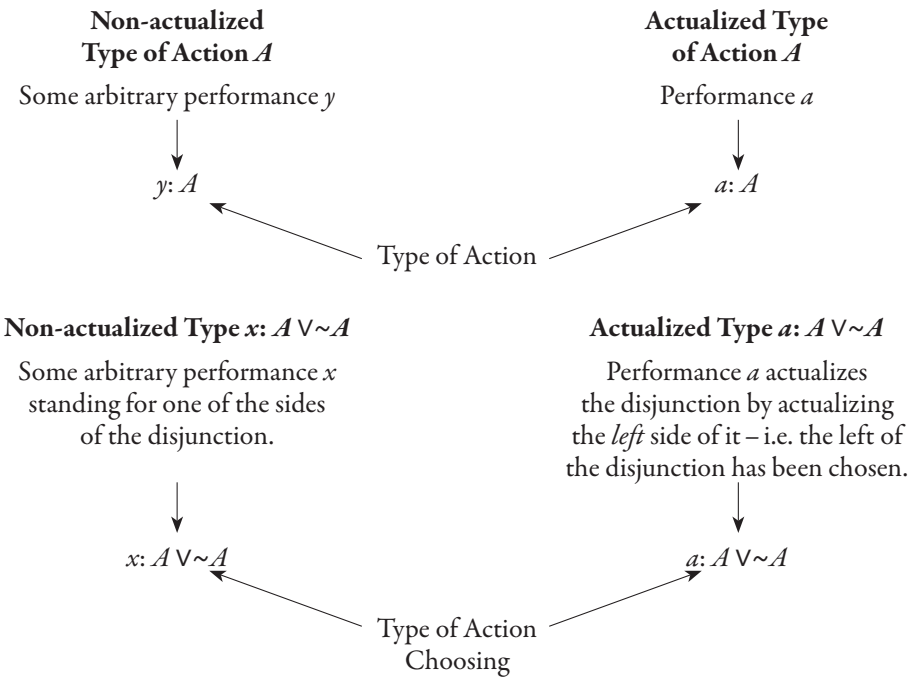
10. Ibn Ḥazm, *Al-Iḥkām fī Uṣūl al-Aḥkām*, ed. Shākir, vol. 3, p. 77; Ibn Ḥazm, *Kitāb al-Taqrīb li-Ḥadd al-Manṭiq wa-l-Mudkhal ilayhi bi-l-alfāẓ al-ʿĀmmiyya wa-l-Amthila al-Fiqhiyya*, ed. ʿAbbās, p. 86 and pp. 83-84.

*then some (legal) procedure **b**, determined by the Law, attributes a form of **sanction** specific to this type of omission.*

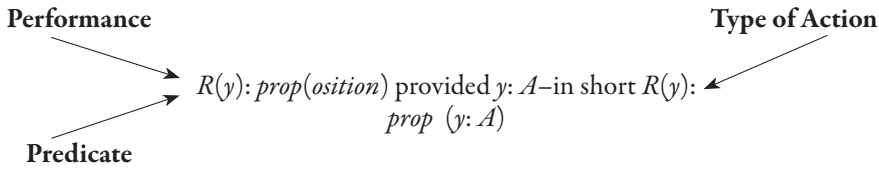
3.3. *Rendering the Conceptual Analysis Explicit*

Before presenting the notation that makes our analysis explicit, let us provide first the main keys for reading the CTT formal terminology employed here and further on in the study:

– If '*A*' stands for a type of action, and '*y*' stands for some not-yet-actualized performance of an action of type *A*, then we will write *y*: *A*. Similarly, the '*x*' in '*x*: *A* ∨~*A*', stands for the performance of a type of action of either *A* or ~*A*. Thus, more loosely, we can say that '*x*' in '*x*: *A* ∨~*A*' stands for the choice of carrying out *A* or intentionally refraining from carrying out *A*.



– If 'y' stands for some not-yet-actualized performance of an action of type A, then 'R(y)' stands for attributing the predicate *Reward* to the performance y of the type of action A. Thus, 'R(y)' is an expression of the type *prop* (i.e. of the type proposition), whereby y is an element of the set A of performances of action. In a more standard terminology, 'R(y)' stands for a propositional function over the set A of performances. Something similar holds for the propositional function "S(z) defined over the set of omissions ~A".



Hypothetical judgment or normative prescription	**Categorical judgment or result of carrying out a prescription**
$R(y)$ *true* $(y: A)$	$R(a)$ *true*
Any arbitrary performance y of A makes it *true that this performance is rewarded*	It is *true* that the performance a is *rewarded*
$d(y)$: $R(y)$ $(y: A)$	$d(a)$: $R(a)$
(legal) procedure $d(y)$ attributes *Reward* to any arbitrary performance *y of A*	(legal) procedure $d(a)$ *attributes Reward* to performance a of A

– If 'x' indicates the choice between carrying out A or intentionally refraining from carrying out A, then "$\mathbf{left}^{\vee}(y) =_{\{H\}} x$" indicates that the result of this choice is to perform A (i.e. it is identical to a performance y of A); and something similar holds for "$\mathbf{right}^{\vee}(z)$".

Accordingly, the first component of the conjunction:

$(\forall y : A)\ \mathbf{left}^{\vee}(y) =_{\{H\}} x \supset R(y)\ \ (x : A \vee {\sim}A)$

reads:

*Assuming that, given the choice **x** of performing or not performing an action of type **A**, performing it has been chosen (i.e. if the left side of the disjunction has been chosen to be performed), then, for any performance **y** of the type of action **A** that is identical to the choice **x**, there follows reward (for performing this action).*

A similar reading applies to the second component:

$(\forall z : {\sim}A)\ \mathbf{right}^{\vee}(z) =_{\{H\}} x \supset S(z)\ \ (x : A \vee {\sim}A)$[11].

Thus, if we pull all this together and employ the abbreviation $_{\{H\}}$ for the hypothesis x: $A \vee {\sim}A$, expressing the fact that the agent can choose to either actualize (perform) A or actualize an omission of performing A, we obtain:

11. The notation for propositional identity, namely "$x =_{\{D\}} y$", standing for "x is identical to y *within the set D*", is closer to what is employed in first-order logic.

$b(x) : [\ (\forall y : A)\ \mathbf{left}^{\vee}(y)=_{\{H\}}x \supset R(y)\ ] \wedge [\ (\forall z : {\sim}A)\ \mathbf{right}^{\vee}(z)=_{\{H\}}x \supset S(z)\ ]$ $(x : A \vee {\sim}A)$.

– Given x: $A \vee {\sim}A$, '$b(x)$' stands for a procedure (expressed by a function) that distributes **reward** and **sanction** depending upon the actual choice made (by the agent ***g***) between performing A or ${\sim}A$ and specific to the type of action chosen to be performed. In short, the function relates performances of actions to the kind of sanction or reward specified by the Law. In our context, the procedure is a legal one carried out by the competent legal authority (in this world, or the next, or both).

The hypothetical can be glossed as follows:

Obligatory action:

Given the choice x between performing A or intentionally refraining from doing so:

If performing an action of type A has been chosen (i.e. if there is a performance y that actualizes the ***left side*** *of the disjunction) – viz.* ***left****(y)= x – then (legal) procedure* ***b*** *attributes a form of* ***reward*** *specific to this type of performance.*

If intentionally refraining from performing A has been chosen (i.e. if there is a performance z that actualizes the ***right side*** *of the disjunction) – viz.* ***right****(z)= x – then (legal) procedure* ***b*** *attributes a form of* ***sanction*** *specific to this type of omission.*

The above yields:

***wājib, farḍ, lāzim*:** Doing A_1 is rewarded. Intentionally refraining from doing A_1 is sanctioned.

$b_1(x) : [\ (\forall y : A_1)\ \mathbf{left}^{\vee}(y)=_{\{H1\}}x \supset R_1(y)\ ] \wedge [\ (\forall z : {\sim}A_1)\ \mathbf{right}^{\vee}(z)=_{\{H1\}}x \supset S_1(z)\ ]$ $(x : A_1 \vee {\sim}A_1)$.

As mentioned, this approach provides a general framework that prevents the kind of puzzles discussed above. It only works, however, if the framework is embedded in a general theory of the meaning of legal norms that also accounts for reasoning involving deontic modalities. It is important to observe that in Islamic jurisprudence there is no theory of deontic syllogisms as such. Rather, within the dialectical theory and practice of *uṣūl al-fiqh,* the distribution and transmission of the deontic modalities described above is governed by argumentation rules on how to justify a claimed legal norm, how to object to such justifications, and how to respond to such objections. In what follows, we will focus on arguments concerning compound premises expressing the causal factors for legal rulings. Inquiring into the causal factors of rulings was regarded as a critical epistemic endeavour, and the rules for challenging arguments in support of a compound causal factor's occasioning of a legal norm were among the more controversial and subtle within Islamic juristic dialectic.

4. Beyond Fallacies of Logical Extrapolation

It is interesting to conjecture as to why extrapolation fallacies involving modal necessity drew the attention of premodern scholars in the Latin tradition but not (at least not explicitly) in the Arabic tradition. From a historical point of view, one possible answer – though perhaps problematic in over-generalizing – might be that Aristotle's modal logic, or at least Aristotle's modal notions as discussed in the *Peri hermeneias* and commented upon by Latin scholars as early as Boethius, took conceptual priority over deontic notions in the Latin tradition, whereas a dominant practice of dialectical legal reasoning in Islamic thought inversed this order of priorities for the Arabic tradition. If true, a possible effect of this might be the fact that, in the further maturation of dialectical theory following the success of al-Samarqandī's *ādāb al-baḥth*, discourse on valid and invalid reasoning with regard to both legal and natural causality appears to have blurred older distinctions and embraced a general causality pattern[12].

Whatever the reason, one might be justified in asking whether fallacies bounded by deontic or legal reasoning can be identified within Islamic disputation theory. The answer is a resounding 'Yes'[13]. This is particularly so in relation to deontic and natural causal necessity in the context of that set of patterns for parallel reasoning called *qiyās*, the basics of which we will now quickly review.

4.1. Qiyās *Basics*

The many debates and elaborations on *qiyās*, which might be translated 'correlational inference' (more often, if less accurately, 'analogy')[14], together constitute one of the finest outcomes of the argumentative approach to legal reasoning within Islamic Law. A particularly lucid example is the systematization of the respected Shāfiʿī theoretician Abū Isḥāq al-Shīrāzī (1003-1083), upon which the following is based[15].

Among the majority of Sunni jurists, *qiyās* consists in a set of methods with varying epistemic grades, the highest of which belongs to that mode, called *qiyās al-ʿilla*, in which the rule-occasioning factor or cause (*ʿilla*) may be identified, located, and proven efficacious. The aim of this archetypal form of *qiyās* is to provide a rational ground for the application of a juridical ruling (*ḥukm*) to a given case, called the branch-case (*farʿ*), which has not been directly and unequivo-

12. See Young 2019 and Young forthcoming.
13. Sources in Young 2019 and Young forthcoming.
14. See Young 2017, p. 10.
15. A landmark on the subject of *qiyās* is Hasan 1986. Young 2017, pp. 110-128 provides a summary of al-Shīrāzī's systematization of *qiyās* and, on this basis, Rahman / Iqbal / Soufi 2019 develop a logical analysis. See also Iqbal 2022.

cally pronounced upon in the primary juridical sources (i.e., the Qur'ān, Sunna [Prophet's example], and *Ijmāʿ* [consensus]). The method starts by attempting to determine the property (*waṣf*) or set of properties in the root-case which constitutes the causal or occasioning factor or factors, or *ratio legis* (*ʿilla*) giving rise to its ruling. If it is 'probable' *(ẓannī*) that this property occasions the ruling, and it is shared by the branch-case, then we may infer that it is equally productive of that ruling in the branch-case[16].

A cardinal feature of al-Shīrāzī's take on *qiyās al-ʿilla* is his particular notion of efficiency (*taʾthīr*), which tests whether the property P purported to be efficient in occasioning the ruling at stake is indeed so. For al-Shīrāzī, *taʾthīr* consists of two complementary procedures:

co-presence (*ṭard*): whenever the property is present, the ruling is also present

and

co-absence (*ʿaks*): whenever the property is absent, the ruling is also absent.

While co-presence examines whether ruling H follows from verifying the presence of property P, co-absence examines whether exemption from ruling H follows from verifying the absence of P[17].

4.1.1. Remarks

1. It is important to bear in mind that *qiyās* procedure involves two primary dialectical steps: one is the epistemological and hermeneutical task of identifying the properties claimed to occasion the ruling, the other – that is, the logical step – assumes that the first has been achieved. The first involves sub-arguments and counter-arguments that are not formal, but contentual or material; the second involves dialectical rules for logical reasoning. Contesting a legal argument might involve contesting a logical mistake, but this concerns fallacies more in line with those in the peripatetic syllogistic tradition, which constitute a group of their own known as *mughālaṭāt*. In the context of legal reasoning, the most relevant form of objection concerns the first step whereby a semantic link between the property and

16. Though admitting a range of degrees, the 'probability' indicated by the term *ẓannī* does not refer to the statistical view conceived by the time of Leibniz. See Hallaq 1997, p. 39. In fact, we might suggest *ẓann* probability is a kind of *qualitative* expression for comparing degrees of acceptability, such as: "there are more indications in favour of accepting that the property **P** occasions the ruling than not".

17. See the preface in Rahman / Iqbal / Soufi 2019. NB: this test of a property's causal efficiency is elsewhere and more commonly called "co-presence and co-absence" (*al-ṭard wa-l-ʿaks*) or 'concomitance' (*dawarān*), and listed among the "modes of causal justification" (*masālik al-taʿlīl*). See Young 2019 and Hasan 1986, pp. 315-330. As for 'efficiency' (*taʾthīr*), such as al-Ghazālī deemed it to be a direct designation of the cause (*ʿilla*) by either univocal source-text (*naṣṣ*) or consensus (*ijmāʿ*), while others held different notions (see Hasan 1986, pp. 272-273 and p. 284).

the ruling is claimed. The sub-arguments and counterarguments concerning this step are not formal and are therefore always prone to further revision. Agreement can be achieved at some point, but the whole process can also start over again[18].

2. In section 5 we will distinguish between *occasioning factors* or *properties*, and *occasioning procedure*: the efficient causation that transforms instances of factors or properties into concrete applications of the ruling. For the time being, however, we will leave this finer distinction aside.

4.2. Kasr *or: "How to Break Apart a Compound Occasioning Factor"*

An important set of recognized, fallacious moves in Islamic legal argumentation theory relates to challenging claims that a property (*waṣf*) or composition of properties constitutes the efficient *ʿilla*, i.e., the occasioning (causal) factor upon which the ruling (*ḥukm*) is grounded. Among other things in this context, what may be subject to fallacy is the form an *objection* takes in challenging a claim that some property (or properties) constitutes the *ʿilla* for a debated case – or the *response* to that objection. Fallacious objections to claimed legal or natural occasioning factors, however, appear not to have been included among the peripatetic tradition's *mughālaṭāt* (usually delimited by *σόφισμα*). Rather, we find them described in legal-theoretical and dialectical works as 'invalid' (*fāsid*, *ghayr ṣaḥīḥ*) objections and responses, while objections and responses that contribute to legitimate refutations and defences are deemed 'valid' (*ṣaḥīḥ*).

Of course, in Islamic legal-theoretical contexts, as in Roman Law, 'valid' bears the sense of "legally sound and effective", as in "a valid contract", while also admitting the deontic reading 'legally allowed'. In Islamic juristic dialectic, however, 'valid' (*ṣaḥīḥ*) and 'validity' (*ṣiḥḥa*) are also applied to arguments – and, by extension, to moves that produce valid arguments – with validity understood as conformity with some set of dialectical rules establishing both the dialectical meaning of the expression involved and the procedural debate-protocol to be followed[19]. On the other hand, 'invalid' (*fāsid*) and 'invalidity' (*fasād*) seems also to relate to the notion of a winning strategy: that is, Q's objection is invalid if, no matter what Q does after some specific move, R wins without further intervention[20].

18. See Rahman / Iqbal / Soufi 2019, pp. 46-47.

19. See Rahman / McConaughey / Klev / Clerbout 2018, pp. 57-105, and p. 279-282 for *refutation*.

20. The use of 'valid' and 'invalid' in argumentative contexts of course relates to logical terminology, but an important caveat must be observed. As mentioned, the notion of *invalidity* – and, by extension, *invalid move* – includes both semantically unsound justifications and objections and the notion of *logically* invalid, whereas its counterpart, *validity*, does not refer to logically valid arguments (at least not in its most common usage). As we shall see below, however, in Islamic legal reasoning an *invalid move* may also refer to a move by Q whereby R wins, no matter what Q attempts to do afterwards (since Q's move contravened the meaning and/or procedural rules established for the the-

Among the most virulent challenges to an *ʿilla*-claim is the charge of *naqḍ* (inconsistency), whereby Q aims to 'destroy' R's claimed *ʿilla* by bringing a case in which it is found without R's claimed ruling (*ḥukm*) – thus failing to exhibit the requisite co-presence (*ṭard*). Notably, a legitimate or valid *naqḍ* brings a parallel case more general than R's, wherein the property claimed to be the occasioning factor is present but R's desired ruling is not, whereas an illegitimate or invalid *naqḍ* may bring a parallel case that in fact constitutes a particularization or further specification of R's root-case[21].

Here we will focus on a special form of *naqḍ* called *kasr*, aimed at destroying a *compound of properties* claimed to be the occasioning factor by 'breaking' apart and disposing of its components. In short, Q objects that one (or some) of the purportedly efficient properties of R's *ʿilla* exists in another case – the *kasr*-case, or "problem-case of breaking" (*mas'alat al-kasr*) – but with a different ruling (*ḥukm*).

Formulating the move is delicate and requires some fine distinctions, beyond the pair *sensus compositus* and *sensus divisus*. It is therefore not surprising that *kasr* triggered long and heated debates over whether it should be admissible at all. The notions of *kasr* discussed here are those elaborated by al-Shīrāzī and al-Bājī, who, well aware of the pitfalls involved, developed two main pathways to 'breaking' an opponent's compound *ʿilla*:

(1) by substitution (*ibdāl*): producing a parallel *kasr*-case, similar to R's root- and branch-case, but wherein one of the properties in R's compound *ʿilla* is replaced by another in the same *maʿnā* (meaning, intension), and that parallel compound is present despite the absence of R's desired ruling (thus, the unaltered property is proven non-efficient in occasioning the ruling);

(2) by removal (*isqāṭ*): producing a parallel *kasr*-case similar to R's root- and branch-case, but wherein one of the non-efficient properties in R's compound *ʿilla* is omitted, while the remainder is present despite the absence of R's desired ruling (thus, the remaining property [or properties] is proven non-efficient in occasioning the ruling).

Each type is further subdivided into subtypes that produce valid refutations of R's *ʿilla* and subtypes that do not. As evident in the above descriptions, a key consideration is that the rules of challenge and defence governing the building of a valid *kasr*-case must be *meaning preserving* in relation to both the form of com-

sis involved), while a *valid move* does not automatically amount to a winning strategy. It refers rather to a legitimate move that can produce the win of one *play* of one of the contenders, but does not prevent finding another play with a different outcome. Put more simply: 'validity' applies to winning just one play, not all relevant possible plays (see the previous reference to Rahman / McConaughey / Klev / Clerbout 2018). Islamic debates – operating primarily in the domain of probability as opposed to certainty – thus also operated primarily at the play level.

21. See Young 2017, pp. 169-173.

position of properties which R claims to constitute the *ʿilla* and to the conceptual dependence between that compound and the ruling.

This of course assumes that the framework is thoroughly dialectical. Indeed, the conceptual links are made explicit by what contemporary dialogicians call *dialogical meaning explanations*. That is, the conceptual links expressed by connectives or other terms, are given by rules of challenge and defence. In the sources, these meaning explanations manifest in the context of debates on the legitimacy of a concrete refutation attempt. It is here that the precise mode of composition assumed by each way of constructing a *kasr*-case is determined.

In the *Sharḥ al-Lumaʿ*[22], al-Shīrāzī appears to be led by this meaning-preservation principle or caveat when dividing approaches to building a *kasr*-case into the aforementioned types and subtypes. Our texts provide sufficient elements to render a precise formulation of the rules governing valid and invalid moves for each type, though the terse examples, aimed at an audience proficient in Islamic substantive law and its points of derivation, are often difficult to follow[23]. In the following presentation we have attempted, with an eye to intrinsic coherence, to reconstruct arguments regarding the classification of types; but this should still be considered work in progress.

Type 1: *Kasr* by Substitution (*ibdāl*) in Dependent Conjunctive Compounds

Type 1a: Valid *kasr* by *ibdāl*

– Valid objection (*iʿtirāḍ*): Given a compound of properties *AB*, claimed by R to constitute the *ʿilla,* a valid *kasr*-refutation by *ibdāl* is one in which Q substitutes *A** for *A*, with *A** being in the same *maʿnā* as *A*, in a *kasr*-case of the same *maʿnā*, and shows the *naqḍ* (inconsistency) of this parallel compound *A*B*.

– Valid response (*jawāb*): Although R might acknowledge that, in principle, the above conditions for a valid refutation have been accomplished by Q, R can nevertheless rejoin if he can produce an argument showing that *A** is *not* in the same *maʿnā* as *A*, and thus Q's *kasr*-case involves a significant deviation from the *maʿnā* of R's root- and branch-cases. R's response thus takes the form of a (counter-) objection by 'disqualifying difference' (*farq*), invalidating Q's *kasr*-case as a proof of R's inconsistency. Importantly, that disqualifying difference also shows that substituting *A** for *A* triggers a change in the meaning of *B* – the *maʿnā* of *B*

22. al-Shīrāzī, *Sharḥ al-Lumaʿ*, ed. al-Majīd Turkī, §§1035-1051, pp. 892-909.

23. The original sources for our study also included al-Shīrāzī, *Al-Maʿūna fī l-Jadal*; al-Bājī, *Kitāb al-Minhāj fī Tartīb al-Ḥijāj*; al-Bājī, *Iḥkām al-Fuṣūl fī Aḥkām al-Uṣū*. Since al-Bājī's classification mostly follows that of his teacher, al-Shīrāzī, and the discussion of *kasr* in the latter's *Sharḥ al-Lumaʿ* is more thorough than the one in his *Maʿūna,* we will base our presentation mostly on al-Shīrāzī's notion of *kasr* as presented in the *Sharḥ al-Lumaʿ*, sometimes completing the presentation of his *Maʿūna* with further details found therein. We will also add some of al-Bājī's remarks should they help clarify the subtypes.

in the context of the new *A* should not be expected to contribute to occasioning a ruling as did the *maʿnā* of *B* in the context of the old *A*.

Type 1b: Invalid *kasr by ibdāl*

– Given a compound of properties *AB*, claimed by R to constitute the *ʿilla*, an invalid *kasr*-refutation by *ibdāl* is one in which Q substitutes *A** for *A*, but *A** (and thus the *kasr*-case itself) is *not* in the same *maʿnā*. (Q's new compound *A*B* may even prove irrelevant for occasioning the ruling of his own *kasr*-case.) Q has therefore failed to show the *naqḍ* of R's *ʿilla AB*, and R is not bound to do anything beyond pointing out that Q's *kasr* is invalid.

– The point here is that the new compound in an invalid (type 1b) *kasr*-case does not preserve the meaning dependence with its ruling in the way that R's original compound had done.

In fact, it is the rule for R's response that signals how his compound is to be understood in the first place. In the next section, we will provide further details of logical analyses of the form of composition *AB* assumed in type 1 and type 2 *kasr* objections. But for the moment let us say that intrinsic to type 1 *kasr* objections and responses are compounds wherein the components are claimed to occasion the ruling *together*, not in isolation. Note that, on the topic of compound *ʿilla*s, W. B. Hallaq observes:[24]

> "The ratio may also consist of more than one attribute, all of which must be considered as 'causing' a normative rule to arise from them. For instance, the ratio of the theft penalty encompasses five attributes: (1) the taking away of something by stealth; (2) the stolen object must be of a minimum value...; (3) the object must in no way be the property of the thief; (4) it must be taken out of custody (ḥirz); and (5) the thief must have full legal capacity. All of these attributes must obtain for an act to qualify as theft (sariqa) punishable by cutting off the hand. Each attribute is necessary; no single one by itself suffices to produce the ratio legis".

More generally, and although not plainly stated in our sources in these terms, we may infer from the examples that, in a successful *kasr*-case, the meaning of property *B* must also be dependent upon the meaning of Q's *A** in such a way that Q's substitution does not alter substantially the meaning of *B* from what it enjoyed when paired with R's original *A*. In short, *B* is a propositional function defined over *A* or its meaning-preserving substitutes. Rejecting a refutation based on *kasr* by *ibdāl* amounts to showing that Q's argument contravenes (partially or totally) the meaning-preservation caveat regarding the dependences inherent to R's compound *ʿilla*.

24. Hallaq 2009, p. 102.

Examples: Type 1

Type 1a: Al-Shīrāzī provides the following example of a valid attempt at *kasr* and its valid response[25].The *ʿilla* of R's *qiyās* is a compound *ʿilla* (*ʿilla murakkaba*) consisting of properties *A* and *B*:

contended branch-case (*far ʿ*):	Sale of what the buyer has not seen…
claimed ruling (*ḥukm*):	is not allowed…
claimed legal cause (*ʿilla*):	because it is [A] an object of sale (*mabīʿ*), and it is [B] unknown of attribute (*majhūl al-ṣifa*) to the contractor at the time of contract…
endoxon root-case (*aṣl*):	like when the seller merely says "I sell you a garment" (which we know, by juristic consensus [*ijmāʿ*], is not allowed).

– Q attempts to invalidate R's *ʿilla AB* by replacing one of its properties (*A, mabīʿ*: *object of sale*) with another (*A**, *mankūḥa*: *object of marriage*)[26] and showing that *B* is thus co-present with the opposite of R's ruling (i.e., it is valid, rather than not allowed) in the seemingly parallel *kasr*-case of marriage.

So, the one disagreeing with him [i.e. Q] says: "This is broken (*inkasara*) by [the fact that] if he marries a woman whom he has not seen, then she will be [[A*] an object of marriage (*mankūḥa*) and] [B] unknown of attribute (*majhūl al-ṣifa*) to the contractor at the time of contract – yet it [i.e. the contract] is valid"[27].

– R responds by invalidating Q's *kasr*-case through an objection of disqualifying difference (*farq*): marriage is not the same as sale with regard to property *B* because the buyer has the option to rescind (*khiyār*) in the case of a sale exhibiting property *B*, but the groom does not have that option in the case of a marriage exhibiting property *B*.

Importantly, Q's *kasr*-case of marriage shares property *B* with the contended case. It is a specific form of ignorance (*jahāla*) – namely, "unknown of attribute" (*majhūl al-ṣifa*) – which, as Hallaq explains, "presumes existence [of the object of contract] but involves lack of reasonable knowledge of the thing's characteristics"[28]. However, 'object of sale' (*A*) and 'object of marriage' (*A**) take different paths in relation to the contractor's subsequent recourse to option (*khiyār*). Thus, the meaning of *B*, 'unknown of attribute', changes if *A**, 'object of marriage', is substituted for *A*, 'object of sale'; and Q's objection of *kasr* is thus nullified.

25. Al-Shīrāzī, *Sharḥ al-Lumaʿ*, ed. al-Majīd Turkī p. 893 and pp. 898-899.

26. Although this is not made explicit in the example in the *Maʿūna*, it is in the example in the *Sharḥ al-Lumaʿ* (p. 893 and pp. 898-899), where (*A*) *mabīʿ* is replaced by (*A**) *mankūḥa*.

27. Al-Shīrāzī, *Maʿūna*, ed. al-ʿUmayrīnī, §127, p. 246.

28. Hallaq 2009, p. 244.

Type 1b: In this example, we begin with the same *qiyās al-ʿilla*, but Q's objection is invalid because his substitution and *kasr*-case are clearly not in the *maʿnā* of R's original property and the contended case.

– Q attempts to invalidate R's *ʿilla* by bringing the *kasr*-case of bequeathing (i.e. in a will) something which is [A*] an object of bequest (*mūṣā bihi*) and [B] unknown of attribute (*majhūl al-ṣifa*) to the contracting party [i.e. the legatee] when the contract was made, but which is nevertheless allowed, in contrast to the contended case of sale.

Such is like when the one drawing indication [R] says: "It is [A] an object of sale which is [B] unknown of attribute to the one contracting at the time of contract, so it resembles when he says: 'I sell you a garment'" So he [Q] says to him: "This is broken (*yankasiru*) by [A*] the object of bequest (*mūṣā bihi*), for it is [B] unknown of attribute to the one contracting at the time of contract, but the bequeathing of it is valid"[29].

Thus, Q has replaced [*A*] with [*A**], then attempted to show the *naqḍ* of this parallel compound [*A*B*] in the *kasr*-case.

– However, Q's replacing of *A* (*mabīʿ*, object of sale) with *A** (*mūṣā bihi,* object of bequest) in the *kasr*-case is invalid. Why? Because *A* and *A** are not in the same *maʿnā*. As al-Shīrāzī explains in the *Sharḥ al-Lumaʿ*:

> Bequest (waṣiyya) is not a parallel case (naẓīr) for sale in terms of lack of knowledge (jahāla), nor is it in [sale's] quality/meaning (maʿnā). Don't you see that no [kind of] lack of knowledge (jahālāt) prevents the validity of the bequest? For this reason, if he were to say: "I bequeath to you a garment, or something, or wealth / property"[30], it would be permitted. But the like of this is not valid in sale.

Thus, Q attempts to replace *A* 'object of sale' with *A** 'object of bequest', despite the fact that a legatee's ignorance of the nature or characteristics of an object of bequest has no efficiency whatever with regard to the bequest contract's validity. According to our understanding, the case is deceptive precisely because, though sharing that property of ignorance (*B*) which is deemed efficient in R's root- and branch-case, its irrelevance in the context of bequest puts Q's substitution and *kasr*-case outside the *maʿnā* of R's original property and cases. The legatee's ignorance of what has been bequeathed to him simply does no work in occasioning that contract's validity or lack thereof, whereas the buyer's ignorance of what has been sold to him does.

29. Al-Shīrāzī, *Sharḥ al-Lumaʿ*, ed. al-Majīd Turkī, 1041.
30. Reading [بخير] instead of [بخبر].

Conclusions regarding Type 1

Our texts indicate that the compounds targeted by type 1 *kasr* involve meaning dependences of the unchanged property (or properties) upon the property replaced. The relation is a conceptual link, whereby the second property [B] in fact constitutes a specification of the first [A]. That is, what is unknown of attribute to the buyer at the time of sale is a specific kind of object of sale – *almost* in the way that a bride unknown of attribute to the groom at the time of marriage is a specific kind of object of marriage (but not quite, since the groom has no option to rescind), and *certainly not* in the way of a legatee's ignorance of the object of bequest (which is in fact a normal state of bequest, not a specific kind).

Type 2: *Kasr* by Removal (*isqāṭ*) in Independent Conjunctive Compounds

In contrast to type 1, a valid type 2 *kasr* works by omitting a non-efficient property of R's compound *ʿilla* and proving the remainder inconsistent via *naqḍ*. Although at this stage of research we have uncovered no explicit positive statements to this effect, we may also conjecture, by sheer logical analysis of the examples discussed in the sources, that type 2 *kasr* was understood (again in contrast to type 1) to target compounds wherein *each of the components was thought (by R) to contribute independently to the occasioning of the ruling*. However our jurists may in fact have conceived of type 2 *kasr*, we will carry this assumption forward into our analyses in order to explore the full potential of this objection and its responses without invalidating it from the start.

Type 2a: Valid *kasr* by *isqāṭ*.

– Valid objection (*iʿtirāḍ*): Given a compound of properties *ABC*, which R claims to be the *ʿilla*, a valid *kasr* by *isqāṭ* is when Q (1) brings a parallel *kasr*-case in which one of R's properties (e.g. *A*) is omitted but the remainder (e.g., *BC*) are present; (2) justifies this *kasr*-case by showing that *A* lacks efficiency (a subsidiary objection of *ʿadam al-taʾthīr*), and can thus be removed; and (3) shows that this *kasr*-case constitutes a *naqḍ*-case for what remains of R's *ʿilla* (*BC*) – i.e. that *BC* is present therein without R's *ḥukm*. Note that Q thus contests R's components in two moves, first denying efficiency to one, and then destroying what remains via *naqḍ*.

– Valid response (*jawāb*): R therefore has recourse to either (1) contesting Q's objection that *A* lacks efficiency (i.e. his subsidiary move of *ʿadam al-taʾthīr*), or (2) contesting Q's *kasr*-case in that it does not demonstrate the *naqḍ* of *BC*.

Type 2b: Invalid *kasr* by *isqāṭ*.

– Given a compound of properties *ABC*, which R claims to be the *ʿilla*, an invalid *kasr* by *isqāṭ* is when Q's *kasr*-case omits *A* despite there being clear evidence that *A* is in fact causally efficient. Having thus wrongly removed the truly

causal portion of R's compound *ʿilla*, the *kasr*-case's subsequent *naqḍ* of what remains (*BC*) does no real harm, having failed to show inconsistency for the causally efficient *A*.

In sum, it may help to think of type 2 *kasr* by *isqāṭ* as Q accusing R of 'stuffing' his *ʿilla* with extra, non-efficient properties. Thus, in subtype 2a, Q rightly jettisons the non-efficient stuffing and asserts *naqḍ* of the efficient remainder, while in subtype 2b, Q wrongly jettisons the efficient component and asserts *naqḍ* only of the non-efficient stuffing. In the following, we will only examine R's response to 2a as discussed by al-Bājī[31], since it yields a complete illustration of this subtype, whereas 2b represents a special case in which Q gets it wrong from the start.

Examples: Type 2

Type 2a: valid *kasr* and valid response

– R asserts that intent (*niyya*) is not obligatory for the minor ritual ablution (*wuḍūʾ*) because *wuḍūʾ* is [*A*] a means (*sabab*) of reaching prayer, and [*B*] not a substitute act (*badal*); therefore, *niyya* is not obligatory, as in the root-case of removing filthiness (*najāsa*), which also has both properties *AB*[32].

In al-Bājī's words:

> As for when [Q] omits a non-efficient property, it is a valid *kasr*. This is like when the Ḥanafī [R], for the [issue] of intent (*niyya*) in minor ritual ablution (*wuḍūʾ*), draws indication that [intent] is not obligatory, because [ritual ablution] is [A] a means of access/mediate cause (*sabab*)[33] by means of which one reaches prayer, which is [B] not in the way of a substitution (*badal*)[34], so intent is not obligatory for it – like [in the case of] removal of impure filthiness (*izālat al-najāsa*).

Thus, R's *qiyās* has the following components:

contended branch-case (*farʿ*):	Intent (*niyya*) in the minor ritual ablution (*wuḍūʾ*)...
claimed ruling (*ḥukm*):	is not obligatory...
claimed legal cause (*ʿilla*):	because *wuḍūʾ* is [*A*] a means (*sabab*) of reaching prayer, and [*B*] not a substitute act (*badal*)...
endoxon root-case (*aṣl*):	like removal of impure filthiness (*izālat al-najāsa*).

31. Al-Bājī, *Minhāj*, ed. al-Majīd Turkī, 436.

32. The relevant substantive legal category here is called *ibdāl*, or substitution, and is when an original, obligatory act of ritual worship, the *aṣl*, is replaced by another kind of 'substitute' act, the *badal*, either because one was not able to do the *aṣl*, or one has neglected/invalidated the *aṣl* and needs to make up for it. In this example, a condition (*sharṭ*) of valid prayer is valid *wuḍūʾ*: the minor ritual ablution with water (constituting the original act, or *aṣl*); but should one find oneself in, e.g. a waterless waste, one may instead perform *tayammum*: ritual ablution with sand or clean earth (constituting the substitute act, or *badal*). *Tayammum* is a *badal* for the originally obliged *aṣl* of *wuḍūʾ*.

33. On the *sabab*, see Hasan 1986, p. 369 sqq.

34. On substitution (*ibdāl*) in general, see Wizārat 1986, vol. 1, p. 140 sqq., s.v. [لادبا].

– Now, in objecting via type 2a *kasr*, Q 'breaks' R's *ʿilla* in three steps:

First, Q brings the *kasr*-case of *tayammum*, the ritual ablution with sand or earth, which contains *A* (being a *sabab*) though not *B* (not being a *badal*)–in fact, *tayammum* is the very *badal* for *wuḍūʾ* – but for which *niyya* is obligatory (the opposite of R's ruling). Property *B* is thus omitted in Q's *kasr*-case (*tayammum* is *A* but not *B*).

Second, Q justifies this omission by claiming that *B* cannot be an efficient property vis-à-vis non-obligation of *niyya* in the contended case, since when it comes to *niyya* whatever holds for the *badal* always holds for the *aṣl* and vice versa – it is never the case that one requires *niyya* and the other doesn't (as is the consequence of R's claim). Q thus attempts to show that *B*, not being a *badal*, has a lack of efficiency (*ʿadam al-taʾthīr*) in occasioning a ruling (*ḥukm*) of non-obligation of *niyya*.

Third, once *B* is shown to be non-efficient 'stuffing', Q's *kasr*-case automatically becomes a *naqḍ*-case for the remaining property *A*. In the case of *tayammum*, we find property *A* (being a *sabab*), but not R's ruling (non-obligation of *niyya*). R's *ʿilla* is present despite the absence of R's *ḥukm* – the very definition of *naqḍ*.

In al-Bājī's words:

> So the Mālikī [Q] says to him: "This is broken (*yankasiru*) by ablution with dry earth or sand (*tayammum*), for it is [A] a means of access / mediate cause (*sabab*) by means of which one reaches prayer, yet it requires intent[35]. This is a valid *kasr*, because the property by way of which [R] guarded against [inclusion of] ablution with dry earth or sand (*tayammum*) [that is, *B*: it is not a substitution (*badal*)] is not efficient in the falling away of intent (*niyya*)[36]. Don't you see that original [acts] (*uṣūl*)[37] and substitutions (*abdāl*)[38] are one with regard to the subject of intent in God's Law?"

– R, however, successfully escapes this objection with a valid response. In short, he revives the possibility that property *B* (not being a *badal*) can have efficiency, by destroying Q's claim of *aṣl-badal* correspondence vis-à-vis *niyya* with counter-examples wherein an *aṣl* does not require *niyya* but its *badal* does. With the possibility of B's efficiency thus restored, R's *qiyās* evades Q's type 2a *kasr* by *isqāṭ*.

35. Note that *tayammum* is itself the substitution done in place of *wuḍūʾ* when there is no water, and so Q has omitted property [B] "not being in the way of a substitution".

36. That is, R likely formulated his compound *ʿilla* with property B (not in the way of substitution) in order to exclude the endoxon case of *tayammum*, which has the opposite (intent is obligatory) of the desired *ḥukm*, but that property B has no efficiency in the negation of intent in the case at hand. So Q's omission of property B in his *kasr*-case of *tayammum* (which is in the way of substitution) is valid, and exposes the inconsistency (*naqḍ*) of the other property A: that is, property A is present in the *kasr*-case of *tayammum*, but without R's *ḥukm*. Thus, it is a valid *kasr*.

37. In this case, minor ritual ablution (*wuḍūʾ*).

38. In this case, ablution with dry earth or sand (*tayammum*).

Conclusions regarding Type 2

Our texts on this type of *kasr* suggest that R implicitly assumes that the compound *AB*, in order to occasion the ruling, must be constituted by a conjunctive antecedent.

Now, if we link an antecedent of the form $A \wedge B$ to R's ruling with some form of implication, and the components of the antecedent do not constitute a chain of dependences as in type 1 *kasr*, then it logically follows that these components distribute **disjunctively** in relation to R's ruling.

This logical fact explains why Q's challenge in type 2a requires both denying efficiency to one property and destroying the other: challenging a conjunction that distributes over an implication requires challenging all of the resultants of the disjunctive distribution. What the dialectical setting adds to this purely logical analysis is the fact that Q's challenges are not of the same form (one claims lack of efficiency, the other builds a *naqḍ*-case). This likewise explains why R can respond by countering one of these different challenges.

Moreover, it is the disjunctive distribution of the conjunction over the implication that explains why, in type 2b, the move that contests the 'redundant' components in the antecedent does not destroy the main claim, given that the component known to be efficient has been omitted from the charge of inconsistency.

Thus, logical analysis suggests that we may think of type 2 *kasr*'s target *qiyās* as being implicitly understood by R to ground such assertions as:

> Performing actions of the types containing *A*, *B* ... is forbidden... iff performing types containing *A* is forbidden or performing types containing *B* is forbidden.

In fact, if we do not assume such a logical analysis, then type 2 *kasr* is wrong from the start. Certainly, we cannot read its target *qiyās* as being of the sort aimed at by type 1 *kasr*, as made plain through considering once more the example of *wuḍū'*. R claims it does not require *niyya* because it is a *sabab* and not a *badal*; but if we read that according to the (meaning dependence) manner of type 1 *kasr*'s target *qiyās* composition, then the attempted *kasr*-case of *tayammum* would be precluded from the outset.

5. Towards a Dialogical Meaning Explanation of *Kasr*

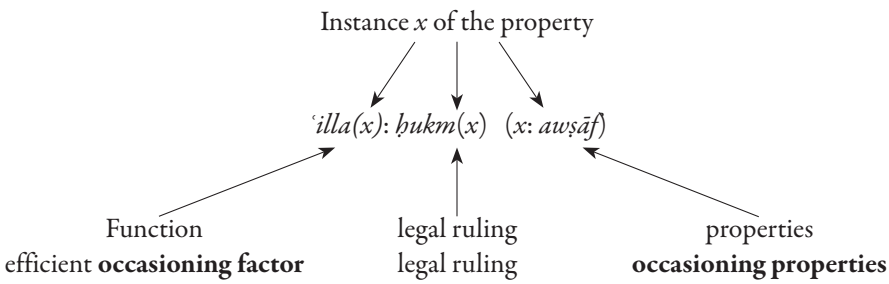
Due to space constraints we will focus on developing a logical analysis that will constitute the building blocks for *kasr*'s dialogical framework. In this study, however, we will neither be able to describe all the required dialogical steps nor to build the actual dialogue.

5.1. *Functional Analysis of the Occasioning Factor*

Rahman and Iqbal have proposed a CTT analysis of the conceptual link between the ruling and the properties constituting the occasioning factor[39]. The idea is that a ruling is conceptually dependent upon the properties that occasion that ruling. Certainly, the *Forbidden* occasioned by intoxicating drinks is not the same as the *Forbidden* occasioned by murder. The legal system attends to this distinction by the quality and degree of sanctions that penalize such acts.

In the context of a CTT framework, functions are the means to express such dependences; and a function is a method or procedure for relating concepts such that one is dependent upon the other. In legal contexts, executing this function is a legal procedure by means of which performances of a type of action occasion the ruling's application to such performances. In concrete cases, it is the legal authority who carries out the procedure.

The upshot of such an analysis is that it allows one to distinguish between the property constituting the *occasioning factor*, and the *actual legal procedure that occasions* the ruling. In other words, according to this view, it is useful to distinguish between the property (Ar. *waṣf*, pl. *awṣāf*) and the properly efficient cause/occasioning factor (*ʿilla*), that is, the procedure that transforms instances of occasioning (causal) factors into applications of the ruling to these instances. If we recall our notation in section 3 we have:



– Notice that the CTT framework allows the introduction of potentially infinite types, each of them intensionally defined – they are not extensional sets. This not only allows one to distinguish between different types of action but between different types of properties as qualities; legal contexts require certain multi-sorted domains. Thus, this multi-sorted language allows one to introduce performances of actions and instances of qualities into the object language. With the term *awṣāf* we may refer to any of these. The distinction between types of actions and types of quality is to be determined in context. And there might be certain

39. See Rahman / Iqbal 2018, pp. 67-132, which is further developed in Rahman / Iqbal / Soufi 2019, pp. 31-40, and in Iqbal 2022.

arguments supporting the reduction, in legal contexts, of all types to types of actions, and in natural contexts, of all types to types of events.

If the above is granted, we might take a step further and delve into the conceptual links constituting compound occasioning factors.

5.2. *Compound Occasioning Factors within* Kasr-*Cases*

As discussed in the previous section the *kasr* method of refutation targets compound occasioning factors: the first type targeting compounds wherein the meaning of one property is dependent upon another, the second targeting meaning independent components.

5.2.1. Dependent Composition

The compound *AB* at work in type 1 *kasr*-cases by substitution may be understood as *B* constituting a *propositional function* over *A*, in the manner described in section 3; that is, the meaning constitution of *B* is based on *A.* (Recall Aristotle's point on the role of 'Good' in 'Good Cobbler')[40]. Notice that in this form of *kasr* Q's objection is based on the idea that both objects of sale and objects of marriage are objects of contractual transaction – i.e. they are in the same meaning (*maʿnā*). Thus, according to Q, they should have the same ruling when they share the property "involving objects unknown of attribute to a contractor at the time of contract", but – and this inconsistency is at the heart of the *kasr* critique – in fact they do not have the same ruling. R's counter-objection, however, specifically targeting Q's *kasr*-case of the unseen bride, demonstrates that his substituting object of marriage (*mankūḥa*) for object of sale (*mabīʿ*) in fact contributes a *kasr*-case marred by a disqualifying difference (*farq*) – i.e. they are not in the same meaning. A commodity whose attributes are unknown to the buyer at the time of a sales contract is relevantly different from a bride whose attributes are unknown to the groom at the time of a marriage contract, as proven by the buyer's right to rescind and the groom's lack thereof. Thus, the ruling of proscription applies only to selling the unseen object of sale, and not to marrying the unseen bride.

In other words, the efficiency of the property "involving objects unknown of attribute to a contractor at the time of contract" in occasioning a ruling of proscription is not evident outside of apt substitutions of similar sets of objects. That is, objects in the same *maʿnā* should allow the property to play the same causal role in Q's opposing *kasr*-case as in R's original, authoritative root-case. Q's substitution should preserve the *meaning dependences* of the original case even as it aims to disprove R's

40. See Arist., *De Int.* 9, 20b35-36 and 21a14-15; *Soph. El.* 20, 177b14-15.

juridical ruling's dependence upon this parallel compound. However, R's counter-objection by disqualifying difference (*farq*) shows that it does not preserve these meaning dependences. This constitutes a novel approach to illicit moves concerning composition. Not only does Type 1a *Kasr* and its unseen bride example occur in a context of determining (legal) causation, but it concerns primarily the *illegitimate substitution* of one of a compound cause's relevant components.

Note the logical structure displays a *double meaning dependence*, namely:

1. In the context of R's root-case, the ruling *not valid* is linked to a cause composed of two properties: *object of sale* and *attributes unknown to contractor at time of contract* (i.e. *attributes unknown to buyer at time of buying*), and not to any other compound cause.

2. In the context of R's root-case, the property *attributes unknown to contractor at time of contract* refers only to objects of sale, and not to objects of any other contract (such as marriage).

Thus, we may begin our formalization as follows:

Selling objects of sale, unknown of attribute, is not valid.

$$(\forall y\text{: } \{\, x\text{: } \textit{Selling} \mid \textit{Involving objects unknown of attribute}\ (x)\ \}\ \textit{Invalid}(y)$$

Any instance y of { those sales x, which involve objects unknown of attribute } { is an invalid y}

In fact, we can be more precise and indicate that what is invalid is actually the *selling of something* unknown of attribute – that is, what is (contractually) invalid is the *left component* of y. Indeed, if:

$$\{\, x\text{: } \textit{Selling} \mid \textit{Involving objects unknown of attribute}\ (x)\ \}$$

is the set of all those sales of objects unknown of attribute, and y is an element of this set:

$$y\text{: } \{\, x\text{: } \textit{Selling} \mid \textit{Involving objects unknown of attribute}\ (x)\ \}$$

then this already assumes the meaning dependence at work:

Involving objects unknown of attribute (x): *prop*
Involving objects unknown of attribute (x) is a proposition

$(x$: *Selling*$)$
provided x is an instance of (carrying out) a *Selling*

With such being the case, then the left component of y is the ***selling x*** (which is of those sales involving objects unknown of attribute)[41], that is:

41. Notice that the set $\{\, x\text{: } \textit{Selling} \mid \textit{Involving objects unknown of attribute}\ (x)\ \}$ is constituted by

left(*y*)=*x*: *selling*,

and the right component of *y* verifies that this selling *x* ***involves an object unknown of attribute***:

right(*y*)=*z*(*x*): *Involving objects unknown of attribute* (*x*).

This leads to the formulation:

(∀*y*: { *x*: *Selling* | *Involving objects unknown of attribute* (*x*) } *Invalid*(*left*(*y*))

The illegitimate use of *kasr* is thus due to the fact that the following substitution does not hold:

(∀*y*: { *x*: *Marrying* | *Involving objects unknown of attribute* (*x*) } ~~*Invalid*(*left*(*y*))~~

And why does this not hold? It does not hold due to a disqualifying difference (*farq*), whereby a telling norm holds true for objects of sale but not for objects of marriage – for commodities, but not brides:

Disqualifying difference (*farq*)

(∀*y*: { *x*: *Selling* | *Involving objects unknown of attribute* (*x*) }
Rescinding-allowed (*left*(*y*))

(∀*y*: { *x*: *Marrying* | *Involving objects unknown of attribute* (*x*) }
Rescinding-not-allowed(*left*(*y*))

This should clarify why type 1 *kasr* can only be carried out by substitution: what is asserted is that the ruling ***forbidden*** applies only to those *Sales y involving objects unknown of attribute*. Substitution should preserve the meaning dependences of original case and also the dependence of the juridical ruling upon this compound.

The approach to the compound *AB* which is at work in type 2 *kasr* cases by *removal* may be understood as omitting from the conjunction the non-efficient component and building an inconsistency-proving *naqḍ*-case for the remainder. As mentioned above, logically speaking such a move is only possible if the meaning of the component that builds the *naqḍ*-case is not dependent upon the one removed[42]. Let us study the logical structure behind this, step by step:

all those sales which have been carried out while the object is unknown of attribute. Thus, for every *x* in this set, one can say that it is a sale involving objects unknown of attribute.

42. Each component expresses a property belonging to some domain of objects (or actions) – for the sake of simplicity we leave the domain tacit. In the context of CTT grammar, such a process is called *sugaring*: it is the reverse of formalization – see Ranta 1994, pp. 7-11. Sugaring procedures are

$$A \wedge B\text{: } prop$$

What makes this proposition true is in fact a pair (or however many components constitute the conjunction) such that the first element of the pair verifies *A*, the second verifies *B*, the third verifies *C*, and so on:

$$(x_1, x_2)\text{: } A \wedge B$$

(Whereby x_1 stands for a function that takes elements of the domain *D* and asserts of them that they are *A*, and something similar holds for $x_{2)}$.

Recall that in our *wuḍū'* example the two members in the conjunction are the properties *being a sabab (intermediate means) for reaching prayer* **and** *not being a badal* (i.e. a substitute for a ritual component missed or contextually unobtainable).

$$(x_1, x_2)\text{: } Sabab \wedge Not\text{-}Badal$$

If we then relate the ruling to the compound, we might be tempted to express it as:

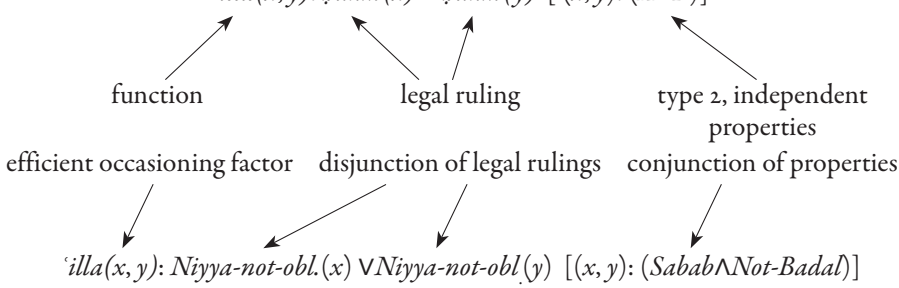
$$\text{'}illa(x,y)\text{: } ḥukm \;\; [(x, y)\text{: } (A \wedge B)]$$

However, this does not express the fact that the second property might be superfluous 'stuffing' – as in fact it is in the example under consideration. Thus, what we need is to make apparent the logical consequence of an analysis that contemplates cases wherein one of the components is not efficient. As mentioned above, this amounts to a disjunctive distribution of the conjunction over the (claimed) occasioned ruling:

$$\text{'}illa(x,y)\text{: } ḥukm(fst(x,y)) \vee ḥukm(snd(x,y)) \;\; [\,(x, y)\text{: } (A \wedge B)]$$

And since *fst(x,y)=x: A, and snd(x,y)=y: B,* we may indulge in the following shortcut:

$$\text{'}illa(x, y)\text{: } ḥukm(x) \vee ḥukm(y) \;\; [\,(x, y)\text{: } (A \wedge B)]$$



This explains why, if an unskilful Q removes a property known to be efficient, the refutation aborts: verification of one of the components is sufficient. More-

those transformations which, step by step, produce a natural language expression. In the case of ambiguity in a natural language expression, one traces bottom-up the meaning-constitution sugaring process that yielded this expression.

over, it explains why, in order to refute R's claim, Q needs to contest the efficiency of one of the components and to build a *naqḍ*-case for the other. In the *wuḍūʾ* example, it amounts to contesting the efficiency of:

ʿilla(wuḍūʾ$_2$*)*: *Niyya-not-obligatory*(*wuḍūʾ*$_2$) (*wuḍūʾ*$_2$: *Not-Badal*)

and building a *naqḍ*-case against:

ʿilla (wuḍūʾ$_1$*)*: *Niyya-not-obligatory (wuḍūʾ*$_1$) (*wuḍūʾ*$_1$: *Sabab*)

by replacing *wuḍūʾ* with *tayammum*. Given:

(*wuḍūʾ*$_1$, *wuḍūʾ*$_2$): (*Sabab*∧*Not-Badal*)

Whereby: *wuḍūʾ*$_1$: *Sabab*

wuḍūʾ$_2$: *Not-Badal*

The above CTT analysis reveals that the two different compositional modes of compound factors occasioning a juridical ruling targeted by a *kasr* refutation decline into two different ways of linking the members of such compounds by means of what, in natural language, we call conjunction. In short, this conjunction either (1) might assume that components display some form of conceptual dependence (as in type 1's target *qiyās*), or (2) might assume that they do not (as in type 2's target *qiyās*).

Note that the dialectic rules established for building a *kasr*-case for both forms of composition require basically the same form of verifiers: namely, a verifier for each of the components (though in the first type [at least], one of the verifiers is a function having as its argument the other verifiers of the compound). This strongly indicates that the two types of compound may be included under a single category we might call 'conjunctive composition'.

Here we may appeal to a famous insight of Avicenna's: namely, that an implication can be reduced to a universally quantified proposition if its components *share content*.

> All conditional and disjunctive propositions, and in particular the conditional in which the antecedent and the consequent share one part, can be reduced to categorical propositions[43].

Accordingly, implication can be generalized as distinguishing, within the category of implications, those for which the content of the antecedent does not occur in the consequent (propositional implications) from those for which it does (universal quantifiers with restricted domains). In CTT and its grammar, the

43. Avicenna, *Al-Shifāʾ : al-Manṭiq : 4. al-Qiyās*, ed. Zāyid, p. 256, ll. 11-15. Reference (and translation) is from Street 2004, p. 533.

category including both forms of implication (but not disjunction) is known as Π-*Type*, also called the *cartesian product of a family of sets*[44].

This Π-*Type* is the dual, i.e. the logical counterpart, of a category we might call *conjunctive* and which includes conjunctions wherein some of the members *share content with the other (*existential quantifiers with restricted domains or subset separation: those elements in A that are B) and those that do not (propositional conjunctions). In CTT, the general category that includes both forms of conjunction is called Σ-*Type* or *disjoint union of a family of sets* Σ-*Type*[45]. The upshot of all this is that whereas our analysis of type 1 amounts to *subset separation* (those elements in A that are B), type 2 corresponds to *propositional conjunction*. Both types of conjunctive composition are at work in the two types of *kasr* we have examined.

Notice that in the case where components of a Σ-*Type* are dependent, the elimination rule holds, but each eliminated component will encode the information of the dependence. More precisely, from the compound *all those A that are B*, the function $fst(y, d(y)) = y : A$ renders "those A that witness B *within* the compound", and $snd(y, d(y)) = d(y) : B(y)$ renders "those Bs that are As *within* that same compound".

And if we again recall our example of the object of sale, $fst(y, d(y)) = y : A$ renders "a sale, involving an object unknown of attribute to the buyer", and $snd(y, d(y)) = d(y) : B(y)$ attributes the property "involving objects unknown of attribute" to instances of sales.

Note that, in fact, interdiction in the sale example has, as its scope, $fst(y, d(y)) = y : A$, i.e. "those acts of selling involving objects unknown of attribute to the buyer". Thus, if *selling a garment* is one specific sale (of an object of unknown of attribute), then $fst(y, d(y)) = y : A$ can be read as the anaphoric construction:

> "If selling a garment is selling one of those objects unknown of attribute to the buyer, then 'this' selling (of a garment) constitutes a law-breaking selling"

5.3. *Dialogical Meaning Explanations*

Rahman and Iqbal, prompted by Young's *Dialectical Forge,* proposed certain first steps towards a dialogical reconstruction of al-Shīrāzī's notion of parallel reasoning, within the dialogical framework called *Immanent Reasoning*[46]. This latter is

44. See Ranta 1994, pp. 45-46. Notice that $((x: A)B$ and $A \supset B$ express the same proposition. However, if B and A "share a part", and, more precisely, if B is dependent upon A, we have $(\forall x: A) B(x)$. So Avicenna is right: every implication can be subsumed under the general universal $(\forall x: A) B[x]$, where the square brackets indicate that 'x', the 'part' of A, might occur or not occur in B.

45. Ranta 1994, pp. 43-45.

46. See Rahman / Iqbal 2018 with further development in Rahman / Iqbal / Soufi 2019. As for immanent reasoning, see Rahman / McConaughey / Klev / Clerbout 2018.

a framework that integrates the fully interpreted language of CTT into Lorenzen & Lorenz's dialogical logic[47], in order to achieve a framework which is sensitive both to content and to the perspective of the meaning play level over the strategic and logical validity level.

Being that we cannot here reproduce all the rules for parallel reasoning developed in Rahman, Iqbal, & Soufi[48], we will restrict ourselves to the case of *kasr*, which was only very schematically and briefly discussed by the authors in 20 lines. Moreover, we will only mention the more crucial steps in the context of the examples discussed in our sources above

5.3.1. Dialogical Meaning Explanation of *Kasr* Type 1

Let us recall al-Shīrāzī's example, where the *ʿilla* of R's *qiyās* is a compound *ʿilla* (*ʿilla murakkaba*) consisting of properties *A*: being an object of sale (*mabīʿ*) and *B*: being unknown of attribute (*majhūl al-ṣifa*) to the contractor at the time of contract, while the claimed ruling (*ḥukm*) is 'not allowed'. Q attempts to invalidate R's *ʿilla* by replacing one of its properties (namely, *A*) with another (*A**, *mankūḥa*: *object of marriage*) and showing that the parallel compound *A*B* is thus co-present with the opposite of R's ruling (i.e. it *is* valid) in the seemingly parallel *kasr*-case of marriage. However, Q's case is based on a substitution that changes the meaning, since the substitute carries with it the additional meaning of *having the option to rescind*. The point, dialogically speaking, is the following:

Interlocutor X observes that if we delve into the meaning of *B*, *involving objects unknown of attribute*, it is apparent that it encompasses more than one kind of "being unknown of attribute" in contexts of contractual transactions, including those with the option to rescind (*khiyār*) and those without. Both are different specifications of *involving objects unknown of attribute*.

So, let *T* stand here for the general set of *acts of transaction* (which include acts of sale and marriage); *B*, as before, for *involving objects unknown of attribute*; R_1 for the property *involving objects unknown of attribute with option to rescind*, as said of acts of Transaction; and R_2 for the property *involving objects unknown of attribute without option to rescind*. Thus, the original proposition and the substitute do not share the same meaning, i.e. they *separate* two sets within the set of transactions *T*, namely:

– the set of those transactions with option to rescind (including acts of selling) involving objects unknown of attribute;

47. Lorenzen / Lorenz 1978.
48. Rahman / Iqbal / Soufi 2019.

– the set of those transactions without this option to rescind (including acts of marriage) involving objects unknown of attribute.

Accordingly, X concludes that R_1 and R_2 yield different rulings. Whereas the former is legally not allowed, the latter is:

X *ʿilla*(*fst*(*x*)): ~*allowed*(*fst*(*x*)) (*x*: (∃*z*: {*y*: *T* | *d*(*y*): R_1(*y*)}) *B* (*z*) })
X *ʿilla*(*fst*(*x*)): *allowed*(*fst*(*x*)) (*x*: { ∃*z*:: {*y*: *T* | *d*(*y*): R_2(*y*)} | *B* (*z*) })

Note that at this point of the debate Y might either concede or demand evidence for the latter assertions; and providing such evidence might commit X to finding cases in the sources, and thus providing rival occasioning factors.

5.3.2. Dialogical Meaning Explanation of *Kasr* Type 2

In relation to our paradigmatic example of removal, let us recall that an original, obligatory act of ritual worship, the *aṣl* of *wuḍūʾ*, claimed not to require intent (*niyya*), was confronted by the case of a 'substitute' act, the *badal* of *tayammum*, which in fact *does* require intent. This proved that whether something is a *badal* or not has no efficiency in relation to requiring intent.

Valid *Kasr* Type 2

As noted, it follows from the logical analysis of claims constituted by a compound antecedent, said to occasion a ruling, that it is the disjunctive distribution of that ruling over the claimed occasioning factor that makes challenges by *removal* even possible. The explicit claim has the form:

X *ʿilla*(*x*,*y*): *ḥukm*(*fst*(*x*,*y*)) ∨ *ḥukm*(*snd*(*x*,*y*)) [(*x*, *y*): (*A*∧*B*)]

This shapes the dialogical interaction. Q must (1) attempt to show that property *B* (not being a *badal*) is not efficient – despite being paired with *A* (being a *sabab*) – and that it can therefore be removed, **and** then (2) proceed to build an inconsistency-proving case against *A*. The disjunctive distribution requires that both kinds of challenges be carried out. R can escape with a valid response, supposing R can restore the removed disjunct by reviving the possibility that property *B* (not being a *badal*) can have efficiency.

6. Overall Conclusions and Work Ahead

The current study is only a first step towards a larger investigation of *kasr*, which will encompass both a comparison between *kasr* and other dialectical objections and analyses of critical discourses from later Muslim legal theorists and dialecticians. In fact, such work should contribute to large-scale research on the notion

of illegitimate moves in Islamic dialectics. Here we will advance, very briefly, some remarks – in fact, working hypotheses – that might motivate or lead such research.

Fallacious – that is, illegitimate – arguments in a specific field of knowledge are arguments that contravene procedural or meaning-setting rules that prescribe what should count as a sound argument in relation to the conceptual framework of that field. From this point of view, the so-called deontic paradoxes or puzzles that we have referred to as extrapolation fallacies find a solution – not by adjusting *ad hoc* some inference rule, but by observing the reasoning patterns for linking concepts established in a specific field.

The setting of what is correct and not correct is implemented by a notion that seems to prefigure what we nowadays in proof-theory call 'admissibility'. The Aristotelian theory of the syllogism formulates a general form of this notion. And one way to interpret the Medieval focus on classifying fallacies relates to the task of identifying one or more admissibility criteria for shaping reasoning patterns. In this light, the Islamic tradition seems to have distinguished admissibility criteria at work in argumentation theory from criteria closer to the perspective on syllogistic fallacies.

The key point, so we conjecture, is that, whereas syllogism-based criteria of admissibility focus on the strategic point of view (namely, that what is admissible is what constitutes optimal moves from the point of view of logical validity), dialectic-based based criteria of admissibility focus on the play-level. In other words, what is admissible is what allows one to carry out dialectical interactions according to rules that establish the meaning relevant to the concepts at work in a debate. An inadmissible move is what contravenes the rules on how to debate, not on how to win the debate in regard to any possible move of the contender. Note that this leaves room for valid moves, that is admissible moves, which are not always successful.

Islamic texts wisely include discussions on both dialectical illegitimate moves (with a focus on the play level) and syllogistic-based illegitimate moves (with a focus on the strategy level of logic). Both points of view are complementary: plays are what strategies are made of. What we should not do is impose a purportedly universal, abstract, strategic admissibility criteria on debates wherein concrete interlocutors dispute content constituted and agreed upon during the interaction. And this, we dare to suggest, is what the dialectical stance in Islamic argumentation theory is all about.

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Abstract: This study examines an Islamic dialectical objection called *kasr*, or 'breaking', as treated by the dialecticians and legal theorists Abū Isḥāq al-Shīrāzī (d. 1083 CE) and Abū al-Walīd al-Bājī (d. 1081 CE). *Kasr* has both legitimate and illegitimate applications, and we confront the fallacious modes denounced by our theorists with deontic paradoxes and puzzles falling under the rubric of *logical extrapolation fallacy*. Our primary claim is that, whereas logical extrapolation produces fallacies or paradoxes by unsafely applying inference rules of standard alethic and/or logical necessity to the deontic realm, the fallacies generated by illegitimate modes of *kasr* in Islamic legal theory (wherein logical rules are expressed dialectically) constitute a genuine source for reflecting on what patterns of reasoning should be endorsed for determining causality in legal – and, perhaps, natural – epistemological contexts. Ultimately, this is the first step towards a larger study of *kasr*, which will compare it with other dialectical objections and treat relevant critical discourses from later Muslim legal theorists and dialecticians.

Keywords: Islamic juristic dialectic (*jadal / munāẓara*); correlational inference (*qiyās*); breaking (*kasr*); logical extrapolation fallacy; compound causes; dialogical logic.

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A Forgotten Mereological Paradox

1. Formulations of the Problem

The problem's formulations can, on the surface, easily be divided into two kinds: those phrased solely in terms of relations (*nisab*) and those in terms of concepts (*mafhūmāt*)[1]. The most sensible way to classify them, however, is instead in terms of the difficulty each identifies.

1.1. *Contradiction*

Consider the totality (*majmūʿ*) of all relations (*nisab*) (call it M). Some of the relations included in M are, for example, the spatial relation between my bookcase and my computer, the causal relation between me and these words, and the relation of dependence between the integers one and two. More important than any of these three for the purpose of the problem, however, is the containment relation between M and each of its constituents. M contains the very containment relation between itself and any one of its constituents.

Now consider the following two facts. (1) The parts of any totality are prior to it. That is, in order to have a totality, one must first have the constituents it is made up of, which is not necessarily a matter of temporal priority but rather one of dependence. Since the containment relation is a part of the totality M, it must therefore be prior to it. (2) Relations are posterior to the things they relate. In other words, a relation can only obtain between things if the things have obtained. But that would mean that the containment relation between M and its constituents must be posterior to M (one of its relata). If these two facts and our

1. I am grateful to Prof. Asad Q. Ahmed and Prof. Timothy Clarke for their comments and criticism. Any errors that remain are, of course, my own.

inferences therefrom withstand scrutiny, a contradiction arises: the same relation is both prior and posterior to M (at the same time, from the same respect, etc.)[2].

The contradiction, however, is not always located in the containment relation's contradictory properties. Instead, it is sometimes identified in the fact that what was assumed a totality of all relations could not be one. The terms of the paradox assume a totality of all relations, yet we learn that the containment relations could not be part of the totality. Therefore, what was assumed to be a totality of all relations is no such totality at all[3].

1.2. *Absurdities Concerning Relations*

Instead of locating a contradiction in the fact that the totality M will be both prior and posterior to the part-whole relation between itself and any one of its constituents, the problem found in this formulation is that it is impossible for a relation to take itself as a relatum or for a relation to be prior to its relatum[4]. Since the part-whole relation is a constituent of M, it will have to relate itself to M, which means that the relation will take itself as a relatum. On another analysis, the containment relation will be prior to its relata: since the totality includes all relations, its containment relation too must be included, and all parts are prior to the whole to which they belong. Therefore, the containment relation must be prior to its relata, which contradicts the self-evident (*badīhī*) fact that relations are posterior to their relata.

1.3. *Absurdity that a Whole Cannot Contain its Contradictory as Part*

What is considered by philosophers in the tradition to be yet another way of formulating the same problem phrases things much more broadly[5]. Now the totality is made up of concepts as opposed to merely relations: Consider the totality of all concepts (*mafhūmāt*) such that none are excluded. The contradictory of this totality is also a concept and must, therefore, also be included in it. But it follows from this that the totality would have to contain its contradictory as a part, which is absurd[6].

2. Muḥammad Bāqir b. Muḥammad Mīr Dāmād, *Muṣannafāt-i Mīr Dāmād* ed. Nūrānī, pp. 686-687, ll. 21-1; Muḥammad Mahdī b. Abī Dharr Narāqī, *Mushkilāt al-ʿulūm*, p. 58, l. 1-4.

3. Amīr Faḍl Allāh Astarābādī, *Ḥall al-mughālaṭāt*, p. 121, ll. 5-10.

4. Walī Allāh Laknawī, *Ḥāshiyat al-Ṣadrā*, ed. Laknawī, p. 102, l. 7; Muḥibb Allāh Bihārī / ʿAbd al-ʿAlī Baḥr al-ʿUlūm, *Sharḥ Baḥr al-ʿUlūm ʿalā Sullam al-ʿulūm*, ed. al-Malībārī, p. 581, ll. 12-14; Mīr Zāhid Harawī, *al-Ḥāshiyah li-Mīr Zāhid ʿalā Sharḥ al-Mawāqif*, ed. Laknawī, p. 75, l. 20.

5. Laknawī, *Ḥāshiyat al-Ṣadrā*, ed. Laknawī, p. 102, ll. 4-7.

6. Bihārī / Baḥr al-ʿUlūm, *Sharḥ Baḥr al-ʿUlūm*, ed. al-Malībārī, p. 581, ll. 1-3 and ll. 8-10.

Depending on the formulation, the problem presents us with one of three difficulties: (1) a contradiction (something is both prior and posterior to something else, or else what was assumed a complete totality could not be so); (2) a relation is identical with or prior to its relatum (whereas we know that a relation must be posterior to its relatum); (3) a whole contains its own contradictory as a part (whereas we know this to be absurd).

2. Attempted Solutions and Objections Thereto

There are at least two kinds of solution pursued in the sources. The first strategy is to reason that there simply could be no such totality, and so there is no problem. The second is instead to explain why no problem would arise even if there were such a totality. I have not organized the solutions according to formulation, since it seems to me that few are in principle restricted to one formulation or another, even if every solution was proposed for only one formulation.

2.1. *There Could Be No Such Totality*

(a) The very concept of such a totality is contradictory

This solution contends that there is no such totality as M since M is contradictory in its very concept. We will recall that M, according to one of the formulations, was specified as the totality of all concepts[7]. The concept 'concept', however, has an undelimited number of instances, and therefore another instance could always be added to its extension. Yet the specification of M runs contrary to this fact by demanding that *all* concepts be included, implying that a limit could be imposed on the number of concepts.

In consequence, to consider "the totality of all concepts such that none are excluded" is to consider something impossible, since one part of the specification (namely, the nature of 'concepts') dictates that the totality be infinite, while the other part ("such that none are excluded") dictates that it be finite. Since this totality is impossible, it could have no instance in the external world. Thus, it is only an impossible concept invented by the mind, and so it is unproblematic if it entails other impossibilities, like something's having its contradictory as a part or a relation's being identical to its relatum[8].

7. Although this solution is phrased in terms of the 'all concepts' formulation, as opposed to the 'all relations' one, it should be clear that it can nonetheless be made applicable to the other merely by exchanging 'concepts' for 'relations'.

8. Ḥamd Allāh Sandīlī, *Sharḥ Sullam Mawlawī Ḥamd Allāh*, p. 148, ll. 5-10; Mīrzā Jān al-Bāghnawī, *Sharḥ Ḥikmat al-ʿayn maʿa ḥawāshī*, p. 293, ll. 6-22.

Objection: Counter-example: The totality of all whole numbers

Consider the totality of all whole numbers such that none are excluded. Although the whole numbers stop at no limit, we can nonetheless make true judgements about the totality of all of them. Therefore, the fact that there are limitless concepts prevents one neither from considering the totality of all concepts nor from passing judgments on it, just as the fact that there are limitless whole numbers prevents one neither from considering the totality of all whole numbers nor from passing judgements on it[9].

(b) There is no such totality

A much simpler version of (a) consists of the claim that there is no such totality either in the external world or in the mind. That there is no such thing in the external world is simply assumed – I have seen no attempt to substantiate it. There is no such totality in the mind because it is only the concept of such a totality, not the totality itself, that obtains in the mind. Indeed, the human mind could not conceive of all the infinite individual parts composing the totality (*bi-l-tafṣīl*) in order for it to exist in the mind[10].

Objection: Counter-example[11]

Both solutions (a) and (b) are susceptible to a theological objection. Consider the totality of all things known to God. Certainly, this is an extant totality encompassing the totality of all concepts and the totality of all relations. Furthermore, since the negation of this totality is also known to God (on pain of divine ignorance), it must contain its contradictory as a part[12].

(c) No such totality could be a relatum

To suppose that such a totality could be a relatum and one of its parts another relatum would be to suppose a contradiction. According to assumption, there is no concept (or relation) left out of the totality; therefore, there could be no relation left to relate the totality to any of its parts, since every relation must be external to its relata, but, by assumption, all are internal to the totality. The paradox amounts to asking, "Is the relation between the totality of all concepts (relations)

9. Ibn Muḥammad Dāʾim Qāḍī Mubārak, *Kitāb Sullam al-ʿulūm wa-ḥāshiyatuhu l-mashhūra bi-l-qāḍī maʿa minhiyyātihi*, p. 258, ll. 7-14.

10. Harawī, *Ḥāshiyat Sharḥ al-Mawāqif*, ed. Muḥammad, p. 75, ll. 20-22.

11. Mīrzā Jān al-Bāghnawī considers an analogous objection regarding the totality of things subject to God's power and makes some attempt to resolve it. See *Sharḥ Ḥikmat al-ʿayn maʿa ḥawāshī*, p. 292, ll. 16-17.

12. Bihārī / Baḥr al-ʿUlūm, *Sharḥ Baḥr al-ʿUlūm*, ed. al-Malībārī, p. 582, ll. 14-19. This objection can easily be modified to apply to the other two formulations of the problem. For a resolution insofar as it applies to one of the formulations, see solution 2b.

and any of its parts – which relation itself is both internal and external to the totality – internal to the totality or external to it[13]?"

2.2. *No Problem Arises from Assuming Such a Totality*

(a) The contradictory properties are both true in different respects

This strategy explains that any contradiction or absurdity is only apparent, since, in fact, the condition of 'unity of subject' for a contradiction to obtain has not been met. Each contradictory property is indeed true of the containment relation but in a different respect. The respect in which each contradictory property is true differs according to formulation.

In the case of formulations where the difficulty identified is that a whole contains its contradictory as part, the solution is phrased as follows: it is only insofar as "the negation of the totality of all concepts" is a concept that it figures in the totality as a part, and only insofar as it is a negation of "the totality of all concepts" that it is the totality's contradictory[14]. Likewise, for the contradiction formulation, it is only insofar as the part-whole relation is a means for considering how a part of M belongs to M that it is posterior to M, and only insofar as it is considered to be a relation that it is part of the totality and so prior to it[15]. Another way to draw the distinction and so perhaps another version of the solution is that it is only insofar as the part-whole relation is specified by its relata that it is posterior to the totality, and only insofar as it is a mere relation that it is part of the totality and so prior to it[16].

Objection: Relations are individuated by their relata

The mere fact that we consider a relation from the respect of being a mere relation and then from the respect of being a specific relation insofar as it has particular relata cannot be a sufficient reason for the fact that it belongs to a totality and so is prior to it or does not belong and so is posterior. Indeed, a relation is individuated by means of its relata, and, therefore, once a relation is specified by particular relata and thus contained in the totality, it could not fall out of it merely by being considered from another respect[17].

13. Narāqī, *Mushkilāt al-ʿulūm*, p. 57, ll. 5-11.
14. Mullā Muḥammad Mubīn, *Kitāb Mirʾāt al-shurūḥ*, p. 124, ll. 12-14.
15. Mullā Muḥammad Mubīn, *Kitāb Mirʾāt al-shurūḥ*, p. 124, ll. 14-15.
16. ʿAlī b. Muḥammad Jurjānī, *Sharḥ Ḥikmat al-ʿayn maʿa ḥawāshī*, p. 292, note 1.
17. Mīrzā Jān, *Sharḥ Ḥikmat al-ʿayn maʿa ḥawāshī*, p. 292, ll. 7-13.

Objection: The totality of relations specified by their relata

The solution cannot accommodate a simple rephrasing of the problem: instead of making it the totality of relations *simpliciter*, make it the totality of relations specified by their relata. In that case, no ambiguity is left open for deploying a distinction of respects between a relation as such and a relation as specified by its particular relata, and the problem remains unsolved[18].

(b) A whole containing its own contradictory as part is not impossible

The last solution is perhaps the only one of its family that cannot obviously be extended to other formulations. It therefore only serves as a solution for the formulation where the paradox is grounded in the intuition that a whole cannot contain its own contradictory as part. The general strategy can be described as one of carefully explaining the relevant terms in order to show that the ostensible problem is rather more prosaic than paradoxical.

The analysis takes the form of a distinction. Only conceptual wholes cannot have their contradictories as a part; external wholes, on the other hand, are constrained by no such limitation. Take, for example, paint, an external whole. It could very well contain its contradictory, not-paint, among its constituent parts. Likewise, granting that the problematic totalities under consideration could exist, the fact that they would contain their contradictories as parts would be no reason to balk, since it is only conceptual wholes that cannot contain their contradictories as a part. It would therefore be perfectly fine for the totality of God's knowledge – recalling the aforementioned counter example – to contain its very contradictory as a part[19].

3. Summary & Analysis

Most authors seem to have understood this paradox as one about relations, which, while correct, is also potentially misleading. All members of a totality are *related* in a certain way such that they can be called a totality at all, and totalities are, therefore, entities constituted by relations. In this sense, the paradox is in fact also one about totalities. Consequently, the paradox is most threatening, and therefore most enlightening, when formulated as the totality of all relations (as opposed to concepts). Formulations given solely in terms of relations immediately block solutions built on drawing a distinction between the troublesome containment relation conceived of as a relation on the one hand and as a concept on the other (see, e.g., some variants of 2a).

18. Laknawī, *Ḥāshiyat al-Ṣadrā*, p. 103, ll. 20-23.
19. Bihārī / Baḥr al-ʿulūm, *Sharḥ Baḥr al-ʿUlūm*, pp. 582-583.

While most of the solutions are plausible, they nonetheless face difficult objections. Solution 1c is an exception. I could locate no objection to it in the literature. According to 1c, it is no problem if absurdities follow from the paradox scenario, since the scenario itself is absurd. Were there such a totality as M, it would be absurd to suppose that M could be related to one of its parts. Since M is, by definition, the totality of all relations, the relation between M and one of its parts could not be external to it, yet it would have to be for M to be its relatum, since every relation is distinct from its relata.

Solution 1c enjoys a great advantage over its relative solution 1a. The latter locates M's absurdity in the fact that it purports to be a totality and therefore to be finite, yet relations and concepts have, by their very nature, an undelimited number of instances. Unfortunately, such a solution throws out the baby with the bath water: by ridding ourselves of the paradox in this way, we also get rid of any totality formed of a description with an undelimited number of instances. Hence, we would be eliminating the totality of all numbers, the totality of all possible things, and the totality of all propositions, among others[20]. While solution 1c follows the same pattern as 1a in that both hold that the very formulation of the paradox conceals absurdity, it differs from it in locating the absurdity in M's role as relatum and what that entails, as opposed to locating it in the contradiction of an infinite totality.

Although it is not raised in the literature, I think there is at least one plausible objection to solution 1c. If what it is for something to be a totality is for its constituents to be related to it by a containment relation, then solution 1c amounts to little more than the mere denial that a totality could be formed of all relations. 1c explains that it is absurd for there to be a relation lying outside of M that relates M to its parts, since we assumed at the outset that M contains all relations. Yet if M could not be related to its parts by a containment relation, then it could not be a totality in the first place. Hence, the solution does little more than assert that there could be no totality of all relations.

According to this criticism, 1c does not account for our intuition that we can form a totality from any description, including relations. We can unproblematically conceive of the totality of all red things, of all chairs, of all stones, etc. – we should *prima facie* be able to do the same for relations and concepts[21]. In addition

20. If the wide-ranging significance of this fact is not plain enough as it stands, recall that one of Ibn Sīnā's proofs for the existence of a necessary being crucially relies on the notion of a "totality of all possible things". If no such totality could be validly formed, the proof would rot at its roots.

21. Although it is not so given in the texts, the paradox can also be formulated using 'things,' 'existents,' 'possibilia,' or 'individuals': Consider the totality of all individuals. There will be a relation between each individual contained and the totality. Each containment relation that stands between them is also an individual and therefore belongs in the totality. All the parts of the totality must be prior to it, by virtue of being parts, but some of the parts, namely the relations, must be posterior to it, since it is one of their relata. This is a contradiction.

to explaining the absurdity that follows on supposing a totality of all relations that is related to each of its parts by yet another relation, it should also explain what it is about relations and concepts themselves that makes them different from other objects, such that they suffer from this problem while others do not.

Indeed, not all objects can be used to formulate the paradox. 'Red things', for example, does not work. Take the totality of all red things. There will be a containment relation between each red thing and the totality itself. This containment relation is *not* a red thing, and hence no absurdity arises. The necessary condition for generating the paradox, therefore, is that the description of the object used to form the totality be true of a relation. Descriptions that can be subsumed under none of the categories – transcendentals – can all be used to generate the paradox (existent, thing, one, possible).

Apparently unproblematic totalities differ from the totality of all relations (or the totality of any predicate true of a relation) in yet another way: the description used to form such totalities can be taken absolutely, without restriction. By contrast, the totality of all relations can be formed unproblematically only when its objects are limited to those which exist or existed before the time the totality was formed. That is, the totality of all those relations existing before the totality itself causes no problem at all. Other totalities (like the totality of all red things), by contrast, can be taken absolutely – we need not qualify the description, 'red things' with 'existing before the totality'.

It is interesting to think about how the paradox bears on our knowledge of the extension of certain terms, even if this issue is not raised by philosophers of the tradition. A common way of thinking of the extension of a term x is as the totality of all objects of which x is true, even if only in potentiality. Yet if the extension of a term is to be thought of as the totality of those objects of which it is true, it follows as a consequence of the paradox that certain terms are somehow flawed: their extensions cannot properly be defined. Hence, a further way that the paradox is paradoxical is that it gives us reason to think that there is something wrong with the term 'relation' and any term true of it (and, by extension, something wrong with totalities).

The next most promising solution is 2a, which argues that the contradictory conclusions that the paradox entails are not genuinely contradictory but only apparently so. They result from a failure to observe the conditions of contradiction. The one contradictory property ('being prior to the totality') belongs to the containment relation considered according to one respect and the other ('being posterior to the totality') belongs to it according to another.

There is some surface-level implausibility to the solution, unfortunately. It is unclear how the fact that it can be considered from one respect as well as from another negates the fact that the very same relation bears contradictory properties. Even if we grant that the containment relation is part of and prior to M on-

ly insofar as it is a mere relation and posterior only insofar as it is individuated by its relata, the fact remains that there is only one relation. How could the same relation be both in and out of the same totality, even if it is in it on account of one description and out of it on account of another? The objection that the very identity of a relation is a function of its relata gets at the same point.

The final solution described, 2b, is also rather plausible. Of course, its plausibility comes at the price of limitation: the solution applies only to those formulations of the paradox in which the difficulty to overcome is that a whole contains its contradictory as part. It consists of a partial concession to the paradox: some wholes *can* contain their contradictories as a part. Only concepts cannot contain their contradictories as a constitutive conceptual part. External wholes, however, can very well contain a contradictory part.

It is worth noting what is, perhaps, only a superficial connection to the Liar paradox. It could be argued that Liar sentences generate their own truth-value[22]. If a Liar sentence is assumed false (or true), that very assumption generates another truth-value, true (or false). Likewise, the totality of all relations generates further containment relations after it has been formed (i.e., the assumption that all containment relations have been gathered itself generates more relations).

4. Historical Development

Kātibī (d. 1276) is the first I have seen pose the problem. He raises it in the section of *Ḥikmat al-ʿayn* on whether necessity is positive or privative. Kātibī, who held that it is positive, is led to grant that, were it positive, it would be a relation, a relation between something's quiddity and its existence. If that were so, however, he would apparently be forced to grant further that the necessity of a necessary being would exist before the necessary being itself does in order for the necessity relation to relate the necessary being's quiddity to its existence, since were the necessary being instead to exist before its necessity, it could hardly be a necessary being[23].

Further, if necessity (a relation) must exist before the necessary being itself (the relatum), it would imply that a relation need not be posterior to its relata. At this juncture, he raises the totality of all relations problem as a counter-example in order to vitiate the objection that a relation need be posterior to its relatum: no impossibility need follow, since the totality of all relations scenario shows that a relation need not be posterior to its relata. Therefore, it is possible for a necessary being (the relatum) to be identical with necessity itself (the relation), and neces-

22. See Rezakhany 2018, pp. 183-220; El-Rouayheb 2019.
23. Najm al-Dīn Kātibī, *Ḥāshiyat Sharḥ Ḥikmat al-ʿayn*, p. 74, ll. 8-9.

sity need not exist posterior to the necessary being in order to relate its quiddity to existence[24].

The earliest commentators of *Ḥikmat al-ʿayn* have little to say about the problem. Ḥillī (d. 1325), who was the work's first *shāriḥ* (commentator)[25], dedicates a succinct 2a-style solution to it, though only insofar as it is a counter-example adduced by Kātibī rather than as an independent problem[26]. Ibn Mubārakshāh (al-Mīrak al-Bukhārī, fl. 1354) is even less interested, not really even proffering a solution[27]. This is perhaps unsurprising, since the problem itself is by no means integral to the discussion in which it happens to figure. It is, after all, merely one way of resolving an objection to a position on the real issue, namely whether necessity is positive or privative. At last, al-Sharīf al-Jurjānī (d. 1413), here Ibn Mubārakshāh's *muḥashshī* (glossator), dedicates a note to the problem, giving a more extensive 2a-type solution[28]. Mīrzā Jān al-Bāghnawī (d. 1587) continues Jurjānī's attitude of treating the difficulty as an independent problem in his supergloss. He seems to have been the first to propose solution 1a.

The paradox appears outside of the *Ḥikmat al-ʿayn* commentary tradition in the work of Mīr Dāmād (d. 1631)[29]. In his *al-Ufuq al-mubīn*, he treats the problem as one in logic[30]. Instead of discussing it in a section on modality, he attempts to solve it as a case parallel to the Liar paradox, in order, apparently, to demonstrate how robust his solution to the Liar is. This is particularly interesting because it seems to indicate that he sees the two problems as closely related. That Russell and some other later philosophers have thought that there is a relationship between Russell's Paradox and the Liar would give further credence to the argument below that the totality of all relations paradox is a mereological relative of Russell's[31].

In Mughal India, the totality of all relations paradox is treated in Mīr Zāhid al-Harawī's (d. 1689) gloss on Jurjānī's *Sharḥ al-Mawāqif fī ʿilm al-kalām*. Mīr Zāhid is the first author I have found to raise the issue in that commentary tradition. It is again in the context of modality that the puzzle is raised and again at the sug-

24. Najm al-Dīn Kātibī, *Sharḥ Ḥikmat al-ʿayn maʿa ḥawāshī*, pp. 292-293.

25. See Jaʿfar Zāhedī's preface to Najm al-Dīn Kātibī / Shams al-Dīn Ibn Mubārakshāh, *Sharḥ Ḥikmat al-ʿayn*, p. 13: "*Īḍāḥ al-maqāṣid* was the first commentary on *Ḥikmat al-ʿayn*".

26. al-Ḥasan b. Yūsuf b. al-Muṭahhar Ḥillī / ʿAlī b. ʿUmar Kātibī, *Īḍāḥ al-maqāṣid min Ḥikmat ʿayn al-qawāʿid*, ed. ʿAlī Naqī Munzavī, p. 78.

27. Kātibī / Ibn Mubārakshāh, *Sharḥ Ḥikmat al-ʿayn*, p. 84.

28. For a full translation, see the appendix.

29. Mīr Dāmād, *Muṣannafāt-i Mīr Dāmād*, ed. ʿAbd Allāh Nūrānī, vol. 2, p. 687.

30. A contemporary of Mīr Dāmād's includes the problem directly after the Liar paradox in a treatise on logical sophisms: Amīr Faḍl Allāh Astarābādī, *Ḥall al-mughālaṭāt*, p. 121. The presence of the problem in this treatise hints that it may have had a more vibrant life in the Safavid world than I have been able here to document.

31. Consensus, however, seems to be on the side of the view that Russell's Paradox and the Liar are not substantively related and are instead genuinely distinct problems. That there is a consensus, of course, does not affect the point at hand, namely that the relation between the Liar and Russell's Paradox has been a matter of philosophical discussion.

gestion that necessity is a relation between existence and a quiddity. While Mīr Zāhid's solution is rather unimpressive (of the 1b variety), his formulation of the problem is interesting and perhaps even novel. Unlike earlier formulations I have seen, where the totality is made up of relations, Mīr Zāhid's is made up of concepts. Besides this, however, the formulation is identical, again invoking the containment relation – which, of course, is also a concept – to undermine our intuition that a relation is inevitably posterior to its relata and never identical with either of them.

Hence, we have here a kind of hybrid formulation: Mīr Zāhid phrases the problem in terms of concepts, but the difficulty he identifies is not the fact that a totality contains its contradictory as part – as is normally the case for the formulation using concepts – it is instead one of the difficulties usually paired with the relations phrasing. Calling this a 'hybrid' formulation may be a bit too charitable, however. It seems more accurate to say that it is a mixed-up or confused formulation, since substituting 'concepts' for 'relations' adds nothing to the problem. If anything, it takes away from it by making solutions of kind 2a more plausible, thereby deceptively simplifying the difficulty it would otherwise present were it phrased solely in terms of relations[32]. At least one later Indian logician and one Iranian reproduce this 'mix-up'[33].

While Mīr Zāhid may have been the first to introduce this problem into the *Mawāqif* commentary tradition, he was perhaps also the last. Neither subsequent Indian glossators of Jurjānī's *Sharḥ* nor Mīr Zāhid's own glossators engage the problem. While the former – e.g. ʿAbd al-Ḥakīm Siyālkūtī (d. 1656/7) – comment without raising the problem, the latter – e.g. ʿAbd al-ʿAlī Baḥr al-ʿUlūm or Faḍl-i Ḥaqq al-Khayrābādī – end their commentaries well before the opportunity for discussion presents itself. It seems that later scholars were interested only in the first section (*marṣad*) of Mīr Zāhid's gloss, which is on the topic of existence and non-existence, and includes extensive discussions of mental existence (*wujūd dhihnī*).

By contrast, the commentary tradition on the *Sullam al-ʿulūm* represents the venue where this puzzle was perhaps most extensively discussed. Unfortunately, I have found no commentator who gives the relations-based formulation that threatens contradiction. Instead, it is only formulations which contravene our intuitions about relations and their relata, and the fact that a whole cannot contain its contradictory as part. Despite this philosophical demerit, the presentation in this tradition is, historically speaking, particularly interesting. The author of the base-text,

32. Compare the 2a solution by Jurjānī with the one by Mullā Mubīn, both translated in the appendix.

33. For example, Mullā Muḥammad Mubīn, *Kitāb Mirʾāt al-shurūḥ*, p. 123, l. 22; Narāqī, *Mushkilāt al-ʿulūm*, p. 58, ll. 1-4. Ḥamd Allāh, Baḥr al-ʿUlūm, Faḍl-i Ḥaqq al-Khayrābādī, and Walī Allāh al-Laknawī all give the standard relations formulation, as do the Safavid authors here studied.

Muḥibb Allāh al-Bihārī, seems to be the first to have formulated the puzzle such that the difficulty to be resolved is that a whole contains its contradictory as part[34].

Still of greater historical interest is the problem's radical shift in location. We will recall that, in its earliest days of discussion, the paradox was found in sections on general metaphysics (*umūr ʿāmma*), embedded in the question on whether necessity was a positive or privative property. Now, in the *Sullam al-ʿulūm* commentary tradition, it is located in a logic textbook in the section on contradiction, and identified as a 'doubt' (*shakk*) or 'difficulty' (*ishkāl*) pertaining thereto. What began, therefore, as a problem in metaphysics is, in the later Indian context, chiefly treated as one in logic. The same was true, as we noticed before, in the case of earlier Safavid philosophers Mīr Dāmād – who discusses the problem as an afterthought to the Liar in the predication section of *al-Ufuq al-mubīn* – and Amīr Faḍl Allāh Astarābādī – who includes it in his treatise on logical sophisms.

Commentators of the *Sullam* held almost every solution devised prior to them and appear even to have devised one of their own. Many authors give at least two, one of which is inevitably the 'contradictory concept' (1a) solution advocated by Bihārī in the *matn* (base-text). Although there were several original contributions made by the commentators, they were obviously acquainted with prior literature. In his *Suddat al-ʿulūm*, al-Anṣārī al-Sihālī reproduces the relevant section from Mīr Zāhid's gloss on Jurjānī's *Sharḥ al-Mawāqif*[35], and Qāḍī Mubārak's commentary leans heavily upon Mīr Dāmād's wording in *al-Ufuq al-mubīn*[36]. Baḥr al-ʿUlūm, inspired by his teacher Qāḍī Mubārak, seems to be the first to formulate solution 2b ("A whole containing its own contradictory as part is not impossible")[37], and Qāḍī Mubārak raises a crushing and, as far as I know, original objection against 1a-style solutions.

The problem does not remain only in books on logic but makes its way into Mughal commentaries on physics (*ṭabīʿiyyāt*), as it did in the earlier Iranian world[38]. Walī Allāh al-Laknawī, one of the many glossators of Mullā Ṣadrā's (d. 1640) *Sharḥ Hidāyat al-ḥikma*, raises the paradox in the discussion on atomism as one difficulty among several facing those who deny atomism and advocate infinite divisibility. Laknawī's choice of phrase clarifies the link between the totality

34. So Baḥr al-ʿUlūm tells us in a marginal note, quoting Bihārī's self-commentary. See *Sharḥ Baḥr al-ʿUlūm*, p. 581, note 1.

35. Aḥmad ʿAbd al-Ḥaqq Al-Anṣārī al-Sihālī, *Suddat al-ʿulūm*, p. 347, ll. 13-17. Compare with Harawī, *Ḥāshiyat Sharḥ al-Mawāqif*, p. 75, ll. 19-21.

36. Qāḍī Mubārak, *Ḥāshiyat Sullam al-ʿulūm*, p. 257, ll. 13 and 17-21. Compare with Mīr Dāmād, *Muṣannafāt-i Mīr Dāmād*, vol. 2, p. 686, l. 21 - p. 687, l. 6.

37. Bihārī / Baḥr al-ʿUlūm, *Sharḥ Baḥr al-ʿUlūm*, pp. 582-583. Compare with Qāḍī Mubārak, *Ḥāshiyat Sullam al-ʿulūm*, p. 257, ll. 5-13.

38. Mīrzā Jān promises a discussion of the totality of all relations paradox insofar as it bears on the issue of atomism in a later section of *Ḥikmat al-ʿayn*: *Sharḥ Ḥikmat al-ʿayn maʿa ḥawāshī*, p. 292, ll. 14-15.

of all relations paradox and atomism: "The upshot of the solution is that the totality of a body's parts such that none are excluded is finite…[39]". This is, of course, the very same formula used to pose the totality of all relations paradox ("The totality of all relations such that none are excluded"). Aside from mere verbal similarities, the two problems have much in common conceptually: both deal with totalities (one of the parts of a single body, the other of concepts or relations and their relata) and infinity. Most significantly, however, Laknawī believes that all the difficulties he raises (including the totality of all relations paradox) are connected by the same solution[40].

The paradox continued to be discussed in the post-Safavid world, though how extensively, it is too soon to say, due to the fact that many post-Safavid logical and philosophical works remain in manuscript and are relatively inaccessible. What can be said for sure, however, is that it does find a place in a potpourri style work by Muḥammad Mahdī b. Abī Dharr al-Narāqī (al-Muḥaqqiq al-Narāqī, d. 1795), which covers difficult questions in exegesis, *ḥadīth*, philosophy, logic, geometry, medicine, Arabic, etc[41].

Narāqī's contribution is noteworthy for a few reasons. The first is that he proposes a solution (1c) I have encountered in no earlier tract. The second is that the paradox is an (extended) afterthought to another, the 'donkey paradox' (*shubha ḥimāriyya*). It goes as follows: Does something that – whether it exists or not – entails that Zayd is a donkey exist or not? In either case, an impossibility is entailed (Zayd is a donkey)[42]. Narāqī was not the first to relate these two paradoxes, however. He was preceded on that point by Mīrzā Jān al-Bāghnawī[43]. Narāqī and Bāghnawī think that this paradox shares the same solution as the totality of all relations paradox, much as Mīr Dāmād thought the Liar did, and Walī Allāh al-Laknawī thought difficulties with anti-atomist positions in physics did[44].

The fact that at least three philosophers link this paradox to another, along with the not entirely unrelated fact that the paradox appears in such a variety of disciplines (metaphysics, physics, logic), suggests that it was particularly slippery. Scholars could not quite figure out how to categorize it. Unfortunately, no scholar ever explicitly explains what it is that unites two such paradoxes. In general, the

39. Laknawī, *Ḥāshiyat al-Ṣadrā*, p. 102, l. 9.

40. Laknawī, p. 102, ll.7-8: "Such are multiple formulations of the difficulty (*shubha*). The solution to all, however, is but one…".

41. Narāqī, *Mushkilāt al-ʿulūm*, p. 1.

42. Narāqī, p. 56, ll. 16-18.

43. Mīrzā Jān, *Sharḥ Ḥikmat al-ʿayn maʿa ḥawāshī*, p. 293, ll. 16-18.

44. A certain Mullā M. N. Shīrāzī connects the problem to both the donkey paradox and anti-atomist difficulties (among others pertaining to infinite totalities) in a one-page entry on the paradox. It is not clear to me if it belongs to some other work and is merely excerpted or if this is simply a free standing note by the author.

only explanation given for the fact that one paradox is linked to another is that they share a solution.

From the thirteenth to the early nineteenth century, the Arabic philosophical tradition developed three significant formulations of and five solutions to the totality of all relations paradox. This activity appears to have been concentrated in later centuries. Until the fourteenth, the paradox remained in gestation: it was barely formulated as an independent problem and had only received a single cursory solution at the hands of Ḥillī (d. 1325). It is with Jurjānī (d. 1413) that the problem is first treated as an important difficulty in its own right, and he spends a few paragraphs elaborating solution 2a. Around a hundred and fifty years later, Mīrzā Jān al-Bāghnawī (d. 1587) criticizes Jurjānī's solution and develops his own (1a). From the seventeenth to the end of the eighteenth centuries, three more solutions, in addition to two more formulations and a couple of objections to prior solutions, are developed mostly by Mughal philosophers (1b, 2b) but also by Qājār philosophers (1c). These last two centuries appear to be, from my limited survey of the literature, the most dynamic.

The extensive treatment the paradox received is all the more surprising in light of the fact that it was likely considered less important than others. One indicator of its minor status is that there appear to be no independent treatises dedicated to it, while this was exceedingly common for the Liar paradox (*shubhat al-jidhr al-aṣamm*) and the paradox of the absolute unknown (*shubhat al-majhūl al-muṭlaq/al-jidhr al-abkam*), among others. Instead, the totality of all relations paradox was always mixed into larger works. Another indicator is that it was frequently treated as an afterthought to some related difficulty. Nonetheless, despite its secondary status, the problem warranted a response from the tradition comparable to those of other more popular paradoxes. Output was robust in terms of solutions, respectable in terms of formulations, and lacked only in objections. For whatever reason, there appears to have been less criticism of alternative solutions than for other paradoxes of self-reference[45].

Given the fact that this paradox was not even posed until the beginning of the post-classical period and that it was not taken seriously as a problem in its own right until the fourteenth century, it could hardly serve as evidence of post-classical decline. Indeed, it is another instance of continued innovation. The later tradition proposed and developed an entirely new problem and then devised and debated potential solutions. What is more, this paradox, like the paradox of the absolute unknown, was apparently indigenous to the Islamic world: there is, as of yet, no evidence of which I am aware to suggest that it was inherited from Greek or any other foreign civilization.

45. Compare with the number of objections in Rezakhany 2023. See also Lameer 2014.

Further, the Islamic world arguably beat Russell to his eponymous paradox – by at least five centuries. While I would not wish to argue that the two paradoxes are structurally identical, there is a strong case to be made that they are functionally identical. By this I mean that both paradoxes upset the same intuition: both show that not every object (or property) can be used to form a sound totality (or set, in Russell's case). Russell's Paradox does this by using the property "being a set that does not contain itself" and the totality of all relations paradox does it by using the object 'relation'. Each involves some auxiliary assumptions that assist in creating the paradox. In the case of Russell's Paradox, these include the assumption that self-containing sets are even coherent; in the case of the totality of all relations paradox, they include the idea that the parts of a whole must be prior to it and that a relation must be posterior to its relata. This being said, I have yet to find an author who explicitly points out that, as a consequence of the paradox, it would seem that the intuition that a totality can be formed from any kind of object is threatened.

In light of the research presented in this chapter, the hypothesis of philosophical stagnation in the post-classical Islamic world appears even more untenable. Perhaps a reasonable objection by a defender of that hypothesis would be that the chronology here given is subject to further research. It may be that some of the solutions, objections, or resolutions I report as being discovered very late (for example, 1c or 2b) were in fact discovered a century or two earlier, in the 16th as opposed to the eighteenth century. While this is possible, it does little to harm the broader conclusion that, between the 12th and 16th centuries, original philosophical research was extensively – even if slowly – conducted in the Islamic world. Furthermore, my review of the literature on this paradox was hardly exhaustive: besides suffering from various gaps in the eastern Islamic literature, it does not account at all for the western Islamic world.

5. Appendix of Translations

Formulation: Contradiction, al-Muḥaqqiq al-Narāqī, *Mushkilāt al-ʿulūm*

Let us take the totality of all concepts such that not a single concept is excluded. Then we relate this totality – from which no concept is absent – to its part. ⟨Upon so doing,⟩ there is no doubt that a relation will obtain between the totality and its part. This relation will be both internal to the totality, since it was assumed[46] that no concept whatsoever was excluded from it, as well as external to it, since relations are external to their relata[47].

46. Reading *li-farḍihi* for *li-ʿaraḍihi*.
47. Narāqī, *Mushkilāt al-ʿulūm*, p. 58, ll. 1-4.

Formulation: Contradiction (of assumption), Astarābādī, *Ḥall al-mughālaṭāt*

Take the totality of all relations such that none are excluded, all are included in it, each is a part of it, and the whole bears a relation to each of its parts. It is necessary that this relation is not ⟨contained in⟩ the whole, since a relation is external to its relata, from which it follows that the totality is not, as assumed, a totality[48].

Formulation: Identity of relation & relata, Baḥr al-ʿUlūm, *Sharḥ Sullam al-ʿulūm*

The formulation: Let us take all relations such that no relation is excluded. This whole has a relation to its parts, among which is the very same relation, and, therefore, the whole bears this relation to the relation itself, from which it follows that a relation can be identical to its relatum, despite what is said about how they must be distinct[49].

Formulation: Relation prior to relatum, Mīr Zāhid, *Ḥāshiyat Sharḥ al-Mawāqif*

If one considered the totality of all concepts such that no concept was excluded – whether ⟨the concept⟩ be considered under the description ⟨'concept'⟩ or not, and whether it has obtained by the time of the consideration ⟨of the totality⟩ or not – the totality would doubtless bear a relation to each of its parts. Now, this relation could not be external to the totality – else, it wouldn't be "such that no concept was excluded" – and therefore it follows that a relation can be prior to one of its relata[50].

Formulation: Contradictory part, Bihārī and Baḥr al-ʿUlūm, *Sharḥ Sullam al-ʿulūm*

If we take the totality of concepts such that none are excluded, which thereby yields a concept*, the negation of this totality is its contradictory*, *and that* negation is also a concept *and is included in the totality. Therefore, the contradictory of the totality figures as its part, which is absurd*[51].

(1a) Ḥamd Allāh, *Sharḥ Sullam al-ʿulūm*

The response, in sum: Since concepts are not infinite in actuality but are rather infinite in the way that numbers, the parts of a body, and the things under God's power are – in the sense that they do not stop increasing at any limit – to consider "the totality of concepts such that none are left out" is to consider

48. Amīr Faḍl Allāh Astarābādī, *Ḥall al-mughālaṭāt*, p. 121, ll. 5-10.
49. Bihārī and Baḥr al-ʿUlūm, *Sharḥ Baḥr al-ʿUlūm*, p. 581, ll. 12-14.
50. Harawī, *Ḥāshiyat Sharḥ al-Mawāqif*, p. 75, ll. 17-20.
51. Bihārī / Baḥr al-ʿUlūm, *Sharḥ Baḥr al-ʿUlūm*, p. 581, ll. 8-10. Bihārī's base-text is in italics.

two contradictory descriptions. "The totality of concepts" demands that it be possible to augment the totality with more concepts, while "such that none are left out" demands that it be impossible. The description, "The totality of concepts such that none are left out" is akin to the description, "The totality that is both infinite and not infinite". Hence, this totality is not instantiated such that it could be a whole that contains its contradictory as part. Indeed, there is nothing even in the mind except the invented concept compounded of contradictory descriptions[52].

(1b) Walī Allāh al-Laknawī, *Ḥāshiyat al-Ṣadrā*

As for talk of the totality of relations, one may respond to this by saying that the relation can be considered in two ways. The first is that it be considered insofar as it takes particular relata. So considered it is posterior to its relata. The second is that it be considered insofar as it is merely a relation, ignoring its particular relata. Under this consideration it is necessarily included in the totality, due to the fact that it is relations insofar as they are relations that ⟨form the totality⟩, not relations insofar as they have specific relata.

Even though this is the solution chosen by *bāqir al-muḥaqqiqīn* ⟨Mīr Dāmād⟩ in his *Qabasāt*[53]*,* it does not completely uproot the matter of the difficulty. One could object by saying that the totality of particular relations such that none are excluded is itself certainly a totality. Each of the relations is a part of the totality, and there is no doubt, furthermore, that this totality has a relation to its parts. Hence, this particular relation considered as being a relation between these two relata is certainly included in the totality, from which follows that a relation can be identical to its relatum, and yet the solution Mīr Dāmād mentioned is no defense here. One can escape this problem, then, using what we have mentioned, namely that the description 'totality of relations' as specified answers to nothing such that it could have a relation to its parts and so entail that a relation can be identical to its relatum[54].

(1c) al-Muḥaqqiq al-Narāqī, *Mushkilāt al-ʿulūm*

The assumption that the totality of all concepts such that no possible concept is excluded is not consistent with the assumption that the same totality is related to one of its parts. Indeed, to assume that the totality is as noted and that it is related to one of its parts implies a contradiction: the fact that the totality is

52. Sandīlī, *Sharḥ al-Sullam*, p. 148, ll. 5-10.

53. This is written as *fatshāʾih*, which is obviously wrong, and has the same consonantal structure as *qabasātih*. I was not able to locate any discussion of the problem in that work. The author probably confused the *Qabasāt* for *al-Ufuq al-mubīn*, which does treat the problem.

54. Laknawī, *Ḥāshiyat al-Ṣadrā*, p. 103, ll. 18-23.

such that it excludes no possible concept implies that it could not be related to anything, while the fact that it is related to its part implies the opposite. It is as if someone were asking, "Is the relation between the totality of all concepts and any of its parts, which relation is itself both external and internal to the totality, external to it or not?"

Yes, one can indeed consider the totality of all concepts which have obtained by the time of its consideration, though it is obvious that the relation between it and any of its parts would not be among them, seeing as how ⟨that relation⟩ would not have yet obtained at the time of the assumption. If, however, one considered the totality of all concepts such that it excludes none that could possibly obtain or be supposed to obtain – even if after the totality has been considered – the assumption of this totality could not be consistent with the assumption that it is related to anything[55].

(2a) Jurjānī, *Ḥāshiya ʿalā Sharḥ Ḥikmat al-ʿayn*

There is no doubt that the mind judges self-evidently (*bi-badīhat al-ʿaql*) that the relation between any two things is, by its very nature (*bi-l-dhāt*), posterior to both of them – whether the two things be real or merely objects of the mind, or one of them be real and the other an object of the mind. This universal proposition is, however, violated by the example given by the author ⟨Kātibī⟩. We must, therefore, either reconcile the universal proposition and the counter-example or give one of them up. Hence, we say the following – and God is the one who grants success:

The totality of relations, insofar as it is a mere object of the mind, exists nowhere besides the mind, and therefore, the totality has no relation between itself and another of its constituent relations except in the mind. Moreover, it is obvious that so long as the mind does not attend to the totality, there can be no relation between it and anything else. Thus, the relation is, in the mind, posterior to its relata insofar as it takes two particular relata, and, insofar as it is merely a relation, ignoring its particular relata, it is contained in the totality. When the mind brings about the totality, it attends to its constituents insofar as they are merely relations, not insofar as they have particular relata. Indeed, it is impossible for the mind to do the latter.

If this is clear to you, then the meaning of the aforementioned universal proposition is as follows: "Every relation is posterior to its relata *insofar as it has particular relata*". Such a proposition does not contradict the fact that a relation could be, from another respect, prior to either of its relata. Know this[56].

55. Narāqī, *Mushkilāt al-ʿulūm*, p. 57, ll. 5-11.
56. Jurjānī, *Sharḥ Ḥikmat al-ʿayn maʿa ḥawāshī*, p. 292, note 1.

(2a) Mullā Mubīn, *Mirʾāt al-shurūḥ*

One could also respond by saying that the concept is a part from one respect and a contradictory from another. The negation of the totality of concepts is included in the totality of concepts *insofar as it is a concept*, and *insofar as it is the negation of the totality*, it is its contradictory. The same can be said of the relation: *insofar as it is a means for attending to the state of the whole and the part*, it is posterior to its relata, and *insofar as it is thought under the description of being a concept*, it is included in the totality. Deliberate on this. Perhaps it is a way of getting at what has been said about how the fact that concepts, considered altogether as a whole (*min ḥayth al-ijmāl*), stop at a limit does not contradict the fact that concepts, considered one by one (*min ḥayth al-tafṣīl*), stop at no limit[57].

(2b) Baḥr al-ʿUlūm, *Sharḥ Sullam al-ʿulūm*

The correct solution is as follows: "the totality of all concepts" is both a mental concept (*mafhūm taṣawwurī*) and extra-mental compound (*murakkab khārijī*). No concept of an extra-mental part can be predicated of its whole, nor can it along with the whole be predicated of some third thing. The most that follows from the scenario is that a mental concept is an extra-mental part of its contradictory, which we refuse to grant is impossible. What is impossible is for both contradictories to be true of the same subject, but that does not follow here. Indeed, the totality of all concepts and its contradictory cannot both be true of the same thing. And, yes, it is true that it is impossible for one thing to be a mental part (*juzʾ ʿaqlī*) of its contradictory, since it entails that contradictories could be true of the same thing, namely their instance, since a thing's essential constituents must all be true of it (*istiḥālat infikāk al-dhātī*)[58].

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Abstract: From the thirteenth century to the nineteenth, philosophers of the eastern Islamic world (Iraq to India) studied a mereological paradox of great philosophical interest. Here are documented many of the important formulations of the paradox, solutions, and objections to those solutions. After the philosophical contributions to the literature are summarized, there follows a section analyzing their merits and shortcomings as well as a section plotting their historical evolution and trajectory. In addition to the article's intrinsic philosophical interest, it is also a touchstone for evaluating historical hypotheses regarding the intellectual decline of the post-classical Islamic world. In the 'Historical Development' section, it is briefly argued that the paradox is functionally equivalent to Russell's. The study ends with an appendix of translations.

Keywords: Paradox; Self-Reference; Relations; Mereology; Russell's Paradox.

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Fallacies and Biblical Exegesis – The Case of Joseph ibn Kaspi

Joseph b. Abba Mari ibn Kaspi was born around 1280 either in Arles, or in Argentière[1]. He spent his life as an itinerant scholar in towns in Southern France and Spain, with the exception of a rather disappointing trip to Egypt to learn the secrets of Maimonides' *Guide of the Perplexed* from his descendants. Kaspi is mainly known for his scriptural exegesis, both of the plain sense of the text and of its philosophical 'secrets', secrets that generally accord with the doctrines of Maimonides and Averroes, his two main philosophical authorities. Outside his scriptural exegesis and philosophical commentaries, he wrote short treatises on logic, ethics, Hebrew language, and essays on Biblical and theological subjects[2].

Of particular interest here is his claim that the Bible cannot be understood correctly without a proper grounding in grammar and logic. The view that knowledge of Hebrew language and its grammar is necessary to understand the Bible is a recurrent motif among medieval Jewish exegetes, especially within the context of Jewish-Christian scriptural polemic, a polemic in which Kaspi engaged. But it is unusual for a Jewish Biblical exegete to require the knowledge of Aristotelian logic, and to apply that knowledge to scriptural interpretation[3]. In this article we shall see how Kaspi appeals to Aristotelian fallacies to explain difficult passage in scripture, both in his scriptural commentaries and in the compendium of logic that he wrote for his son Solomon, *A Bundle of Silver (Ẓeror ha-kesef)*[4].

1. 'Kaspi' literally means 'of silver', so it stands to reason that he, or at least his family, hailed originally from Argentière.

2. For a summary of Kaspi's life, works, and principle doctrines, see Kasher / Kahan 2019. Kaspi decided later in life to include the word *kesef* ('silver') in all his book's titles.

3. Kaspi's contemporary, Shemariah ha-Ikriti also claimed that knowledge of logic was necessary for understanding sacred scripture, and he, too, produced a compendium of logic for that purpose. See Manekin 1997. In the introduction to his dictionary of Hebrew roots, *Sharshot kesef*, Kaspi criticizes his grammarian predecessors for their ignorance of logic. See Kahan 2018, p. 154.

4. An edition of *A Bundle of Silver* is under preparation.

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A Bundle of Silver is one of Kaspi's later works, written in Spain in 1332. According to the introduction, Kaspi felt the need to compose a short work in the art of logic, despite the fact that short treatises by al-Fārābī and Averroes on the subject were available (in Hebrew). His book would be simpler and more comprehensive than the short treatises, and hence would be more appropriate for his contemporaries. By 'more comprehensive' he apparently meant that he would add material from Jacob Anatoli's Hebrew translation of Averroes's Middle Commentaries on the Organon, for this is what he does. Then he states that because the study of logic is necessary for understanding sacred scripture, he is only including in his compendium material to that end. After summarizing the doctrines of the first five books of the expanded *Organon*, he omits sections on the *Topics*, *Rhetoric*, and *Poetics*, but not the Fallacies. Kaspi recommends to his son that he begin his study with the *Logical Terms* of Maimonides and continue with *A Bundle of Silver*. There will be no need for him to study other works, since *A Bundle of Silver* includes all one needs to know[5].

A Bundle of Silver is the first compendium of logic written originally in Hebrew and, to judge from the twenty-eight extant manuscripts, it was apparently a popular work throughout the Middle Ages, even longer among the Karaites of the Crimea, who studied the work until the beginning of the twentieth century[6]. It is mostly an intelligent, if drastic, abridgement of two sources: Anatoli's translation of Averroes's Middle Commentaries on the Organon and the aforementioned short treatises of al-Fārābī. Kaspi also uses sparingly Averroes' Epitome of the Organon in the Jacob b. Makhir translation. At times Kaspi lifts whole passages from his sources, even neglecting to remove the 'He said' from the appropriated texts. One can only speculate why he considered the subject of *Topics*, *Rhetoric*, and *Poetics* "unnecessary for the understanding of scripture"; a century later the Italian Jewish scholar, Judah Messer Leon, would write a textbook on rhetoric claiming that the origins of the art lay in scripture, and Kaspi himself refers occasionally to the *Topics* and the *Poetics* in his writings. The answer may lie in Kaspi's view of scripture as a source of truth for the elite and of religion for the multitude; he does not appear to be interested in disputation, rhetoric, or poetry for its own sake. There may be a more prosaic reason: since manuscripts of the short treatises of al-Fārābī and the Epitome of Averroes generally position the *Fallacies* directly after the *Posterior Analytics* and before the *Topics*, *Rhetoric*, and *Poetics*, one is not skipping over anything in choosing the first six books for a compendium.

5. See Rosenberg 1983, esp. pp. 290-291 (Hebrew).

6. This is based on the testimony of the extant manuscripts, for which see Steinschneider 1893, § 42.

Shalom Rosenberg, who edited the section on the fallacies from *A Bundle of Silver*[7], showed that although Kaspi draws much of his compendium from Averroes' middle commentaries, the section is based on a Hebrew translation of al-Fārābī's *Fallacious Topics*[8], translated in Hebrew as *Sefer Safastania* or *Sefer ha-Hat'a'ah*. It appears that when Kaspi refers in his scriptual commentaries to *'Sefer ha-Hat'a'ah'* he means the *Fallacious Topics* rather than Averroes's middle commentary, which was translated into Hebrew in 1313 by Qalonymos b. Qalonymos of Arles[9]. It is not clear whether Kaspi knew of Qalonymos's translation, although he certain knew of Qalonymos after the latter addressed a highly critical letter to him[10]. Of course, Kaspi would have gained his general knowledge of fallacies from earlier Jewish works by Maimonides, Samuel Ibn Tibbon, Jacob Anatoli, etc. as well as from Arabic sources untranslated into Hebrew[11].

1. Kaspi's section on fallacies in *A Bundle of Silver*

Rosenberg suggested that Kaspi for the most part greatly adapted his source, replacing al-Fārābī's own examples often with Biblical ones[12]. This is an exaggeration; most of the section, including the examples, is taken often verbatim from the Hebrew translation of the *Fallacious Topics.* This translation is extant in nine medieval manuscripts, a preliminary investigation of which indicates that there were revisions in the translation of terms and passages[13]. Mauro Zonta, and Steinschneider before him, claimed that the manuscripts represented independent translations, the earliest of which stems from Spain in the twelfth century. Evidence of an early translation date can be found in terminology not associated with the

7. Rosenberg 1983, pp. 279-290. The edition is based mostly on ms. Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. ebr. 283. Rosenberg lists ms. Paris, Bibliothèque Nationale de France, héb. 673 as '367', correcting Steinschneider 1893 (see p. 278 n. 2), but p. 673 is correct.

8. Al-Fārābī, *Kitāb al-'Amkina al-mughlaṭa*, ed. al-'Aǧam, pp. 131-164.

9. For example, his discussion of synonymy in his commentary on Exodus 10:2 has examples that appear in the *Fallacious Topics*.

10. The letter has been edited recently by Freudenthal / Kasher 2021. Rosenberg 1983, p. 275, claims that Averroes's Middle Commentary on the *Fallacy* had not been translated by Qalonymos before Kaspi composed *Ẓeror ha-kesef*, but this implies an implausibly early date for the latter.

11. On Kaspi's knowledge of Arabic, and his possible use of Arabic literary sources, see Aslanov 2002.

12. Rosenberg 1983, pp. 275-276.

13. Ms. Jena, Universitätsbibliothek, rec. adj. f. 10/10; ms. München, Bayerische Staatsbibliothek, hebr. 26/6, hebr. 110/5, hebr. 244/5; ms. Paris, Bibliothèque Nationale de France, héb. 929/5, héb. 972/3; Ms. Parma, Biblioteca Palatina, Parm. 2761/2; ms. Città del Vaticano, Biblioteca Apostolica Vaticana, vat. ebr. 421/2; ms. Wien, Oesterreichische Nationalbibliothek, hebr. 53/7; ms. St. Petersburg, Institute of the Oriental Studies of the Russian Academy, D 69/4, is a modern copy of the Vienna ms. with Vienna's pagination written in the margins. The Jena ms. is the earliest dated (1408) and includes all the known logical writings of al-Fārābī translated into Hebrew. See Steinschneider 1893, §15.III.6.

Tibbonide family of translators, and in transliteration of some Arabic terms. But some of that non-Tibbonide vocabulary is found in all of the manuscripts, which date from the fifteenth and sixteenth centuries. Either the manuscripts by that time combined independent translations, or they recorded translations and their subsequent revisions.

Al-Fārābī's treatise is in three chapters; the short introductory chapter places fallacious topics in the context of formal disputational practices (e.g., questioning, responding, resisting, rebutting); the second chapter enumerates and elucidates fifteen fallacies of language; the third chapter does the same (but at greater length) for eighteen fallacies of (extra-linguistic) notions (*maʿanin / ʿinyanim*). Kaspi omits entirely the context of disputation, both in his abbreviated introduction and in his treatment of the fallacies having to do with the desired thesis (*al-maṭlūb / ha-mevukash*), which he limits to *petitio principii*. In fact, his abridgement of al-Fārābī's introductory chapter is as follows:

> "We should speak of those sorts of things that cause an inquirer to commit a fallacy, either [himself] deviating from the correct path or causing another to err, such that occasionally the deceiver against whom a claim is made appears as the claimant, the lender as a borrower, the one who has not delivered from error as one who has been delivered Hence in all such cases we should provide ways to preserve man from error, whether in his own reasoning, or in his reasoning with another"[14].

The phrase "or as the lender when he is the borrower" is not in al-Fārābī, and its placement after the passage about the claimant suggests that Kaspi saw the context as juridical rather than disputational. In any event, both authors hold that the purpose of learning about fallacies is to preserve the inquirer from error.

A detailed analysis of Kaspi's abridgement in relation to its main source is beyond the scope of this essay; here we make a few comments and elaborate more in the Appendix. We first give Kaspi's list of ten fallacies of language, which combines some and omits others of al-Fārābī's list of fifteen.

L1. *Fallacy of homonymy*, by virtue of the imprecise use of polysemous nouns (e.g., metaphorical, homonymous, transferred, univocal, distinct, derived, etc.) and prepositions and prepositional particles such as *la*, *ba*, or *mim* in Arabic, *lamed*, *bet*, or *mem*, in Hebrew.

L2. *Fallacy of grammatical form,* e.g., a masculine form referring to a feminine object, or a plural form signifying a singular object. Kaspi informs us that there are many of these in Hebrew.

L3. *Fallacy of amphiboly*, e.g., the same term signifying different things depending on its placement in the sentence, or how the sentence is parsed or interpret-

14. Rosenberg 1983, p. 279.

ed. Kaspi includes within this category the homonymy of the particles of likeness, e.g., *kemo* ('like'); indeed, at first glance he seems to reduce amphiboly to this homonymy, but his examples do not always contain 'like'. In any event, whoever understands this homonymy, writes Kaspi, will understand that there is no contradiction between saying, "Do not answer a fool like his folly" (Prov. 26:4) and "Answer a fool like his folly" (Prov. 26: 5) because in each verse the particle 'like' is referring to a different kind of fool[15].

L4. *Fallacy of synonymy*, where there are slight differences in meaning between synonymous terms. According to Kaspi, most synonyms are like this, and he refers to al-Ghazālī for support; this claim is not found in al-Fārābī, at least not in the *Fallacious Topics*. As we shall see, Kaspi uses this as a principle of scriptural.

L5. *Fallacy of replacement*, e. g., a term with its definition or description.

L6. *Fallacy of combination and division*.

L7. *Fallacy of vocalization*, relevant to languages such as Arabic and Hebrew, which are written without vocalization, and where ambiguity arises because of the various possibilities of vocalization.

L8. *Fallacy of the order of the discourse,* as when we replace "It is not possible for him to do this" for "It is possible for him not to do this"[16].

L9. *Fallacy of inflection, gestures*, etc. Examples are provided in the scriptural commentaries, as we shall see below.

L10. *Fallacy of changing a sentence's pause and accentuation.* It seems that Aristotle's fallacy of intonation has been split by Kaspi into the fallacy of vocalization and the fallacy of inflection, to which we will return below.

Al-Fārābī's third chapter deals with fallacious topics concerning extralinguistic notions. Kaspi's list of eight combines some and omits others of al-Fārābī's list of eighteen, virtually ignoring the discussion of the last half of the chapter.

N1. *Fallacy of taking the essential for the accidental and vice-versa.* In addition to employing al-Fārābī's examples[17] Kaspi expands upon the examples drawn from the syllogism and refers the reader to Averroes's *Book of the Syllogism*[18]. This fallacy also includes taking an accidental cause to be an essential one, though it does not mention the taking of a non-cause to be a cause, which al-Fārābī treats separately, and which Kaspi omits.

N2. *Fallacy of the concomitant accident*, e.g., taking the yellow of bile to indicate honey because honey is yellow, or the woman with an extended belly to indicate being pregnant because a pregnant woman has an extended belly. This fallacy,

15. See the appendix below for his interpretations of these apparently contradictory verses.

16. Kaspi pushes al-Fārābī 's example in the direction of the treatment of modal opposition in the *De Interpretatione*, to which he refers the reader.

17. One he formulates in the style of Biblical Hebrew.

18. I.e., the Middle Commentary on the *Prior Analytics*.

explains Kaspi, is due to the mistaken conviction that universal premises convert universal. He connects the fallacy with the topic from the sign in rhetoric and with the view that two affirmative premises yield in the second figure, "the cause of many mistakes in the arts and in ordinary conversation".

N3. *Fallacy of qualification*, i.e., *simpliciter* and *secundum quid*. Kaspi claims that are many marvelous examples in scripture of such fallacies, but the only examples he cites here are from al-Fārābī.

N4. *Fallacy of conjoined predicates*, which produce either falsehoods ("This is a son; this is yours; therefore, this is your son") or superfluities ("Reuben is a man; Reuben is a white man; therefore, a man is a white man".)

N5. *Fallacy of many questions*, an example of which is Zeno's paradox of motion.

N6. *Fallacy of the failure to observe conditions of contradictory opposition*, a version of *ignoratio elenchi*; Kaspi provide examples of this in his scriptural commentary, as we shall see below.

N7. *Fallacy of begging the question*, i.e.., when one takes the desired thesis to be proven as a premise in the syllogism, which often results from synonymous terms. He refers again to Averroes's *Book of the Syllogism* and omits the more extensive discussion of fallacies having do with the *maṭlūb* in al-Fārābī's *Fallacious Topics*[19].

N8. *Fallacy of circular proof.* Kaspi refers the reader to what he wrote about circular proof in the section on the syllogism of *A Bundle of Silver*.

2. Fallacies in Kaspi's commentaries on scripture

According to Kaspi, knowledge of some of the aforementioned fallacies enables the intelligent reader of scripture to resolve apparent contradictions, fathom the prophetic message, and understand the Biblical narrative. Indeed, the Biblical authors were well aware of the fallacies. Here are some of the fallacies the exegete finds in scripture.

L1. *Fallacy of homonymy*

In the *Table of Silver*, one of his collections of homilies on scriptural topics, Kaspi discusses the question whether prophets ever fail to tell the truth. He relates that according to the Spanish Biblical exegete Abraham Ibn Ezra, although prophets uttering prophecy always tell the truth, prophets in their daily intercourse may

19. To take one example: al-Fārābī writes of a fallacious topic based on "transposition and substitution" in which he warns of the deceptions involved in imagining things in the soul that substitute for the thing itself. One of the examples that al-Fārābī cites is "our representation of what is before the world – for it immediately occurs to our souls an infinite time before it". Marwan Rashed has argued that this must be an interpolation because otherwise al-Fārābī comes off sounding like Philoponus or al-Kindī. If that is so, then the interpolation is already attested in the Hebrew manuscripts of the *Fallacious Topics*. See Rashed 2008, esp. pp. 33-34.

lie if they have a good reason[20]. This appears to be the plain sense of scripture, as when Abram informs Pharoah that his wife Sarai is his sister, or when Isaac does the same for Rebecca before Abimelekh, or when Jacob deceives Isaac; the examples are many. But Kaspi holds that were prophets to utter a statement that cannot be interpreted as true, that would undermine their credibility as prophets. So in the aforementioned and other cases, he appeals to logic, specifically, to homonymy, to explain what the prophets say. With respect to Isaac telling Abimelekh that Rebecca is his sister he writes:

> "The truth has escaped many of the scriptural commentators due to their ignorance of the art of logic, whose whole purpose is the alignment of inner and external speech...When Isaac says of his wife, 'She is my sister' (Gen. 26:7), it is correct, because the Arabic language understands 'brother' and 'sister' to be said of friend and fellow homonymously, or in general and in particular[21]. Hence when Abimelekh claimed, saying to Isaac, 'She is really your wife! Why did you say, 'She is my sister?' (26:9), the meaning was: Why did you use an ambiguous term which can be understood in multiple ways? We understood it literally. Isaac replied: 'Because I thought I might lose my life on account of her'.(26:9) It's good for me that you interpreted it thus, and it is enough for me that I was preserved from saying a falsehood"[22].

Kaspi does not use the technical term for fallacy here, but he does elsewhere when he refers to the polysemous feature of the prepositional particles *lamed*, *mem*, and *nun*. For example, in Isaiah 1:29, where the prophet castigates Israel, "Truly, you shall be shamed because of the terebinths you desired, and you shall be confounded from the gardens (*me-ha-ginot*) you coveted", he writes:

> "The purport of the [letter] *mem* here is like *min*, i.e., 'on account of'. The sages of logic already listed as one of the places of error and fallacy the multiple significations of the particle *min*. And Aristotle wrote of this matter subtly in the beginning of the *Physics*"[23].

In the *Silver Table*, in commenting on Daniel's introduction to his interpretation of Nebuchadnezzar's dream, Kaspi writes:

20. Joseph ibn Kaspi, *Shulḥan kesef*, ed. Kasher, 80, p. 145, and commentary on p. 146.
21. Cf. *Millot ha-higgayon* (*Logical Terms*) attributed to Maimonides, ch. 13.
22. *Millot ha-higgayon* (*Logical Terms*), 148.
23. Joseph ibn Kaspi, *Adnei kesef*, ed. I. Last, to Isa. 1:29, 84. The reference is probably to Aristotle's discussion of one thing coming *from* another thing in *Physics* 1.7, with which Kaspi would have been familiar via Averrroes' Middle Commentary. Kaspi makes the same point and refers to both the *Fallacy* and the *Physics* in his commentaries on Isaiah 24:22 (Joseph ibn Kaspi, *Adnei kesef*, ed. Last, 123), Nahum 3:5 (Joseph ibn Kaspi, *'Asarah kelei qesef*, ed. Last, vol. 1, p. 115), and Lamentations 3:21 (Joseph ibn Kaspi, *'Asarah kelei qesef*, ed. Last, vol. 2, p. 21).

> "What Daniel said to Nebuchadnezzar – 'My lord, the dream is *for* your enemy and its meaning *for* your foe!' – was undoubtedly true. Its interpretation is not what our predecessors thought. Rather, its interpretation is 'for the sake of your despisers and enemies'. [...] Now [Daniel] did not wish to say explicitly 'to your detriment', but it is implied [...]and [Daniel] explained at length the matters, as they are written, and he didn't explain how the dream would be for the benefit of his enemies until the end of his speech. This is the splendid wisdom of the masters of logic and sciences"[24].

According to Kaspi, Daniel wished eventually to inform the king that his dream prophesied good for his enemies, but he used the ambiguous particle *lamed* to avoid having to say explicitly at the outset that the dream predicted Nebuchadnezzars' downfall.

L7. *Fallacy of vocalization*

As an example of a fallacy of vocalization al-Fārābī cites *ʿa-l-y* from *Surat al-Hijr* 15:41, which can be read as *ʿalayya* or as *ʿalīy*[25]. The Hebrew translates the Qur'anic verse literally twice with *ʿa-l-y* appearing without vowels. Kaspi alludes to this example in his interpretation of the Genesis 22: 14: "And Abraham called the name of that place The Lord shall see (*yir'eh*)". Therefore it is said to this day, In the mount of the Lord it shall be seen (*yera'eh*):

> "Who will enlighten me as to the change between the first *y-r-*' and the second *y-r-*'? Only the Men of the Great Assembly by means of their vocalization! And we have noted this at length in other places. And this is one of the types of ambiguity. And if we say the *yod* with a *ḥiriq* and the *alef* with a *qamatz* (e.g., *yir'ah*) it will mean something else. And this has already been enumerated in the *Book of the Fallacy*, like '*a-l-y* with a *qamatz* in the *lamed* and *a-l-y* with a *tzere* in the *lamed*. And there are many examples besides this one"[26].

The Great Assembly to which Kaspi refers was believed to have been a council of leaders and prophets that restored Jewish practices upon the return from the Babylonian exile in the late sixth century B.C.E. According to medieval scriptural exegetes, most notably Abraham Ibn Ezra, probably Kaspi's source, one of the tasks of this group was to fix the vocalization of the consonantal text. This is an anachronism, since that task was performed by the Masoretic scribes in early medieval Palestine and Babylonia. But Kaspi's commentary provides an example of how an Aristotelian fallacy, illustrated by a Qur'anic example, becomes appropriated in a Jewish exegetical context to illustrate the functions of a legendary ancient council.

24. Joseph ibn Kaspi, *Shulḥan kesef*, ed. H. Kasher, 90, p. 155.

25. The latter reading was used by some Shi'ites to show that 'Alī is mentioned in the Qur'an. See Lalani 2006.

26. Commentary on Gen, 22:14, in Joseph ibn Kaspi, *Mazref la-kesef*, ed. I. Last, p. 63.

L9. *Fallacy of inflection, gestures,* etc.

Kaspi solves several problems of exegesis by suggesting that scripture want us to focus on the speaker's manner of uttering words, or accompanying gestures, to fathom his intent, and in this regard he often refers to a Hebrew saying *mi-pi soferim ve-lo mi-pi sefarim*, "from the mouth of scribes, not from the mouth of books", a saying cited in Hebrew already in Judah Ha-Levy's *Kitāb al-Khazārī*, but which Kaspi occasionally attributes to rabbinic sages. One might call this speaker's meaning as opposed to semantic meaning.

For example, there is a well-known difficulty in the Balaam narrative: why was God angry with Balaam's decision to accompany the Moabite dignitaries when He had previously told the seer, "If these men have come to call you, then rise and go with them" (Num. 22:20). Kaspi's answer is that God said these words to Balaam in anger:

> "Our soul is fed up with the lengthy explanations of our predecessors. I say that it has been explained in the books of the philosophers, and especially in the *Book of the Fallacy*, that a statement can have opposing meanings, depending upon the suggestion of the speaker, whether placating or not. There are many like this, for example, 'Shall Saul be king over us?' (I Sam. 12:12). Hence, it has said, 'From the mouths of scribes, and not from the mouths of books'. Although God said to Balaam previously, 'Rise and go with them', the form of the utterance issuing from His mouth was in the manner of anger and implacability"[27].

Another example: Joshua appears to contradict himself when he offers the Israelites the option of serving other gods if they wish (Joshua 24:15) but then says that they will not be able to serve the Lord (24:19). According to Kaspi, each statement was said in its own special manner, "and this has been explained in the *Book of the Fallacy*, and there are many of this sort in all the prophetic writings"[28].

And in response to the question what was the point of scripture recording Pharoah's own version of his dream after the Biblical author had already communicated its contents, Kaspi writes:

> "It seems to me that when the dreamer narrates his dream before the interpreter slowly and moderately, the interpreter's faculty of estimation will be moved by this moderate listening, when he hears the words come from the mouth of the dreamer, for the words will have a new significance according to the speakers, as they stated, 'From the mouths of scribes and not from the mouths of the books'"[29].

27. Joseph ibn Kaspi, *Mazref la-kesef*, ed. Last, on Numbers 20:22, 269. Cf. Joseph ibn Kaspi, *Tirat kesef*, ed. Last, p. 163.

28. Joseph ibn Kaspi, *Adnei kesef*, ed. Last, on Joshua 24:15, 9. Cf. Joseph ibn Kaspi, *Adnei kesef*, ed. Last, on Is. 7:12, 95.

29. Joseph ibn Kaspi, *Tirat kesef*, ed. Last, p. 124.

In other words, Joseph could not have easily interpreted the dream had he not heard the dreamer himself relate it, and it was important to emphasize this for the reader. Here the point is not so much the fallacy of inflection, but the principle of speaker's meaning underlying it.

N1. *Fallacy of taking the essential for the accidental and vice-versa.*

Kaspi writes of this fallacy in *A Bundle of Silver* that "there are wondrous matters in the sacred books; the seeker shall find them". This suggests that Kaspi saw them among the secrets of the Torah, one of which he point to in his work, *A Silver Goblet*. There he writes that the Torah attributes causal efficacy to accidental causes for the sake of the proper beliefs and behavior of the multitude. For example, the Bible is full of the belief that blessings and curses pronounced by human beings are efficacious. Moses arranged the Israelites on Mount Ebal and Mount Gerizim to hear the blessings and curses, and indeed, the early books of the Torah are full of such stories. Since Balaam was renown for this ability to effectively bless and curse people, Moses had to reinforce the belief that his curses actually turned into blessings. For had the people believed that they were cursed, their imagination would have induced a plague among them, "as is well known to physicians". Kaspi sums this up this sort of 'noble fallacy' by writing that because the Biblical authors wrote narratives for the sake of the proper political order, "they write of notions that were accidental in such a way as the multitude will view them as essential"[30].

A similar idea appears in his commentary on the Pentateuch, where scripture considers Aaron and his sons to be the essential causes of blessings when they pronounce the priestly benediction. In fact, they are accidental causes – the essential cause is God – but because they are accidental causes they can be truly described as "the ones who bless". This secret is explained by God Himself when he says, "So they will put My name on the Israelites, and I will bless them" (Num. 6:27). Since Aaron and his sons place God's names on their lips, "God is in fact the one who blesses"[31]. Elsewhere Kaspi writes that God's agency should not be construed as proximate and without intermediaries. He appeals to the principle he finds *Guide of Perplexed* 2:48, that although scripture describes God as a proximate agent, He is in fact the remote cause of all agency – natural, voluntary, and accidental. So, to say that God is the essential cause of blessings is simply to say that as cause of causes, he is the remote cause of all blessings, and, indeed, of everything that exists[32].

N2. *Fallacy of the concomitant accident*.

30. Joseph ibn Kaspi, *Gevia' ha-kesef*, ed. Herring, 20, pp. 42-43.

31. Joseph ibn Kaspi, *Mazref la-kesef*, ed. Last, on Num. 6:27, 246.

32. Cf. Joseph ibn Kaspi, *Mazref la-kesef*, ed. Last, on. Ex. 7:3, 152-3, where a similar point is made about human choice.

In Joshua 9 the story is told how the Gibeonites tricked the Israelites into entering in a covenant with them by claiming that they had come from afar. To prove this they showed them their worn clothes and crumbling provisions. Kaspi writes:

> "*They [the Israelites] took of their provisions* (Joshua 9:14). They took their proof from the provisions, and from the other things, that they had come from a far land. They erred in what is known from the *Book of the Fallacy* as 'the fallacy of the concomitant accident', and its main point is that universal statements do not always convert universally"[33].

As we shall see below in the Appendix, the same example of this fallacy is brought in *A Bundle of Silver*[34].

N3. *Fallacy of qualification.*

In the case of fallacy of qualification Kaspi's interest lies less in pointing out deliberate fallacious reasoning by the Biblical author and more in resolving textual problems. He uses the qualified/unqualified distinction to claim that sometimes scripture asserts that an attribute applies without qualification, when in fact it is true relative to something else[35]. For example, we are told that Jacob also loved his wife Rachel (Gen. 29:30), i.e., in addition to loving his wife Leah, and that Leah was hated (v. 31). Resolving the apparent contradiction, Kaspi writes:

> "By saying 'also', [scripture] indicates that [Jacob] loved Leah after he lay with her. Nor should you raise an objection to the knowers of Hebrew and logic from the subsequent statement, 'that Leah was hated'. For attributes are at times said without qualification and at times in relation. Hence it is true to say of Leah that she was loved and hated [by Jacob], i.e., loved without qualification, and hated in relation to Rachel. Hence two contraries are true of the same subject at the same time from two aspects. All this is well-known to those who know how to speak"[36].

When scripture refers to the tree of the knowledge of good and evil in the garden of Eden, Kaspi writes, "But the tree of knowledge without qualification and absolutely (*bi-stam u-be-muḥlat*) is the same as the tree of life"[37]. In the medieval Jewish Aristotelian tradition, eternal life is understood as the survival of the intellect, and so eating of the tree of life would be a parabolic way of saying something like the intellectual conjunction with the active intellect. By contrast when scripture speaks of the 'intellect' (3:6) attained by eating of the tree of the knowl-

33. Joseph ibn Kaspi, *Adnei kesef*, ed. Last, on Joshua 9:14, 6.

34. Rosenberg 1983, p. 284.

35. Kaspi makes frequent use of this principle in his scriptural exegesis and claims that "the books of wisdom are full of this". See Joseph ibn Kaspi, *Maẓref la-kesef*, ed. Last, on Gen. 31:24, 74.

36. Joseph ibn Kaspi, *Maẓref la-kesef*, ed. Last, on Gen. 29:30, 73. "Knowing how to speak" may be translated as "knowing the manner of Logic".

37. Joseph ibn Kaspi, *Maẓref la-kesef*, ed. Last, on Gen. 2:9, 18.

edge of good and evil, writes Kaspi, that is to be identified with the practical intellect discussed in the (Aristotle's) *Ethics* and not intellect without qualification[38].

N6. *Fallacy of the failure to observe conditions of contradictory opposition*.

For this fallacy we return to the *Table of Silver* and to Kaspi's claim that prophets never lie. Moses was particularly careful to formulate his statements to Pharoah in such a manner as not to give even the appearance of deception, but Pharoah, in his ignorance of logic, made false inferences from them. After giving a few examples, Kaspi writes:

> "Moreover, Pharoah fell into another prodigious error, for which Moses was not at all guilty. God commanded Moses to tell Pharoah, 'We shall go a distance of three days in the wilderness'. [Ex.8:23]. Now Pharoah thought that the intention of this statement was that 'and afterwards we shall return to you' or 'and we won't go any further'. Is there greater madness than this?! And his ignorance of logic was responsible for all this. For he did not know the matter of the affirmation and negation of contradictory opposites, and that sub-contraries are also both true together. Now the Lord gave wisdom to Moses and thwarted the plans of Pharoah"[39].

There is nothing incompatible between affirming that the Israelites will travel a distance of three days in the wilderness and that they will continue their journey for many days afterwards; they can be true together, as "some men are writing" and "some men are not writing" can be true together.

In conclusion, Kaspi's use of logic in his scriptural exegesis in general, and his use of the fallacies in particular, is a fascinating and, at least in the Jewish tradition, unique application of fallacy theory. Kaspi claims that he was criticized for innovating his method of applying logical doctrines to scripture, but he was convinced that the method resolved many problems both of scriptural exegesis, and, in a broader sense, of the importance of scripture for instructing true beliefs and establishing a virtuous society. The Biblical authors wrote allegorically for the intellectuals but they also wrote for the multitude, and as experts in Aristotelian logic, they knew the proper use of fallacious topics. It is quite clear by now that this application of fallacies is quite removed from the sphere of formal disputation, and to some extent, from the area of demonstration.

38. Joseph ibn Kaspi, *Mazref la-kesef*, ed. Last, on Gen. 3:6, 22.
39. Joseph ibn Kaspi, *Shulḥan kesef*, ed. Kasher, 98, p. 162.

3. Appendix

Joseph ibn Kaspi's Section on Fallacies from *A Bundle of Silver*, ed. Rosenberg, p. 275-295.

We should speak of those sorts of things that cause an inquirer to commit a fallacy[40], either [himself] deviating from the correct path or causing another to err, such that occasionally the deceiver against whom a claim is made appears as the claimant, the lender as a borrower, the one who has not delivered from error as one who has been delivered Hence in all such cases we should provide ways to preserve man from error, whether in his own reasoning, or in his reasoning with another.

We say that one may commit a fallacy in many ways:

[L1.] First, by virtue of the various types of a term's common meaning, such as the ambiguous, the metaphorical, and the like, as is explained elsewhere. For there may be a meaning that contains two things, and because one mistakenly believes[41] these two things to be the same, one sees no difference between taking the one or the other. For example: "The vegetable is taller than the earth; what is taller than something is bigger than it; hence, the vegetable is bigger than the earth". The common meaning here is 'big' and 'tall', since one mistakenly believes that their meaning to be the same.

Likewise, "One benefits from evil; what is beneficial is good; hence, evil is good". For 'evil', 'benefits from', and 'good', are said in different ways.

Likewise, "Place is in something; what is in something is in place; hence, place is in place". This is so because the use of [the particle] *bet* is said in different ways. And the Hebrew grammarians have noted that *bet* serves different purposes.

Likewise, "Your child is yours; what is yours is a possession; hence, your child is a possession". That is because the expression, 'yours', and in general, the particle *lamed* is said in many ways[42]. The grammarians have already enumerated this. Likewise, the word *min*, or the [letter] *mem* that replaces it, indicates different meanings. For it can mean 'after' as in "The time was clear *min* being cloudy"[43]; or indicates an agent, as, "The wound is *min* the curse"; or indicates matter, as "The

40. *yit'eh*, lit., 'errs'. The term acquires a technical sense in these contexts, and occasionally I translate it as 'commits a fallacy'. I similarly vary the translation of the *yat'eh*, 'misleads', 'deceives', etc., and the Tibbonide *hat'a'ah*, 'fallacy' or 'misleading'.

41. *Yidmeh*, translating the Arabic *yawhamu*.

42. The example is not found in al-Fārābī and should be distinguished from the example below of the fallacy of accident, implied already in Aristotle: "This is a son; it is yours; hence, it is your son". What generates the fallacy here, according to Kaspi, is the equivocality of the term 'yours' [lit., 'to you'] which may mean 'belongs to you' or 'related to you'. Aristotle (*Sophistici elenchi*, 24, 179b39-180a7) explicitly rejects the notion that the solution for the fallacy of accident lies in the equivocality of 'yours'.

43. The Arabic and the Hebrew translation lack the word 'the time' and this appears to be Kaspi's addition to signify a temporal use of *min*. It should be noted that the Hebrew particle *min*, as opposed to the Arabic term *'an*, does not normally signify 'after' in time.

chair was *min* wood". Most deceptions have to do with metaphorical terms, such as those occurring to the readers of Plato's books when he called matter 'mother' 'wet-nurse' and 'female', and form, 'male' and 'man' since the female desires and yearns for the male, or for her man.

[L2.] One likewise commits a fallacy by virtue of the [grammatical] forms of words, such as when [a word] signifies an action [but is not a verb], or a plural term indicates something singular, or a feminine something masculine, and vice-versa. There are many of these in Hebrew[44].

[L3.] One likewise commits a fallacy by virtue of the commonality [homonymy] found in particles of likeness, such as "What Reuben said is the case is like what he said; Reuben said he is a stone; hence, Reuben is a stone". Or: "What a man knows is what he knows. A man knows an ox; hence he is an ox". This is because the word 'like' applies to special types [of like things]. And the word 'is' can indicate the knower and the thing known. Therefore, somebody who recognizes the commonality in the particle 'like' will understand that there is no contradiction between "Do not answer a fool like his folly" and "Answer a fool like his folly"[45].

[L4.] One likewise commits a fallacy when one considers there to be no difference between two terms that possess one meaning, such as sword, javelin, dagger, knife. In fact, when one examines precisely most synonyms one discerns slight differences in meaning, although they are called in inquiry synonyms. This was already said by Abu Hamid [al-Ghazālī] in another book[46].

44. Kaspi fails to provide examples from the Hebrew translation, perhaps because the Hebrew translation transliterates and explains the Arabic examples.

45. This fallacy is not presented in al-Fārābī as having to do only with particles of likeness, but rather with homonymy caused by syntactic context, the remedy for which is paying attention to that context. Kaspi's interpretation of the apparent contradiction between verses 4 and 5 of Proverbs 26 varies slightly from text to text. In one version of the commentary on Proverbs 26:4-5 (found in ms. Paris, Bibliothèque Nationale de France, héb. 184, *'Asarah kelei qesef*, vol. 1, p. 74), Kaspi refers the reader to the rabbinic distinction between answering the fool in matters of Torah but not answering him in mundane matters (*b. Shabbat*, 30b). However in the other version of the commentary Kaspi writes: "Most of the sentences of this book [Proverbs] are possible, and there are three species of possible [less frequently possible, more frequently possible, equally frequently possible]. The consequence is that subcontraries will be true together with every matter [i.e., concrete terms], even more evidently when the matter is possible", i.e., "Some man is white" and "Some man is not white". It is hence likewise true to say that "Some fool should not be answered" and that "Some fool should be answered". Hence Solomon did well to command us, "Don't answer a fool" and "Answer a fool", for there was no need for him to mention 'some', since the matter is transmitted to the sages which species of fool one should not answer and which one should, for 'fool' is the genus of innumerable species" (ms. München, Bayerische Staatsbibliothek, hebr. 265, *Asarah kelei kesef*, vol. 1, p. 118). This latter explanation seems closer to the fallacy of ignorance of the conditions of opposition than the synonymy of the particle 'like'. But in the commentary on Isaiah 16:13 (Joseph ibn Kaspi, *Adnei kesef*, ed. Last, p. 113), Kaspi says that what preserves the two verses from being contradictories is the synonymity of the particle 'like'.

46. Kaspi emphasizes the subtle differences in meaning between synonyms in *Sharshot kesef*, a dictionary of Hebrew roots, and in his *Retuqot kesef*, on the principles of the Hebrew language. See Kahan 2015. He has a small chapter in *Retukot kesef* "making known the distinctions between syno-

[L5.] One likewise commits a fallacy when one changes a term with its discourse, such as replacing 'man' by 'looking above' or 'erect of stature'[47].

[L6.] Among the fallacious topics is changing composition to division, as "Five is the combination of even and odd; hence it is even and odd; hence five is even, and what is odd is even". Likewise, what is opposite of this, i.e., replacing division by composition, as "Reuben is blind; therefore he is blind-hearted"[48]. Likewise, the replacement of composition by composition, as "Reuben is seeing in his eye [*pikeaḥ be-ʿeino*] and skilled in his healing [*pikeaḥ bi-refuato*]" for each [instance of *pikeaḥ*] indicates something that the other doesn't indicate. If we grasp one in place of the other, or just say *pikeaḥ* without qualification, one will err, for occasionally it might happen that an inferior doctor possesses a good eye, for which he is called a physician who is *pikeaḥ* [seeing], and it will be thought that he is *pikeaḥ* [skilled] in healing.

[L7.] Among them are topics of vocalization, as '*alay*', '*alei*', for one can vocalize the *lamed* by *pataḥ* or by *ẓerei*, and there are various examples of many similar things[49].

[L8.] Among them is the changing the orders of the discourse in priority and posteriority. For occasionally this changes the meaning, as "It is not possible to do this" and "It is possible not to this", as is explained in *On Interpretation*. There are many like this[50].

[L9.] Among them is changing the voices and gestures adopted by the speaker, such as [speaking with] the winking of the eyes or the moving of the lips, or scoldingly, moderately, or in astonishment, or in anger, or in exaggeration, or laughingly, and many distinctions like this. For this reason [the sages], of blessed memory said, "from the mouth of the authors and not from the mouth of the books"[51].

nyms made by al-Ghazālī". Now although al-Ghazālī mentions synonyms in the *Intentions of the Philosophers*, a well-known text in its Hebrew translations, he does not make the point there about the subtle differences between synonyms. But he makes a similar point in reference to divine names in *al-Maqṣad al-asnā: fī sharḥ asmāʾ Allāh Al-Ḥusnā*, 37 (trans. Burrell / Daher 1992, p. 25: Regarding "some of the names close to another in meaning we should believe that there is a dissimilarity between the meanings of the two words"; some others "are in a class with 'sword' [*sayf*] and 'sword made in India' [*muhannad*] or 'sharp sword' [*ṣarim*]". The resemblance with the examples in al-Fārābī is striking.

47. Rosenberg 1983, p. 281, n. 12, suggests that the problem of substitution arises when the discourse (predicate) is composed of accidents like 'looking above' or 'erect in statute'. But even if the discourse is a definition, it can lead to fallacies of substitution.

48. So also the Arabic, but the Hebrew translation has "Reuven is blind [and] possesses a heart; hence, he has a blind heart", which makes more sense.

49. The Arabic cites two verses from the Quran 7:156 and 15:41, where a change in vocalization affects the meaning. The Hebrew paraphrases 15:41 *u-derakhekha alay yashar* but gives no indication of the change in vocalization. This is provided by Kaspi without citing the Qur'anic paraphrase.

50. Kaspi skips over the fallacy caused by grammatical inflections (*tasārīf*), since the Hebrew translator views this as Arabic-specific.

51. Judah ha-Levi, *Kuzari,* 2:72. (Kaspi attributes this saying to the Sages, but Halevy writes, "It is said".)

[L10.] Of them is changing the pauses and connections in an utterance, whether spoken or in writing. All this is evident, and there is no need to cite examples. In sacred scripture there are wondrous matters, but this is not the place to mention them.

He said further that there are many topics that cause fallacies virtue of notions:

[N1.] This [occurs] when what is accidental is considered to be essential, and vice-versa. For example, "And lightening flashed, and he slaughtered the ox, and it died, and rain fell upon it"[52]. Now 'it died' applies to 'he slaughtered' essentially, but the other events are accidental. Of this there are wondrous matters in sacred scripture that will be found by one who seeks them. For this reason one finds several predicates combined of one subject, none of them said of others essentially, as in "Reuben is a white, fat, builder" or "Reuben is an author and a physician". Predicates that are ordered together accidentally can be fallacious in several ways, as when one says, "This is a child; he belongs to you; hence, he is your child". These are true separately but are fallacious when they are combined because some are said of others accidentally. Of these are "Reuben is a man and Reuben is not Simeon"; from which it seems to follow that "Whoever is a man is not a man" because the one who is not Simeon is a man. Now it is not the case that Reuben is not Simeon because he is a man. Likewise, "Man is living; living is a genus; hence, man is a genus". This is false because living, which is an attribute of man, happens to be a genus[53]. And from what is accidental, as "Every man is possibly walking; and everything walking is moving", and the conclusion is "Every man is necessarily moving". This follows if the condition is understood in the conclusion, i.e., "Every man in so far as he is walking is necessarily moving". And in this way one find a true conclusion proceeding from false premises, for it does not follow essentially but it just happens to be the case. This is explained at length in Averroes' *Book of the Syllogism*. In general, the great number of fallacies of this sort lie in the four causes, mostly in the efficient cause, because one considers to an essential cause what is really only an incidental one, or vice-versa. It follows therefore that we should always distinguish what a thing is from what it is essentially, and what it is from what it is incidentally, so that at times we understand that something may be incidentally possible but essentially impossible, as is explained in the books on nature[54].

52. The example is that of al-Fārābī , but Kaspi formulates it in Biblical Hebrew.

53. Cf. *Fallacious Topics*, ch. 3 (140, l. 13 and the Hebrew translation, ms. MÜNCHEN, Bayerische Staatsbibliothek, hebr. 26, f. 446a): "Frequently, something that follows from a discourse follows from a characteristic that is accidental to it, but people think that it follows from the discourse with the characteristic, as when Protagoras thinks that 'It is necessary that man is an animal' follows from 'Man walks' and 'Whatever walks is an animal'. But it is not necessary [because of the premises] that man is an animal".

54. The claim that something can be incidentally possible but essentially impossible is based on al-Fārābī 's discussion in the *Fallacious Topics* of the fallacy in Parmenides' argument against change.

[N2.] Among the fallacious topics is [the topic by virtue of] a thing's concomitant accident, i. e., when a notion is present in a thing, and one knows this through sense-perception or otherwise, and later, when this very notion is present in another thing, one reasons that the second thing is the first. For example, when we see yellow in honey, and then we see it in bile, we think immediately that [bile] is honey. Likewise, when we see Reuben wearing a black turban and later Simeon comes along wearing a black turban, we think immediately that he is Reuben. Likewise, when we see a pregnant woman with a swollen belly, and later we see another woman with hydropsy whose belly is swollen, we think immediately that she is pregnant. The reason for this is that when a notion is consequent upon something, one thinks that the latter will convert in predication. And the root of this error is that one thinks that a universal affirmative statement converts [universal]: since all honey is yellow and every pregnant woman has a swollen belly, one thinks that the converse is true, but that is not always so. In sum, the inquirer thinks that because honey is yellow, what is not honey is not yellow, just as "What is not yellow is not honey" is true.

Now because of this topic our forefathers erred when they took proof from the provisions of the inhabitants of Gibeon: "And the men took of their provision" [Joshua 9:14], i.e., they took proof from the nature of their provision [by reasoning that] because bread that comes from a distant land is moldy, what does not come from a distant land is not moldy, just as "What is not moldy does not come from a distant land" is true. In sum, they thought that affirmative universal proposition converts. If so, the concomitant accident causes a fallacy in one of two ways: the first is that one mistakenly believes in the truth of the converse in predication, as one said that since all honey is yellow, he mistakenly believes that everything yellow is honey. The second is that one mistakenly believes that the converse of the converse of the contradictory is true, because the converse of the contradictory is always true. For the contradictory of "What is honey is yellow" is "What is not honey is not yellow", and the converse of that is "What is not yellow is not honey", but he mistakenly believes that what is not honey is not yellow[55]. From the first way are composed syllogisms that are called 'sign'. For example, a

According to al-Fārābī, it is incidentally possible but essentially impossible for the existent to be generated from the non-existent (matter). The argument is relevant to physics, rather than to logic. For a different interpretation see Rosenberg 1983, p. 283, n. 20.

55. What Kaspi, following al-Fārābī, calls the "converse of the contradictory" (*'aks al-naqīd / hefekh ha-soter*) is the same as our contrapositive. Hence if a) "Every a is b" is the original statement, then the converse of the contradictory is b) "Every not-b is not-a". Kaspi identifies c) "Every not-a is not-b" as both the contradictory of a) and as the converse of the converse of the contradictory of a). In fairness to him, the Hebrew is garbled here. Al-Fārābī writes: "The second [mode of fallacy] is that one mistakenly believes the truth of the converse of its contradictory [i.e., of the universal converted proposition]". The Hebrew has "The second is that he mistakenly believes the truth of the converse of the predication of its converse (*hefekh nesiat hofkho*)". In any event, both modes of fallacy occur from illegitimately considering *a* and *b* to be coextensive, hence universally convertible.

deviant adorns himself; Reuben adorns himself; hence, he is a deviant[56]. Likewise, "A thief walks at night; Simeon walks at night; hence, Simeon is a thief". By virtue of this topic the combination of two affirmations in the second figure is occasionally thought to yield [a valid conclusion], which is the cause of many fallacies in the arts, likewise in one's conversation with one's neighbor. And because of this wise men thought that the sinews lacking pulse that grow from the liver are thicker. The cause of this error is from what we see with respect to plants and trees, for what is nearer to the place of growth is thicker, which leads them to believe that everything thick is always close to the place of growth, as every place of growth is thick. The second aforementioned way of the fallacy of the concomitant accident is also the cause of many errors, e.g., the statement of some of the Pythagoreans that every existent is in place because everything that is non-existent is not in a place. And they supposed that everything not in place is non-existent. So, too, the argument of Melissus: If the existent comes into being then it has a beginning; but the existent does not come into being; hence it has no beginning. This matter also reduces to the conjunctive conditional syllogism, since it is known that positing the contradictory of the consequent implies the contradictory of the antecedent, but it is not always true that positing the contradictory of the antecedent yields the contradictory of the consequent[57].

[N3.] Another fallacy concerns what pertains to notions that are connected to something, whether time or place or a state, in sum, what is peculiar to some thing, whatever that is. For this misleads and deceives one into thinking that they come to be in an absolute and unqualified manner. Of this sort there are wondrous notions in sacred scripture. But the proper example according to logic is when we say of the dead Elyakim[58], "Elyakim is a poet; therefore, Elyakim is"; likewise, "Reuben is not Simeon; hence, he is not". Likewise, "What already was exists now in thought; hence, it exists now". Likewise, "The slaughterer of an animal is dead; hence, he is alive; hence, the dead is alive". Another fallacy is that we mistaken believe that what is present in some is present in all and without qualification, as one who thinks of two affirmative premises in the second figure that they yield [a valid conclusion] without qualification, because it is the case with some matters [concrete terms].

[N4.] Among these [fallacies is] what one mistakenly believes from conjoining[59] many notions [serving as] predicates; this occasionally produces a falsehood or a superfluity. An example of a falsehood was already provided for you[60]. An

56. Reading with ms. Paris, Bibliothèque Nationale de France, héb. 929, f. 232b.

57. Connecting the fallacy to the conjunctive conditional syllogism is Kaspi's addition.

58. Arabic: *'Umeris*; the mss of the Hebrew translations have various corruptions, but ms. München, Bayerische Staatsbibliothek, hebr. 26 has *Elkayam*, the closest to Kaspi's *Elyakim*.

59. Reading *mi-heksher* with ms. Oxford, Bodlein Library, Rawl. Or. 34, f. 56a.

60. The Arabic and Hebrew translation provide the aforementioned example: "This is a son,

example of a superfluity is "Reuben is a man, and Reuben is a white man; hence Reuben is a white man".

[N5.] One of the fallacious topics is when a premise is taken to be one although it is actually many. The fallacy of this topic lies in the subject[61] in which two opposite determinations in two different characteristics adhere, and which is grasped without qualification[62]. This is like Zeno's doubts regarding motion. Zeno says that when a mover[63] traverses a certain distance at a certain time, it is well-known that it traverses half the distance before it traverses all of it, and that it traverses half of the half before it traverses half. And since a body is divided into infinite halves, it follows that the mover traverses an infinite course in finite time, which is false. This follows because 'infinite', whether in distance or in time, is in one of two ways: in length or in division. Now the mover cannot traverse a course infinite in distance in finite time, nor can it traverse a course infinite in division in finite time, and vice-versa. Because Zeno understood the course infinite in division and time finite in length, he erred and mistakenly believed the falsehood. But when the aspects are grasped uniformly, whether the course is finite or infinite in time, the mover will be found to have traversed either a finite course in a finite time or an infinite course in an infinite time. For we will grasp the first in length and the second in division, and neither of them is false.

[N6.] Of fallacious topics is the failure to take opposites in such a manner as to fulfill the conditions of opposition, which is as we have said, that they not belong to the same subject at the same time and from the same aspect[64]. This is evident and examples would be unnecessary were it not that occasionally one considers something to be true that is false.

[N7.] Of fallacious topics is the assumption of the primary desired thesis, and that is when the desired thesis is taken as a premise in a syllogism. Most of these fallacies result from their being synonymous terms, which was explained in Ibn Rushd's *Book of the Syllogism*[65].

and his is yours; hence this is your son" and explicitly say that this is a fallacy from unqualified statements, which is curiously omitted in Kaspi's restatement.

61. Kaspi's text has predicate, following the Hebrew translation, but the original Arabic has *mawdū*, which makes more sense.

62. The example given by al-Fārābī is: "Is clay water and dirt or not?"

63. Kaspi: *ne'etaq*. The Hebrew translation has *ma'atiq*, a close rendering of *mutanaqqil*, listed as a variant reading in el-Ajam's apparatus, p. 148, n. 5.

64. Cf. *Ẓeror ha-kesef*, ms. Oxford, Bodlein Library, Rawl. Or. 34, f. 10b: "The definition of opposites in general is two things that cannot possibly belong to the same subject at the same time in the same aspect".

65. See Averroes, *Sefer ha-heqqesh*, ms. Paris, Bibliothèque Nationale de France, héb. 920, ff. 14a-15b: "The assumption of the primary desired thesis, which is the act of the one who errs or misleads, occurs in two ways: first, when the desired thesis itself is assumed as the premise demonstrating itself, which occurs when the subject or the predicate of the desired thesis are both synonymous terms...". Averroes goes on to explain this in the subsequent pages.

[N8.] Of[66] fallacious topics is the circular proof, and that has already been mentioned by us in the *Book of the Syllogism*[67].

[Kaspi] said. The fallacies and deceptions are many, and not every man is wise, i. e., possessing the first species of wisdom mentioned by the Guide, which alone is truly wisdom[68]. Likewise, not every man is worthy of [various] wisdoms. One who persists in error is of the domesticated animals, not made in the image and likeness of God. Aristotle has said that just as 'man' is said equivocally of the figure drawn on the wall or made of stone, so too of one who is not expert in the sciences. Yet one who knows the ways of logic to the extent that he is preserved from every error, and knows the Torah and the Prophets in the true and correct manner – then he is a man, and an individual or angel, when he will be perfect. May God place us among the Ishim[69] and even the angels; hence says the Lord. But for that, what is written in Sifrei[70] on the verse "...and the LORD thy God will bless thee in all that thou doest" (Deut. 15: 18) is necessary: "One might think [that the blessing will be bestowed on the idle?" scripture teaches, "in all that thou doest". We shall seek help in the Lord in all.

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66. Reading *u-min* with ms. Oxford, Bodlein Library, Rawl. Or. 34, f. 57a.

67. See ms. Oxford, Bodlein Library, Rawl. Or. 34, f. 42a-b.

68. Cf. Maimonides, *Dalālat-al-Ḥā'irīn*, ed. Munk / Joel, 3, 54; translation in Pines 1963, p. 632: "[Wisdom] is applied to the apprehension of true realities, which have for their end the apprehension of Him, may He be exalted").

69. *Ishim* refers to the rank of angels that communicate with prophets; for Maimonides, this is the rank of the active intellect; see *Yesode ha-Torah (Code of Law)*, 4:6.

70. See *Sifrei al Sefer Devarim*, ed. Finkelstein, 181 (based on a 2); my thanks to Dr. Avigail Manekin Bamberger for the reference. The verse is cited by Kaspi imprecisely.

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Abstract: The scriptural exegete, intellectual, and itinerant scholar, Joseph ibn Kaspi (1280-1344, Southern France and Spain) believed that a knowledge of logic was essential for understanding the Bible. For that reason he wrote for his son a compendium of logic, *A Bundle of Silver*, based mostly on his abridgement of Hebrew translations of the commentaries and short works of Averroes and al-Fārābī. The compendium was the first such work written in Hebrew and enjoyed a measure of popularity subsequently; twenty-eight manuscripts of the work are extant, from as late as the turn of the twentieth century. The present article considers the section on fallacies from the compendium as well as Kaspi's use of Aristotelian fallacies in his explanations of scriptural passages; the Appendix presents an annotated translation of the section on fallacies.

Keywords: Joseph ibn Kaspi; Hebrew logic; Jewish philosophy; fallacies; *A Bundle of Silver.*

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Fallacies in Rabbinical Thought, in Medieval Jewish Philosophy, and in the Treatise on Talmudic Methodology by Abraham Elijah Cohen

False arguments challenged Jewish scholars in the central disciplines of Jewish literature: law, ethics, theology, science, and even methodology. The present paper will focus on argumentation criticism in talmudic and rabbinical literature, Jewish philosophy, and in a fourteenth or fifteenth century treatise combining these two disciplines.

Talmudic and midrashic literature includes a large variety of works originating in different times and places. It summarizes and analyses works, interpretations, lectures, homilies, legal rulings, and folk stories of earlier generations. In this sense, the talmudic-midrashic literature is based on two aspects – a traditional aspect and a logical aspect. The Talmud conceives of both tradition and logical argumentation as reliable and binding sources but nevertheless differentiates between them: Reliable tradition must be accepted unconditionally and uncritically, while logical arguments are to be examined through critical tools.

The Mishnah states:

> אם הלכה – נקבל. ואם לדין – יש תשובה
> "If this is a legal tradition (*halakha*) – we will accept it. But if it is a logical inference (*la-din*) – there is a reply" (*Mishnah, Kareitot* 3, 9; *Yevamot* 8, 3)[1].

Talmudic literature may thus be viewed as an accumulative literature in which the words of one generation are passed on to future generations. But alongside tradition, the words of earlier sages are also tested and examined – validated or refuted – using a wide range of methodological criteria. In this sense, a large part of the debates that fill long pages in talmudic literature, constitute critical exami-

1. On the meaning of 'halakhah' see Elon 1997, pp. 83-84 and Urbach 1984, p. 8. On the meaning of 'din' see Elon 1997, p. 240 and Urbach 1984, p. 67.

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nations of the soundness of arguments and an attempt to identify errors with regard to both content and structure[2].

The ability to draw convincing conclusions from the words of the ancients, as well as to refute the positions of colleagues and to identify misleading arguments is highly valued in talmudic thought. Apart from their appreciation of the ability to produce sound arguments, the sages even valued the ability to formulate fallacies.

The Babylonian Talmud expresses this in the following way:

> "Rav Yehudah said in the name of Rav: 'They place on the Sanhedrin (the great assembly of scholars) only one who knows how to render a carcass of a creeping animal pure by Torah law'. Rav said: 'I will deliberate this law and render it pure: If a snake, which kills and increases impurity, is pure, then a creeping animal that does not kill and increase impurity, isn't it logical that it should be pure?!' (The Talmud answers:) But this is not so, rather it is similar to the case of an ordinary thorn (that causes injuries and may kill like a snake, and yet it is ritually clean. Namely, there is no connection between 'increasing impurity' and 'being pure') (*Sanhedrin* 17a-b)"[3].

The carcasses of the creeping animals mentioned in Lev. 11, 29-30 are rendered ritually unclean by the biblical and talmudic traditions, hence an argument that renders them pure is nothing but a deception. Yet, the very ability to formulate

2. Consider, for example, the following passage in the *Mishna, Baba Kama* 2:5: "An ox that did damage on the premises belonging to the plaintiff, how so? If he gored, pushed, bit, lay down on, or kicked while on public ground, he pays half the damage; if while on the premises of the plaintiff, R. Tarfon holds that he pays the whole damage; the rabbis, however, say half the damage. Said R. Tarfon to them: In a case in which the law is lenient with the 'tooth' and 'foot' on public ground, making them not liable, it decrees rigorously if the same happened on the premises of the plaintiff, namely, that the whole damage must be paid; in a case where it decrees rigorously that the 'horn' on public ground must pay half, is it not a logical inference that we ought to strictly adjudge the same, if on the premises of the plaintiff, liable for the whole? They said to him: It is sufficient that the result derived from the inference be equivalent to the law from which it is drawn, viz., as if on public ground only half damage, so also if on the premises of the plaintiff half damage". (translation Danby 1959, pp. 333-334). The principle applied by R. Tarfon is an *a fortiori* argument – this is the first formal principle in the list of principles according to which the Torah is expounded as worked out by Hillel the Elder and R. Yishmaʿel. The Sages restrict the application of this principle through a formal principle: "It is sufficient that the result derived from the inference be equivalent to the law from which it is drawn". The Mishnah, here as in many other cases, reveals a formal critical sense towards the validity of an argument. On the *a fortiori* argument in talmudic literature see Sion 1997, pp. 47-62.

3. In *Eruvin*, 13[b]: "Rabbi Aḥa bar Ḥanina said: It is revealed and known before the One Who spoke and the world came into being that in the generation of Rabbi Meir there was no one who was his equal. Why then didn't they establish the *halakha* in accordance with his opinion? It is because his colleagues were unable to ascertain the profundity of his opinion. As he would state with regard to a ritually impure item that it is pure, and display justification for that ruling, and likewise he would state with regard to a ritually pure item that it is impure, and display justification for that ruling [...] It was taught: There was a distinguished disciple at Yavne who could purify the creeping animal, adducing one hundred and fifty reasons in support of his argument. Ravina said: I will deliberate and purify it: Just as a snake that kills and thereby increases ritual impurity is ritually pure, a creeping animal that does not kill and increase impurity, all the more so. But it is not so, rather it is similar to the case of an ordinary thorn".

logically misleading arguments is appreciated by the Talmud inasmuch as it testifies to the creativity and sophistication of the scholar.

However, fallacies in the realm of law obviously cannot be accepted as binding. A talmudic argument must be coherent with the legal tradition. An interesting example of this is found in the tractate *Derekh Eretz Rabbah*, where the following casuistic debate takes place:

> "He who had intercourse with a married woman is permitted to marry her daughter' (this is the law that is being discussed). This is the question that R. Jose ben Taddai of Tiberias put to Rabban Gamliel: 'If I, who may cohabit with my wife, am prohibited from marrying her daughter, how much more should I be prohibited from marrying the daughter of a married woman, seeing that I am prohibited from cohabiting with her'(i.e. with the married woman)! He (i.e. Rabban Gamliel) replied to him: 'Go and provide me [a wife] for the High Priest of whom it is written: 'A virgin of his own people shall he take to wife' (Lev. 21:14), and I will provide you [with wives] for Israel'. Another explanation: 'We do not rely on logical argumentation to abrogate a specific law of Scripture'. Rabban Gamliel then excommunicated him!' (*Derekh Eretz Rabbah*, chapter 1, 55b)".

R. Jose ben Taddai argues: My wife's daughter is forbidden to me even though her mother, who is my wife, is permitted to me. All the more so should the daughter of another married woman be forbidden to me since her mother is forbidden to me. The conclusion of the argument is therefore that it is forbidden to marry a woman who is the daughter of a married woman, and the practical implications of this are that it is almost forbidden to marry at all[4].

Rabban Gamliel does not explain why the argument of ben Taddai is false[5]. In his reply he argues that Ben Taddai's argument contradicts the law according to which the High Priest is permitted to marry only a virgin. In the second version of Rabban Gamliel's reply he formulates a general principle purported to establish a proper balance between tradition and legal sophistry: "We do not rely on logical argumentation to abrogate a specific law of Scripture", meaning that we do not accept argumentations, not even sound ones, that conflict with the traditional and accepted interpretation of the Torah. He also excommunicates the disciple.

Also, in Jewish philosophy, the tendency towards a critical approach is discernible. Jewish philosophers criticized the words of their predecessors and colleagues in order to prove them false[6]. In this realm, the work that influenced the

4. Except, perhaps, for in exceptional cases such as the daughter of a widow or a divorced woman, as pointed out by R. Yaakov ben Baruch Neumburg in his commentary *Naḥalat Yaʿaqov*, (*Derech Eretz Rabbah*, 55b).

5. See an explanation in Mielziner 1968, p. 140.

6. Within the limited framework of the present paper we will neither be able to demonstrate all the critical uses of fallacies made in the writings of Jewish philosophers, nor will we distinguish between different types of fallacies (such as the distinction between an argument that is deceptive be-

theory of fallacies was, of course, Aristotle's *Sophistical Refutations*. Medieval Jewish scholars fluent in Arabic could have been familiar with this work through its ninth and tenth centuries Arabic translations[7]. Hebrew readers could have used the Hebrew translation of Averroes's middle commentary on Aristotle's work or his *Epitome of Logic*[8]. Aristotle's doctrines of fallacies are, of course, also discussed by additional Muslim and Christian authors, whose works constituted important channels for Jewish familiarity with the theory of fallacies[9].

The title of the Arabic translation worked out by ʿĪsā Ibn Zurʿa (d. 1008), which was used extensively by students of fallacies in the Muslim world[10], is *Kitāb Sūfisṭīqā* (Book on the Art of Sophism). The titles of the other two Arabic translations also use the transliteration *Sūfisṭāʾīīn* or *Sūfisṭāʾiyyah*[11]. However, when describing the content of the book, Jewish and Arab authors used words derived from the verb ghālṭa (to mislead or deceive), hence the book also became known as *Kitāb al-Mughālaṭah*, and in Hebrew: *sefer ha-haṭʿaʾah*.

This is how Maimonides describes the subject of fallacies in his *Treatise on the Art of Logic*:

> "There is also a kind of syllogism used for deception and falsehood (مغالطة وتمويه), where one or both premises are such wherewith or wherein a man errs or falsifies in any of the syllogistic moods. Such syllogisms are called sophistic (سوفسطائية), and the making thereof and the knowledge of the ways in which people deceive and utter falsehood (يغالط ويموه) constitute what is called sophism (الصناعة السوفسطائية)"[12].

Following the above-mentioned work of Aristotle, we find numerous uses of the terms *Mughālaṭah* (in Arabic) or *haṭʿaʾah* (in Hebrew), as well as verbs derived from these two roots, in the writings of Jewish philosophers criticizing various philosophical systems.

cause it relies on false assumptions and an argument that is invalid from the perspective of its structure. Or between an argument that is nothing but a mistake and a fallacy that pretends to be a sound argument). A comprehensive study of the ways in which Jewish philosophers used the theory of fallacies is most certainly a *desideratum*.

7. On the Arabic translations of the *Book of Fallacies*, see Endress / Hasper 2020.

8. The Hebrew translation of Averroes's middle commentary on this book was carried out by Kalonymos ben Kalonymos (fourteenth century). The *Epitome of Logic* was translated into Hebrew by R. Jacob ben Makhir (thirteenth century). The Hebrew translation of Al-Farabi's treatise on *Sophistical Refutations* was carried out by an anonymous translator in the first half of the thirteenth century.

9. *Cf.* Zonta 2011, pp. 26, 50, 54 and 67.

10. See Endress / Hasper 2020, pp. 74-75.

11. The one by Ibn ʿAdī and the one attributed to al-Nāʿimī carry the titles: *Kitāb tabkīt al-Sūfisṭāʾīīn* (*Book on the Rebuke of the Sophists*), and *Kitāb [...] fī al-tabṣīr mughālaṭat al-Sūfisṭāʾiyyah* (*Book on Enlightening the Fallacies of the Art of Shophism*), respectively.

12. Efros 1938, pp. 48-49; Arabic: "Mūsā Ibn-i Maymūn'un Maḳāla fī Sināʿat al-Manṭiḳ'i", 50. Compare Al-Farabi, *Catálogo de las ciencias*, 37-38 (Arabic); Medieval Hebrew translation in: Zonta 1992, p. 10.

For example, the first Jewish Aristotelian (or better: Avicennian) in twelfth century Spain – R. Abraham Ibn Daud, criticizes R. Solomon Ibn Gabirol in the following words (as translated into Hebrew by R. Samuel ibn Maṭuṭ)[13]:

> ולא הייתי עושה אותו מטרה לחצי לולי שהוא הטעה את האומה הטעאה גדולה וידע זה מי שיעמוד על ספרו
> "I would not have turned him into a target of my arrows had he not deceived the nation with a great deception (*haṭ'a'ah gedolah*); anyone reading his book will notice this (Abraham Ibn Da'ūd, *Exalted Faith*, 106-107)".

According to Shlomo Pines, the Arabic original for "deceived [...] with a great deception" was probably the verb *ghālaṭa* (or: *aghlaṭa*) – 'deceived' or 'formulated a fallacy'[14].

Ibn Daud's younger contemporary, Maimonides, viewed deceptions as something that 'defiles' rational comprehension (he uses the term *najāsah*, which means 'ritual impurity' or 'filth')[15]. In his opinion, knowledge of metaphysics must be acquired through an orderly moral and intellectual step-by-step preparation, which includes being cautious of fallacies[16]. Indeed, in Maimonides's criticism of the Muslim theologians (*mutakallimun*), he accuses them of not identifying fallacies correctly. According to Maimonides, the *Kalām* proves the existence of God exclusively on the basis of the conclusion that the world was created. However, in Maimonides's view, it cannot be demonstrably proven that the world was created, hence if we follow the method of the *mutakallimun* also God's existence cannot be proven.

13. Ibn Daud's book *The Exalted Faith* was originally written in Arabic, but the Arabic original was lost. It has reached us in two different Hebrew translations worked out by Solomon ibn Lavi and Samuel ibn Maṭuṭ (both from the end of the fourteenth century). The translation of Ibn Lavi is the earliest of the two translations, and the translation of Ibn Matut seems to be a stylized adaptation of Ibn Lavi's translation. On the two translators and their translations, see Eran 2019, pp. 60-69.

14. Pines 1977, pp. 61-67. For the misleading translation of Ibn Lavi, seeAbraham Ibn Da'ūd, *Exalted Faith*, p. 106: ולא הייתי מגנה דבריו לולי שדבר סרה גדולה על האומה, ידעה מי שעמד על ספרו "I would not have condemned his words had he not spoken very offensively about the nation, and anyone reading his book will notice this". For a different English translation, see Samuelson / Weiss 1986, p. 40.

15. According to Maimonides, "the nobles of the children of Israel' sinned by not preparing themselves properly for the study of metaphysics and were therefore punished". Maimonides adds: "This having happened to these men [= the "nobles of the children of Israel"], it behoves us, all the more, as being inferior to them, and it behoves those who are inferior to us, to aim at and engage in perfection our knowledge of preparatory matters and in achieving those premises that purify apprehension of its taint (אלמטהרה ללאדראך מן נג'אסתה), which is error (אלתי הי אלגלטאת)" (Pines 1963, p. 30; Arabic: Qāfiḥ 1972, vol. I, p. 33).

16. For example, in the *Guide of the Perplexed* 1, 5 he writes: "In the same way we say that man should not hasten too much to accede to this great and sublime matter at the first try [...] when, however, he has achieved and acquired knowledge of true and certain premises (מקדמאת חקיקיה יקיניה) and has achieved knowledge of the rules of logic and inference (קואנין אלקיאס ואלאסתדלאל) and of the various ways of preserving himself from errors of the mind (אגאליט אלד'הן), he then should engage in the investigation of this subject" (Pines 1963, p. 29; Qāfiḥ 1972, I, p. 32).

Maimonides writes:

> "When I considered this method of thought, my soul felt a very strong aversion to it, and had every right to do so. For every argument deemed to be a demonstration (ברהאן) of the temporal creation of the world is accompanied by doubts and is not a cogent demonstration, except among those who do not know the difference between demonstration (אלברהאן), dialectics (אלג'דל), and sophistic argument (אלמגאלטה)"[17].

The term *al-burhān* here clearly denotes a demonstrative proof, discussed by Aristotle in the *Posterior Analytics*. The term *al-jadal* is a dialectical proof, discussed by Aristotle in the *Topics*. And *mughālaṭah* is a fallacy, which is the subject matter of *Sophistical Refutations*[18].

In the second half of the paper, we will present a hitherto unpublished text that focuses on the philosophical analysis of fallacies in the realm of talmudic study. The text is a commentary on the thirteen hermeneutical principles through which the Torah is expounded. These thirteen principles are ancient rabbinical methods of hermeneutics and inference frequently applied in talmudic literature in order to expound the Written Law and to connect the legal tradition to Scripture[19]. This commentary was authored by a scholar named R. Abraham Eliyahu Cohen, and is preserved in the ms Vaticanus Hebraicus 37 (IMHM, f 153) folios 74r-84r.

The manuscript is written in Italian script. In the bibliographic literature, its dating moves between the fourteenth and sixteenth centuries[20]. Aaron Freimann

17. Pines 1963, p. 180 ; Qāfiḥ 1972, vol. I, p. 193.

18. According to Maimonides, deceptive arguments defile one's thought, but only when they are not identified as such. However, as an educational tool Maimonides viewed fallacies as being both effective and desirable. In his legal *magnum opus*, *Mishne Torah* (Laws of Torah Study 4:6), he rules that pedagogical methods should include a component of intentional deception, meant to test the student and to develop his critical sense. He says: "The teacher should lead the students astray (להטעות את התלמידים – meaning to use fallacies) through his questions and through the deeds that he carries out in their presence, to sharpen [their powers of thought] and to test whether they remember what he taught them or not". Maimonides also appreciated the ability to cope with fallacies. In his commentary on the virtues of the wise enumerated in Mishnah *Avot* 5:6, he writes: "If he (i.e. the wise) was deceived by someone skilled in the art of sophistications (אד'א גאלטה מגאלט באלצנאעה אלספסטאויה), he should not be surprised and frightened [...] but quickly sense wherein the fallacy lies and expose it. This is what is meant by the words [of the Mishnah]: 'And is not afraid to answer' (Maimonides, commentary on *Avot* 5:6: *Mishnah ʿim pirush Rabbenu Moshe Ben Maimūn*, *Neziqin*, 299]).

19. For the thirteen principles see *Sifra*, 11r-3r (transl. Neusner 1988, I, p. 57). For analysis of the principles cf. Mielziner 1968, pp. 126-176.

20. The catalogue of the Institute for Microfilmed Hebrew Manuscripts dates the manuscript to the fifteenth century, while in the list of Nehemia Alony and David S. Levinger (Alony / Levinger 1968, p. 17, par. 37) it is dated to the fourteenth or fifteenth centuries. Naftali Ben-Menahem and Umberto Cassuto dated the manuscript to the fifteenth century (Ben-Menahem 1954; Cassuto 1956, pp. 50-51). Aaron Freimann dated it to the sixteenth century (Freimann 1917, p. 113). In the new catalogue of Hebrew manuscripts in the Vatican, the manuscript is dated to end fifteenth century Italy. See Richler Beit-Arié / Pasternak 2008, p. 25.

adds that it may have been written in Crete but did not substantiate this claim[21]. On page 1b of the manuscript, a table of contents written by two different persons is found[22]:The first one is Caroli Friederici Borromaei, and the second one is [Giulio] Bartolocci. In the seventeenth century this Borromaeus authored lists of Hebrew manuscripts found in the library of the Vatican[23], and Bartolocci was his contemporary[24]. From here we learn that the manuscript, and hence also the commentary of Cohen, was found in Rome in the seventeenth century.

Cohen's commentary is abundant with medieval logic. The author applies the doctrine of the categories, the doctrine of the quality of propositions (which distinguishes between positive and negative propositions), and an interesting type of modal doctrine of propositions. In his discussion of the principle of *gezerah shavah*, the author applies the doctrine of the five predicables while paying special attention to the doctrine of accidental predicate and its different types. The author's discussion of the principle of *binyan av* is replete with distinctions between the genus, the species, the difference, the property, and the accident.

The influence of logic stands out in Cohen's treatise especially in the area of fallacies. A great part of Cohen's commentary on *a fortiori* arguments (*qal va-ḥomer*) teaches the student how to identify a sound and valid *a fortiori* argument, and how to distinguish it from an *a fortiori* argument that only appears to be sound or valid. Cohen's instructions rely not only on the examination of the formal structure of this kind of argument, but also on a meticulous examination of the talmudic terminology and the different kinds of wording used to present *a fortiori* arguments in the Talmud. In his commentary, Cohen applies theories discussed by the scholastic logicians of the thirteenth and fourteenth centuries, and even his terminology reflects a few Latin terms.

Cohen fiercely criticized a group he called the *balbelanim* (lit. 'those who confuse', but the term can also be understood as a mocking disruption of *mefalpelim* – those who apply the method of *pilpul* – casuistry), or more maliciously: "The *balbelanim* of contaminated minds". Cohen did not clarify the essence of the casuistic method he attacked and why he attacked it, but from his words it may be inferred that, in contrast to the self-understanding of the casuists, he did not consider them experts in "the Torah and its ways". He also viewed them as being unable to distinguish valid inferences from fallacies. From his words it ap-

21. The manuscript (folios 2b-30b) contains a portion of Rashi's commentary on the Torah with a translation into Italian. It is possible that it was this translation that made Freimann think the manuscript was written in Crete. See also Richler 1994, p. 192.

22. See Cassuto 1956, pp. 50-51.

23. See Richler 1994, p. 192. This Borromaeus should be confused neither with the founder of the Bibliotheca Ambrosiana in Milano in 1609 (Richler 1994, p. 121), nor with Borromaeus the archbishop of Milano (1538-1584).

24. On Borromaei see Brisman 1977, pp. 6-8.

pears that he also criticized the casuists because he considered them void of logical-philosophical education.

Similar criticism of casuistry (*pilpul* method) in the name of logic is found in two additional works, presumably from the fourteenth or fifteenth centuries. One is *Shaʿarei Ṣedeq (attributed to Gersonides*), and the second is *ʿAlilot Devarim*[25]. The author of *ʿAlilot Devarim* is one of the fiercest and most conspicuous spokesmen for combining logic with traditional (halakhic as well as non-halakhic) Jewish studies, and many parallels exist between his work and that of Cohen: Both authors passionately criticize the casuists for their inadequate knowledge of logic, and both incline towards an interdisciplinary integration of logic into talmudic studies. Robert Bonfil leaned towards the opinion that *ʿAlilot Devarim* was authored "in South Italy or Candia, which in those days [the second half of the fourteenth century] were meeting places for scholars of an orientation similar to that of the author of *ʿAlilot Devarim*" (Bonfil 1980, p. 261). In light of these findings, it is reasonable that also the work of Cohen was authored in or around Italy at the turn of the fourteenth century or in the beginning of the fifteenth.

The examples brought by A. E. Cohen in order to demonstrate the thirteen principles are, in a certain sense, 'formal'. By this I mean that they deal exclusively with the structure and meaning of the talmudic arguments, but not with their real legal content. Cohen is not interested in the practical implications for Jewish law. The only purpose of the many examples brought in this treatise is to train the student to identify fallacies. As stated by the author:

25. On *Shaʿareii Ṣedeq* see Ravitsky 2009, pp. 174-204. On *ʿAlilot Devarim*, see Bonfil 1980. It seems that the criticism levelled by the two authors was directed against casuists the kind of which we find among the תוספות גורניש (Tosafot Goornish – the exact pronunciation of this word is still subject to debate among scholars). On the Tosafists of Goornish, see Ta-Shema 1971 and 1976. According to some scholars, the Tosafists of Goornish developed a method of learning aimed mainly at identifying several alternative interpretations of the talmudic discussion, interpretations that appear to be true even though they are not, the purpose of the study being to beware of these interpretations. According to Ephraim E. Urbach, "the spellings גורניש ,"גורניץ" ('goornitz', 'goornish'), together with the contents of the works, led me to find in them the Spanish word 'guarnicio', sometimes also spelled as 'gornicio' 'gurnisio', 'gornisio', meaning to fortify, to guard in order to protect, and to beware of something" (see Urbach 1986, II, p. 770). See also Breuer 2003, p. 187, n. 92 who tends to think that it should be vocalized גֻרניש like 'in German G[e]warnis, which is very close in meaning to 'guarding' (cf. Kupfer 1976). The 'balbelanim' scholars attacked by A. E. Cohen appear to have been casuists the type of which we find among the 'Tosafists of Goornish'. From his words we learn that the 'balbelanim' considered themselves experts in the identification of misleading arguments. However, there might be a difference between the attack of the author of *Shaʿarei Ṣedek* against casuistry and that of Cohen. The author of *Shaʿarei Ṣedek* attacked the very casuistic method of interpretation. A. E. Cohen, on the other hand, confessed to the basic premises of the casuistic method, according to which the ability to differentiate between true and misleading arguments should be a dominant tool in shaping the ways in which the Torah is studied. Cohen's disagreement with the casuists concerned the ways and the means to achieve this, that is, it touched upon the question how deep one's logical training should be in order to be able to identify fallacies and accordingly to interpret the Talmud correctly.

> I shall therefore further explain this to you by way of examples that will clarify my intention to you concerning these matters, and from them you will learn how to distinguish between a true *a fortiori* argument (קל וחומר אמיתי) and a false *a fortiori* argument (קל וחומר מטעה), and between an *even more so* argument (קל וחומר של לא כל שכן) and an *all the more so* argument (קל וחומר של אחת כמה וכמה), and between an *a fortiori* argument and an argument of *we hear from it* (ממילא משמע) that are accustomed in the language of the Talmud, if God will help you understand these notions, which you will find explicitly stated in the Torah, none of them is crooked or twisted. (Ms. Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. Heb. 37, ff.75r-v).

Cohen's commentary helps the student training his ability to identify sound and false *a fortiori* arguments, as the author explains to him the logical errors characterizing false arguments. First, he demonstrates a valid *a fortiori* argument (the first example below), and then he presents different kinds of fallacies (examples 2-5). He writes as follows[26]:

[1] A sound *a fortiori* argument (קל וחומר אמתי):

A	B
If an ox which is stronger than an ass	is not stronger than a man
C	
Then a cat which is not stronger than an ass	is obviously not stronger than a man.

The author explains the diagram:

There are four elements in this *a fortiori* argument: man, and ox, and ass, [and cat]. We already know that an ox is stronger than an ass, let us say that we observe this empirically. Also the second premise is a generally accepted opinion among us, let us say that we observed a man and an ox fighting and [it was seen that] an ox is not stronger than a man. Also the third premise is a generally accepted opinion among us, for we saw a cat fighting with an ass and [it was seen that] it is not stronger than it. From these three premises we would like to conclude that the cat is not stronger than the man – although we have never seen them fighting with one another. And do not let these words of mine surprise you, since I have presented to you an easy understandable parable in order to clarify my intentions to you, and you will think they are self-evident. But it is not really as you think. Because in accordance with the regular way of learning you will think that the *a fortiori* argument that I will present to you is true, but this is not really so, although this is what the sophists of our nation (המתחכמים באומתנו), the casuists (הבלבלנים) who think themselves to know the Torah and its ways, although they are far from it like the sky is far from the earth and the East from the West.

26. The graphic arrangement of the arguments reflects their arrangement in the manuscript itself.

[2] And the following is an *a fortiori* argument that seems sound in the eyes of the masses (להמון) but is in fact a false argument:

If man who is able to kill a mole	is unable to kill a lion
A mole who is unable to kill a man	can obviously not kill a lion

And the author explains the logical mistake inherent in this argument:

This [argument] is obviously false, since there are only three elements in it, as will be demonstrated below: A man is unable to kill a lion even though he is able to kill a mole, because he kills it [i.e. the mole] by force and not by cunning. But a mole, who uses cunning, is [nevertheless] able to kill a lion. This is so because it kills it through cunning and not through force. It gets on it and enters its ear, and from there it kills it.

[3] Behold, also the following [argument] is false although it will appear true to you, despite the fact that it cannot be refuted:

If a sack which is big	is found in the room
A fish which is not big	is obviously found within the room.

This *a fortiori* argument cannot be refuted but is nevertheless false, because it is not really an *a fortiori* argument (Vat. Heb. 37, f. 75v).

One can think of two solutions to this fallacy. The first one is that it ignores the equivocality of the words: "is found in the room": It is actually found in the room (the sack), and: It is possible for it to be found in the room *vis-à-vis* its size (the fish). The second one is that being in a room is not just a matter of size but of other characteristics as well. A sack can actually be found in a room, but a living fish cannot survive in a room outside the water[27].

[4] A similar false example:

If a basket containing a fish	is found within the room
The fish which is in the basket	is obviously found within the room

Also, this [argument] cannot be refuted but is nevertheless false, because the fish that is in the basket is not within the room, rather it is in the basket and the basket is within the room (Ms. Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. Heb. 37, ff. 75v-76r).

Here the fallacy is based on the meaning of "being in the room" – it could mean that the fish, like the basket, is in the room directly, but it could also mean that the fish is in the room only indirectly (since it is in the basket, which is in the room).

27. Nevertheless, later on in his discussion Cohen adds that the mistake in this argument is in its structure, since if the first premise in a sound *a fortiori* argument is positive, all the rest must be negative. Hence, says Cohen, even without a refutation of its meaning, this *a fortiori* argument must be false.

[5] Likewise, the universal concepts of physics are subject to debate between the logicians, for example:

If [the category of] living beings,	is included in [the category of]
which includes man,	substance,
Then man, who is included in	is obviously included in
[the category of] living beings,	[the category of] substance.

This is also false inasmuch as [the category of] living beings is included in [the category of] substance, since substance is the proximate genus of it. But [the category of] man is not included in [the category of] substance without mediation; hence, the fallacy lies in the phrase 'is in', which refers to unmediated inclusion.

In contrast, the claim "since man is a living being, and living beings are substances, it follows that man is a substance" – is true. However, this is not an *a fortiori* argument, it only [teaches about] the logical predications and the consequence [that follows from] their structure. Indeed, we do not define this [kind of argument] as an *a fortiori* argument; rather, when such a matter is clarified in talmudic terminology, the terms used are "we hear from it" (ממילא משמע) or "it is plain" (פשיטא) (Vat. Heb. 37, ff. 75v-76r).

Following this list of fallacies, the author explains the principles of the *a fortiori* argument, providing the reader with further examples of how one should be cautious not to formulate the arguments in a false way. He writes as follows:

> [6] You must thoroughly observe how an *a fortiori* argument is formed, so that you do not repeat your words in order to clarify what you have already said, as this is not an *a fortiori* argument. For example, if you argue as follows:
>
> | | |
> |---|---|
> | If an ox, which is stronger than an ass, | is not stronger than a man |
> | Then a man, who is stronger than an ox, | is it not more so that he is stronger than an ass?! |
>
> Behold, already in the second premise you made clear that an ox is not as strong as a man, meaning that if the bull is equal to him or smaller than him, then the man is stronger than it, and there is therefore no need for a third premise. And if he [the man] is stronger than it [the ox] – then it is immediately implied, once you articulated it in your speech, that he is stronger than the ass, and there is no need for any additional inference. But then its value is exactly that of propositions with immediate implications, of which we already spoke. (Ms. Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. Heb. 37, ff. 76r-v).

According to Cohen, in this example the structure of the argument is erroneous, as the second and third premises teach exactly the same, hence one of them is superfluous. The conclusion of the argument is, of course, true, but it cannot be considered an *a fortiori* argument.

I will conclude the discussion with A. E. Cohen's final two examples of fallacies. In the first example (example #7), the predicates in the premises do not be-

long to the same category, while in the second example (example #8), the author distinguishes between an argument the conclusion of which is immediately and obviously implied in its premises, and an argument in which the conclusion is reached through a deduction that relies on the premises:

> [7] One must always argue according to the following rule: The *a fortiori* argument should always relate to what is affirmed or negated in the first premise. If the premise presents a quantitative affirmation, then the conclusion must present a quantitative negation. And if the premise presents a qualitative negation, then the assertion of the conclusion must also be qualitative. Do never confuse different categories of the ten categories that are clarified in logic, because if you do so your conclusion will be false even though the premises are true. And even though you have presented the premises and their content in an orderly fashion, with no [formal] refutation you will easily be able to judge whether the concluding [process] is invalid or valid, that is, when the conclusion and the premises do not belong to the same category, and your words are pure nonsense.
> An example of this:
> If a man, who is intelligent, is unable to live at the depth of the sea
> Then a whale, which is not small, is obviously unable to live at the depth of the sea. (Ms. Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. Heb. 37, f. 76v).

In the margins, the following addition is found:

Behold, the first premise affirms that human beings are wise, as wisdom belongs to the category of quality, but then in the third premise you said: "A whale, which is not small", which relates to the category of position. Thus, even if you have fulfilled the rest of the mentioned conditions, you have achieved nothing but lies and falsehood and the like. However, this lie is inherent, because this is not an *a fortiori* argument [...] as the premise and the conclusion do not belong to the same category (Ms. Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. Heb. 37, 78r-v).

> [8] What remains for us is to discuss and expand a little on the talmudic terms משמע ממילא ('we hear from it [by its mere expression]') or פשיטא ('it is plain'). This matter should be examined, for if we claim: 'Man is a living being, and living beings are substances', then we immediately[28] draw the conclusion that 'man is a substance'. Hence, when you assert that 'man is a living being', it also appears that 'living beings are men', which is like saying that 'the coin is money', since coins and money are not separate matters, and the mind immediately compares between them so that they become identical, to the extent that if you say 'money' it will be understood as

28. Literally: "the intellect runs immediately". Cf. the way Thomas Aquinas describes the act of inferring: "Tertius vero actus rationis est [...] discurrere ab uno in aliud". Cf. Thomas de Aquino *In Aristotelis Libros Posteriorum Analyticorum*, ed. Spiazzi, p. 147 (transl. Larcher 1970, p. 1-2).

> meaning 'coin'. Likewise, if you claim that a certain class is more inclusive than another, for example, 'man is included in living beings, and living beings are included in substances', immediately the mind concludes that 'men are included in substances'. But this is not arranged in such a way that one can possibly formulate it as an *a fortiori* argument, neither is it a syllogism in the eyes of a logician, because in this way there is no need for syllogism. And even if it was not understood otherwise, the syllogism that one would inferred from it would not be true. This is similar to the example we brought above:[29] The fish is in the sack, and the sack is in the room [in the margins is added: This does not mean that the fish is in the room; rather, the fish is in the sack, and the sack is in the room]. But according to the rules of logic, if you say that 'every man is a living being', it is understood that there is no part of the essence of living beings that is not found in man also, and there is nothing in the essence of substances that is not found in living beings, hence it follows that 'every man is a substance'. In this way, the mind does not immediately grasp the conclusion, and there is also no way of refuting it, hence the inference has made you understand something that you didn't already know. (Ms. Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. Heb. 37, f. 78v).

In this example, the author explains that the inference "man is a living being, and living beings are substances, hence man is a substance" can be understood in different ways. One way is that the propositions identify the subject with the predicate: "Man is a living being" – to be a man means to be a living being. That is to say, men and living beings are identical realities. Likewise, the meaning of the proposition "living beings are substances" is that living beings and substances are identical. Cohen claims that according to this understanding, the mind immediately grasps that to be a man means to be a substance.

The second way of understanding is that the propositions convey a structuralistic meaning: "Man is a living being" means that structurally the category of men is included in the category of living beings; and "living beings are substances" means that structurally the category of living beings is included in the category of substances. Cohen claims that also according to this understanding, the mind immediately grasps that structurally the category of men is included in the category of substances. However, Cohen adds, such an understanding constitutes neither an *a fortiori* argument, nor a syllogism, because the argument is misleading, just like in example [4] above. While the category of living beings is directly included in the category of substances, the category of men is only indirectly included in the category of substances. This means that the chain of propositions is misleading.

The third way of understanding the argument according to Cohen is not entirely clear, but his intention appears to be as follows: The proposition "men are living beings" means that living beings do not possess essential traits that are not found in men also. Likewise, the proposition "living beings are substances" means

29. See example [4] above.

that substances do not possess any essential traits that are not found in living beings also. And from here it follows that the conclusion "men are substances" means that substances do not possess any essential traits that are not found in men also. It is possible that Cohen means to convey the idea that in order to understand the three propositions as a syllogism, they must be explained in accordance with *Dictum de Omni*. According to such an understanding, says Cohen, the premises do not cause the mind to grasp the conclusion in an immediate way, hence a valid syllogism is formed and not a false argument.

It is highly possible that this rabbinical commentary was written under the influence of the growing preoccupation with the doctrines of Aristotle's *Sophistical Refutations* among the Christian schoolmen of the 12th century[30]. Likewise, Cohen's attention to semantics (i.e. the subtle differences between seemingly similar talmudic expressions) and his application of fallacies in order to teach talmudic principles, may have been influenced by the prominent attention to semantics and the use of *sophismata* as a method of analyzing philosophical issues in scholastic logic.

As has been demonstrated by Mauro Zonta, fourteenth century scholastic philosophy influenced Jewish thinkers mainly through oral discussions[31]. At least one clear piece of evidence of the impact that scholastic interest in fallacies exerted on Jewish logicians in the fourteenth century through oral encounters is known to us. The fourteenth century Jewish Provencal scholar R. Moshe Narboni, in his commentary on Al-Ghazali's *The Intentions of the Philosophers*, informs us as follows[32]:

> "I would like to demonstrate here three fallacies on which a great Roman sage (חכם מופלא מן הרומיים) has tested me, and I will explain their solution in accordance with what he taught me. He said to me: You have eaten what you bought; you bought a living fish; from here it follows that you ate a living fish. Behold, the two premises are true, but the conclusion is false. The solution is that 'what' refers to the substance, whereas 'living' is a quality [...]".

In this case, the encounter with the scholastic discipline of fallacies was oral, and Jewish scholars like R. Abraham E. Cohen may also have engaged in philosophical discourses with schoolmen on logical issues[33].

30. Kneale / Kneale 1962, pp. 226-227; Ebbesen 1987, pp. 110-112.

31. Zonta 2006, pp. 9-10.

32. Edition in Chertoff 1952, II, pp. 99-100.

33. R. Kalonymos ben Kalonymos, in his criticism of his contemporaries, described those who boasted about their logical education as priding themselves in their ability to construct fallacies: 'And if he has studied an entire page on the art of logic, including what is written in the margins, he will let his voice be heard publicly, saying: "I am an expert in the ways of logic, I have understood the modes of הטעמה (lit. 'emphasis' – it should however be read ההטעאה [fallacy], and indeed in the edition of Michael Wolf (J.M. Stand, Lemberg 1865), p. 42, we find the version ההטעה), and the thoughts of the cunning. I am able to permit what is forbidden and to prohibit what is permitted many times, I know how to purify a creeping animal in three hundred ways, through convincing inferences. I will declare the [ritually] unclean to be clean and welcome it [lit. I will explain its faces, or: modes]"

To conclude: The work of the fourteenth century Jewish scholar R. Abraham E. Cohen on talmudic methodology can be considered as a work that introduces the prevalent preoccupation with logic among contemporary scholastic logicians into the framework of talmudic study. In this sense, the little-known but extremely interesting treatise of Cohen connects between the two topics examined in this paper – the place of fallacies in rabbinic literature, and their application in medieval Jewish philosophy.

These two currents testify to the developed critical sense that characterized Jewish scholars in the talmudic and medieval periods, and to a prevalent recurring motif in Jewish literature – testing the truth of arguments and identifying fallacies. The halakhic literature and the Jewish philosophical literature shared some common aspects of tradition and continuity, but at the same time they were both far from being dogmatic. Scholars of Jewish law and Jewish philosophy possessed a developed critical sense and knew well that even the arguments of great and important scholars could actually be false.

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(Haberman 1956, p. 42). From here it appears that logic was conceived as a tool to be applied mainly in the realm of fallacies. A broad application of this realm is found in *Sefer Taʿalumot Ḥokhmah* (Mysteries of Wisdom) by R. Moshe ben Yehuda Galiano. The author was the disciple of R. Eliyahu Mizraḥi, who lived in sixteenth century Constantinople. This book is a collection of examples of different kinds of fallacies known from Aristotelian logic. The examples relate to the entire complex of the author's broad education, both theoretical knowledge like medicine, astronomy, physics, and metaphysics, and historical and practical knowledge like events and facts of life. The work is replete with logical terms, attempting to examine all these examples in the light of the Aristotelian doctrine of fallacies. In certain places, it even appears that the work strives to exemplify the practical as well as the theoretical value of the doctrine of fallacies – it may benefit the sciences as well as life itself. On this work, see David 1995.

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Abstract: Jewish scholars' interest in fallacies spans across the main disciplines of Jewish mediaeval literature (Law, Ethics, Theology, Science). The paper will focus on the rabbinical critical appraisal of flawed or misleading arguments, with special reference to fourteenth and fifteenth century treatises, amongst which a hitherto unpublished text. Abraham Eliyahu Cohen's commentary on the thirteen principles of Talmudic inference is ripe with methodological concerns about sophistical deception and, in its effort to sort out valid and invalid patterns of argumentation, displays remarkable logical insights and terminology highly reminiscent of the Latin logical tradition.

Keywords: Talmudic logic; fallacies; Abraham Eliyahu Cohen.

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Are Zeno's Paradoxes of Motion Fallacies? Evidence from the Hebrew Aristotelian Logical Tradition

I was unable to attend in person the conference on fallacies in Lille in May 2021, at which the papers in this volume were presented. In order for me to have come to Lille, I would have had to come half way to Lille first. But in order to do that, I would have had to come half way to half way to Lille. And in order to do that, I would have had to come half way to half way to half way to Lille and so on *ad infinitum*. My absence in person from the conference can thus be explained by appeal to Zeno's problem of dividing motion into halves, outlined, e.g., in Aristotle's *Physica* 239b9-14. This problem can be understood to prove that all motion over divisible space is impossible, since any divisible space can be divided into infinitely many halves. If motion is impossible, then my absence from Lille is easily explained[1]. This excuse is not unlike the argument we find at Aristotle's *De Sophisticis Elenchis* 172a8-9 that 'one could deny that walking about after a meal is rather good, because of Zeno's argument'[2]. Such a claim, says Aristotle, is 'not doctorly' (οὐκ ἰατρικός), thereby giving us the impression that the contentious quarreler (ὁ ἐριστικὸς) here is recommending against exercising after dinner on the grounds that motion is impossible. While I will not deny having viewed this claim with some sympathy after some of my larger dinners, on the whole I would have to agree with Aristotle that it is too universal (κοινὸς) for the argument in question[3]. The same can be said of my excuse for not coming to Lille: denying all motion to

1. Zoom provided a modern-day solution to Zeno's paradox, allowing the conference to take place amid the Covid-19 outbreak. I thank the organizers for putting together a wonderful and stimulating conference even in the face of great obstacles.

2. τις μὴ φαίη βέλτιον εἶναι ἀπὸ δείπνου περιπατεῖν διὰ τὸν Ζήνωνος λόγον. Aristotle's Greek text is taken from W.D. Ross 1958 edition. Unless otherwise noted, all translations are my own.

3. The Greek here does not, in fact, contain the comparative, 'too', but has only κοινὸς γάρ, "for it is universal". My addition of 'too' is based on the implication of the statement's context that the suitable argument here would be more particular. We shall see below that Averroes gives much more detailed criteria for precisely what kind of particular argument should be made in this context.

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explain my absence would work, but it involves making an excessively universal claim. While excessive universality is improper under certain circumstances, it is hardly fallacious reasoning in itself. What makes it improper is not that it is contrary to reason, but that it does not fit in the context of the contentious argument.

Aristotle's proper arguments against Zeno's paradoxes are, in fact, to be found at *Physica* VI. His mentions of the paradoxes in *De Sophisticis Elenchis* and *Topica* do not explain why Zeno's reasoning is not correct, but apparently refer the reader to the *Physica* for the full treatment. Medieval and Renaissance Hebrew treatments of Zeno's arguments in commentaries and super-commentaries on Aristotle, however, argue for the fallaciousness of Zeno's arguments in the context of the *Topica* and *De Sophisticis Elenchis* without directly referring the reader to the *Physics* in those works. This is because the authors of the Hebrew works view Zeno's arguments as properly dialectic and accordingly treat their refutation in works dedicated to dialectic. This shift in the context of the arguments is present in the primary sources for the study of the *Topica* and *De Sophisticis Elenchis* in Hebrew: Al-Fārābī's *Art of Dialectic* and *Sophistical Refutations* and Averroes' Middle Commentaries on Aristotle's *Topica* and *De Sophisticis Elenchis*. Aristotle's works, *Topica* and *De Sophisticis Elenchis*, have to this day never been translated into Hebrew. I shall argue that Al-Fārābī and Averroes respond to Zeno in detail in these commentaries on dialectic and sophistic because the paradoxes of infinity associated with Zeno were used by the Muslim *Kalām* as important parts of the arguments for atomism. This will also explain why the four paradoxes of motion are more or less assimilated into the Paradox of the Stadium, which Al-Fārābī calls "The Question of the Halves"[4]. The Hebrew treatments of Zeno are, as we shall see, heavily indebted to Al-Fārābī and Averroes.

In what follows, we shall first turn to Aristotle's presentation of Zeno in the *Physica*, *Topica* and *De Sophisticis Elenchis*, before examining how Al-Fārābī and Averroes differ in their approach from Aristotle. We shall focus on the Hebrew translations of these works and then turn to the Hebrew commentary tradition and how it viewed these works in the absence of serious engagement with *Kalām* atomism among Hebrew thinkers in Southern Europe in the 12th-16th centuries.

1. Aristotle

Aristotle briefly discusses the paradoxes once in the *Topica* and twice in the *De Sophisticis Elenchis*[5]. We have already seen that in one of these places (*De Sophisticis Elenchis* 172a8-9), Aristotle does not argue the fallaciousness of the claim that mo-

4. For an overview of Zeno's paradoxes, and their continued importance for philosophy and mathematics to this very day, see Huggett 2019.

5. See *Topica*, 160b6-10 (below), *De Sophisticis Elenchis* 172a8-9 (mentioned above) and 179b20-

tion is impossible, but only that the argument in one specific case is too universal for its context. In the short mention of Zeno at *Topica* 160b, Aristotle discusses raising objections (ἔχων ἔνστασιν) and making counterarguments (ἀντεπιχειρεῖν) to universal propositions. Upholding universal propositions without objection is, according to Aristotle, to act peevishly (δυσκολαίνειν), but one would seem to be even more peevish to reject universal propositions without raising objections and counterarguments. Aristotle continues:

> "Yet this is not enough. We have many arguments that are contrary to accepted opinions and which are difficult to solve, like Zeno's argument that motion is impossible or that the stadium cannot be traversed. But it is not the case that we should not accept the [arguments] opposite to these on this account"[6].

That is, the addressee of Aristotle's *Topica* is not expected to be able to solve Zeno's argument, but he does not have to accept the impossibility of motion. Arguments of this kind, which Aristotle calls difficult (χαλεπός), are an apparent counterexample to the peevishness normally incurred by one who cannot answer objections or bring counter-arguments. Clearly, Aristotle would not have us admit Zeno's argument against motion, but showing its fallaciousness is beyond the scope of the dialectical argumentation discussed in the *Topica*. That Zeno's arguments are contrary to accepted opinions may also play a part in Aristotle's not giving full arguments in the *Topica*. Indeed, the audience of the *Topica* would appear to be expected to include both that motion exists and that the stadium can be traversed among their accepted opinions (δόξαι). In this case, there is no need to refute views contrary (ἐναντίοι) to them in the *Topica*.

Aristotle's second mention of Zeno's paradox of motion in the *De Sophisticis Elenchis* (at 182b26) is similar in that it too offers no obvious logical solution to the paradox. Instead, it would seem to imply that the reader and the interlocutor with sophists would not be able to expose the false deduction made by Zeno or his followers and accordingly would not have a true refutation of Zeno's argument against motion. Aristotle says:

> "There is nothing to prevent the same argument from having a number of flaws; but it is not the exposition of any flaw that constitutes a solution; for it is possible for a man to prove that a false conclusion has been deduced, but not to prove on what it depends, e.g. in the case of Zeno's argument to prove that motion is impossible. So that even if anyone were to try to establish that this is impossible, he still is

24. Aristotle also refers to Zeno at *De Sophisticis Elenchis*, 182b26, but this concerns the paradox of being, viz. Zeno's claim that all being is one.

6. *Topica*, 160b6-10 (Pickard-Cambridge 1984a's translation): καίτοι οὐδὲ τοῦθ' ἱκανόν· πολλοὺς γὰρ λόγους ἔχομεν ἐναντίους ταῖς δόξαις, οὓς χαλεπὸν λύειν, καθάπερ τὸν Ζήνωνος ὅτι οὐκ ἐνδέχεται κινεῖσθαι οὐδὲ τὸ στάδιον διελθεῖν, ἀλλ' οὐ διὰ τοῦτο τἀντικείμενα τούτοις οὐ θετέον.

> mistaken, even if he has deduced it ten thousand times over. For this is no solution; for a solution is an exposition of a false deduction, showing on what its falsity depends. If then he has not made a deduction, whether he is trying to establish a true proposition or a false one, to point this out is a solution"[7].

This is not to say that Zeno's arguments were not fallacious, but only that the reader of the *De Sophisticis Elenchis* is not expected to be able to show on what their falsity depends.

Indeed, in his main discussion of Zeno's paradoxes of motion in *Physica* VI Aristotle explicitly says both ὁ Ζήνωνος λόγος ψεῦδος λαμβάνει, 'Zeno's argument takes up something false' (233a21-22) and Ζήνων δὲ παραλογίζεται· 'Zeno argues fallaciously' (239b5). Aristotle explains Zeno fallacious reasoning in both chapters 2 and 9 of *Physica* VI, but it is the first account that addresses in greatest detail what later came to be called 'the Question of the Halves'. Aristotle's explanation is:

> "It is impossible for a thing to pass over or severally to come in contact with infinite things in a finite time. For there are two senses in which length and time and generally anything continuous are called 'infinite': they are called so either in respect of divisibility or in respect of their extremities. So while a thing in a finite time cannot come in contact with things quantitatively infinite, it can come in contact with things infinite in respect of divisibility: for in this sense the time itself is also infinite: and so we find that the time occupied by the passage over the infinite is not a finite but an infinite time, and the contact with the infinites is made by means of moments not finite but infinite in number"[8].

This explanation relies on a distinction between different types of continuity: continuity in divisibility, i.e. divisibility into things always divisible[9], and continuity in extremities, i.e. when the extremities touch and are one[10]. It would, indeed, take an infinite amount of time to pass over an infinite set of things whose extremities touch and are one, which Aristotle here calls 'quantitatively infinite'.

7. *De Sophisticis Elenchis*, 179b17-26 (Pickard-Cambridge 1984b's translation). οὐδὲν δὲ κωλύει τὸν αὐτὸν λόγον πλείους μοχθηρίας ἔχειν, ἀλλ' οὐχ ἡ πάσης μοχθηρίας ἐμφάνισις λύσις ἐστίν· ἐγγωρεῖ γὰρ ὅτι μὲν ψεῦδος συλλελόγισται δεῖξαί τινα, παρ' ὃ δὲ μὴ δεῖξαι, οἷον τὸν Ζήνωνος λόγον, ὅτι οὐκ ἔστι κινηθῆναι. ὥστε καὶ εἴ τις ἐπιχειρεῖ συνάγειν ὡς δυνατόν, ἁμαρτάνει, κἂν [εἰ] μυριάκις ᾖ συλλελογισμένος· οὐ γάρ ἐστιν αὕτη λύσις· ἦν γὰρ ἡ λύσις ἐμφάνισις ψευδοῦς συλλογισμοῦ παρ' ὃ ψευδής. εἰ οὖν μὴ συλλελόγισται, †εἰ καὶ ἀληθὲς ἢ ψεῦδος† ἐπιχειρεῖ συνάγειν, ἡ ἐκείνου δήλωσις λύσις ἐστίν.

8. *Physica*, 233a21-31 (Hardie / Gaye 1984's translation): τὸ μὴ ἐνδέχεσθαι τὰ ἄπειρα διελθεῖν ἢ ἅψασθαι τῶν ἀπείρων καθ' ἕκαστον ἐν πεπερασμένῳ χρόνῳ. διχῶς γὰρ λέγεται καὶ τὸ μῆκος καὶ ὁ χρόνος ἄπειρον, καὶ ὅλως πᾶν τὸ συνεχές, ἤτοι κατὰ διαίρεσιν ἢ τοῖς ἐσχάτοις. τῶν μὲν οὖν κατὰ τὸ ποσὸν ἀπείρων οὐκ ἐνδέχεται ἅψασθαι ἐν πεπερασμένῳ χρόνῳ, τῶν δὲ κατὰ διαίρεσιν ἐνδέχεται· καὶ γὰρ αὐτὸς ὁ χρόνος οὕτως ἄπειρος. ὥστε ἐν τῷ ἀπείρῳ καὶ οὐκ ἐν τῷ πεπερασμένῳ συμβαίνει διιέναι τὸ ἄπειρον, καὶ ἅπτεσθαι τῶν ἀπείρων τοῖς ἀπείροις, οὐ τοῖς πεπερασμένοις.

9. See, e.g., *Physica* 232b24-25: λέγω δὲ συνεχὲς τὸ διαιρετὸν εἰς αἰεὶ διαιρετά. Cf. 231b15.

10. See, e.g., *Physica* 227a10: λέγω δ' εἶναι συνεχὲς ὅταν ταὐτὸ γένηται καὶ ἓν τὸ ἑκατέρου πέρας οἷς ἅπτονται, καὶ ὥσπερ σημαίνει τοὔνομα, συνέχηται.

Yet, says Aristotle, that which is infinitely divisible does not require an infinite amount of time to pass over. This is probably a consequence of Aristotle's earlier claim that infinite divisibility exists as a potential that cannot be actualized all at once[11]. Time, too, Aristotle points out, is infinitely continuous in divisibility and its infinite divisions which exist potentially could be taken to correspond to the potentially existing infinite divisions of space[12]. Zeno's fallacy, then, is of a kind discussed repeatedly in the *Topica* and *De Sophisticis Elenchis*: when a term, in this case 'infinite' or 'infinitely continuous', is said in many ways, it can be misunderstood according to the wrong meaning. Zeno understands it one way, when in fact he should have understood it in another.

Still, Aristotle does not address Zeno's paradoxes of the stadium, of the halves, and of motion in general in any detail in dialectic or sophistical refutations, but rather in physics. Undoubtedly, Aristotle does this because he does not want to include extensive discussions of infinity, continuity, or divisibility in the *Topica* and *De Sophisticis Elenchis*. Indeed, when it comes to equivocal terms, Aristotle prefers to discuss terms like 'sharp' (ὀξύς), whose meaning in music is easily distinguished from its meaning in describing objects[13]. The background required to disambiguate the terms of *Physics* VI takes much longer to explain. Indeed, Aristotle may also be using his refutation of Zeno as a pedagogical opening to encourage his readers to turn to the *Physics* in order to grasp those complicated concepts. It is also possible that Aristotle does not think that the readers of the *Topica* and *De Sophisticis Elenchis* and perhaps regular Greeks in general include the denial of motion or the other issues Zeno raised among their δόξαι; accordingly, there is no great need to refute these claims in the dialectical works[14].

Nevertheless, in the Arabic Aristotelian tradition and consequently in the Hebrew Aristotelian tradition, Zeno's paradoxes were, in fact, given serious treatment in commentaries and super-commentaries on Aristotle's *Topica* and *De Sophisticis Elenchis*. In what follows, I shall trace in outline how Zeno's paradoxes came to be seen as part of dialectic and sophistic through the commentaries of Al-Fārābī and Averroes and then Hebrew commentaries on those works. This move often led to simplifying the paradoxes or assimilating them into what is often known as the paradox of the stadium. Further, I shall argue that for Al-Fārābī and Averroes, this change served to remove dialectical considerations, especially those of Kalamic atomists, from physics. Jewish commentators later followed this trend, thereby perpetuating the distinction between Aristotelian science and religious dialectical argumentation.

11. See *Physica* 206a14-b3. See, e.g., Bostock 1972 and Hintikka 1966.

12. For a recent comprehensive and clear account of these issues see Sattler 2020, pp. 277-334.

13. See, e.g., *Topica* 106a12-14.

14. Sattler 2020, pp. 124-175 argues that even Zeno himself did not believe that motion is impossible, but only that it is beyond human knowledge. Indeed, she argues, Zeno joins other Eleatics in maintaining that physics cannot properly be known by man.

2. Al-Fārābī

Al-Fārābī discusses Zeno's paradox(es) of motion twice in his commentary on Aristotle's *Topica*, the *Book of Dialectic* (*Kitāb al-jadal*)[15] and once in his commentary on the *De Sophisticis Elenchis*, the *Book of Refuting the Misleading* (*Kitāb al-ʾamkana al-mughalaṭa*)[16]. The first discussion in the *Book of Dialectic* occurs within an account of the benefits of dialectic for philosophy (paragraph 20) and the second discussion echoes the conclusions of the first (paragraph 88). For Al-Fārābī, the first benefit (نافع) of dialectic for the study of philosophy is primarily in testing traditionally received opinions, i.e. those opinions one encounters first and which are inculcated through education or acculturation (أدب) and habituation (عود); these opinions would apparently include religious views. Such tests, says Al-Fārābī, are not possible without 'opposition' (عناد), which in turn is not possible without the art of dialectic (صناعة الجدل). Indeed, Al-Fārābī dedicates a large part of his *Kitāb al-jadal* to describing how in debates questioner and respondent put forward various opposing views and arguments whose resolution brings them closer to practicing philosophy. Still, rather than bring an example of a test of traditionally received opinions, Al-Fārābī brings the example of Zeno's paradox, which shows, he says, that dialectical arguments can even bring people to become skeptical (حمل ... على الاسترابة) about the sensibles (بالمحسوسات). Indeed, Al-Fārābī attributes knowledge of three types of propositions to a beginner who has not yet studied philosophy: widely held opinions (الآراء المشهورة), traditionally received opinions (الآراء المقبولة), and sensible opinions (الآراء المحسوسة). The implication is that if dialectical arguments such as Zeno's paradox can make one skeptical of sensible opinions (e.g. that motion exists), then they can also make one skeptical of widely-held opinions and traditionally received opinions. The resolution of such dialectical difficulties is accordingly the first task of dialectic[17].

15. An edition and French translation of Al-Fārābī's *Book of Dialectic* is in D. Mallet's unpublished 1992 doctoral thesis (Mallet 1992). A recent English translation of the work can be found in Di Pasquale 2019. Di Pasquale's translation lists the page and paragraph numbers of Mallet's edition. For the anonymous, probably twelfth century, Hebrew translation of Book I of Al-Fārābī's *Book of Dialectic* see Y. Halper and G. Weber's 2022 edition, which also follows the paragraphing of Mallet's edition. Citations from Al-Farabi's commentary will accordingly be to paragraphs of Mallet's edition.

16. An edition of Al-Fārābī's *Kitāb al-ʾamkana al-mughalaṭa* is in R. Al-ʿAjam's 1987 edition of Abū Naṣr Al-Fārābī's *Al-Manṭiq ʿinda al- Fārābī* (vol. ii, p. 131-164). An anonymous medieval Hebrew translation of this text, probably from the thirteenth century, is extant in at least six manuscripts: mss. München, Bayerische Staatsbibliothek, hebr. 110, ff. 219r-223v and hebr. 244, ff. 213r-222r; ms. Wien, Österreichische Nationalbibliothek, hebr. 53, ff. 27v-40v (second pagination); ms. Paris, Bibliothèque Nationale, héb. 929, ff. 227v-241r; ms. Parma, Biblioteca Palatina, Parm. 2761, ff. 118v-131r; ms. Jena, Universitätsbibliothek, Rec. adj. f. 10, ff. 45v-53r.

17. Abū Naṣr Al-Fārābī, *Kitāb al-jadal*, p. 52: ويحمله ذلك على أن يمتحن الآراء المقبولة التي كان لقيها أوّلا وأدّبها وعوّدها، حتّى أنّه ربّما حمل كثيرا من الناس في كثير من الأوقات على الاسترابة بالمحسوسات وامتحانها كما عرض لبرمنيدس ولزيئن (= ولزنين) إلى أن قالا في الحركة إنّها غير موجودة وإنّ الكثرة غير موجودة وإنّ الموجود واحد ورأوا أن يتبعوا ما توجبه المشهورات التي كانت هي المعقولات عندهم وأن يستريبوا بالمحسوس إذ كانت المعقولات أخصّ بالإنسان من المحسوسات.

In the *Book of Refuting the Misleading*, Al-Fārābī says the following:

> "Zeno's doubts about motion … include the question of the halves: it is known that one who transverses some distance has crossed half of that distance prior to crossing the entire distance and has crossed half of that half prior to crossing the entire half. Now, when a body is infinitely divisible into halves, it necessarily follows that the one in motion crossed an infinite distance in an infinite amount of time. But this is false. Now this necessarily follows because a distance is infinite in one of two ways: in length or in division. Accordingly, it is not possible for one to cross a distance that is infinite in length in a time that is finite in length. Nor is it possible for one to cross a distance that is finite in length in a time that is infinite in length. Nor is it possible for one to cross a distance that is infinite in division in a time that is infinite in division. Or vice versa. And since he took a distance that is infinite in division and a time that is finite in length he was led to error and imagined there to be an infinite time because of the infinite distance"[18].

This argument, which focuses on the distinction between infinite in division and infinite in length, is clearly based on Aristotle's *Physics* 233a21-31, which I quoted in full above. Al-Fārābī's treatment, though, avoids the concept of continuity altogether. Instead, it concentrates on elucidating cases of crossing a distance[19] in time, based on whether the distance or time is finite, infinite in division, or infinite in length. The latter is apparently equivalent to Aristotle's 'quantitatively infinite'. Al-Fārābī singles out three cases which are impossible, but maintains that it *is* possible to cross a distance that is infinite in division in a time that is finite in length, even though that time is infinite in division. Zeno apparently did not see this because he was misled by the ambiguity of the infinite and so conflated infinite in division and infinite in length. That is, Zeno fell pray to the fallacy of ambiguity and thus judged something impossible that was in fact completely possible.

18. Abū Naṣr Al-Fārābī, *Al-Manṭiq ʿinda al- Fārābī*, vol. ii, p. 148: وشكوك زنين في الحركة ... منها مسألة الإنصاف. وهو أن المنتقل إذا قطع مسافة ما فظاهر أنه قطع نصف تلك المسافة قبل أن يقطعها وأنه قطع نصف ذلك النصف قبل أن يقطع تمام نصفها. وإذا كان الجسم ينقسم أنصافاً غير متناهية لزم أن يكون المتحرك قطع مسافة غير متناهية في زمان متناه, وذلك محال. وإنما لزم من قبل أن المسافة تكون غير متناهية بإحدى جهتين إما في الطول وإما في القسمة. وكذلك الزمان والمتحرك لا يمكن أن يقطع مسافة غير متناهية في الطول في زمان متناه في الطول, ولا أن يقطع مسافة متناهية في الطول في زمان غير متناه في الطول, ولا أن يقطع مسافة غير متناهية بالقسمة في زمان متناه بالقسمة. وكذلك بالعكس, ولمّا أخذ المسافة غير متناهية بالقسمة وأخذ الزمان متناهياً في الطول غلط وأوهمه أن تناهي الزمان من جهة لا تناهي المسافة. I am reading المنتقل rather than Al-ʿAjam's التنقل in accordance with the three manuscripts in Al-ʿAjam's footnote 5 and the Hebrew translation. The Hebrew translation, as found in Jena, Universitätsbibliothek, 10, f. 49r has: וספקות זנין בתנועה ... מהם שאלת החציים והוא שהמעתיק כשילך מהלך מן המהלכים בידוע שהלך חצי המהלך ההוא קודם שילך כל המהלך והלך חצי חציה קודם שהלך כל חציה. וכאשר יהיה הגוף מתחלק לחציים שאין לו תכלית יתחייב שיהיה המתנועע הלך מהלך שאין לו תכלית בזמן שיש לו תכלית. וזה שקר. וזה יתחייב מפני שהמהלך יהיה באין תכלית באחד משני צדדים באורך או בחלוקה. וכן הזמן והמתנועע אי איפשר שילך מהלך שאין לו תכלית בארך בזמן שיש לו תכלית בארך. ולא שילך מהלך שיש לו תכלית בארך בזמן שאין לו תכלית בארך. ולא שילך מהלך שאין לו תכלית בחלוקה בזמן שאין לו תכלית בחלוקה. וכן ההפך. ומפני שלקח המהלך בלא תכלית בחלוקה נלקח הזמן בתכלית באורך הטעה ודמה שהתכלית הזמן מצד לא תכלית המהלך.

19. Or, perhaps, 'interval'. The Arabic مسافة is used to translate the Greek διάστασις at *Physics* 202b17-18. See *Glossarium Græco-Arabicum*, مسافة. Διάστασις refers to an 'interval' or an 'extension', at *Topics* 142b5 (see also Plato, *Timaeus,* 36a). Still, the term, مسافة, does not here refer to an interval of time and so I have preferred to translate it 'distance'.

In sum, Al-Fārābī's discussions of Zeno leads us to two conclusions. One is that Zeno's paradox is actually a sophistical fallacy. It is accordingly properly placed in the *Sophistical Refutations* which he sees as a continuation of dialectic. Second, refutations of this kind are somehow connected to testing received opinions, which include religious opinions. That is, Al-Fārābī apparently views Zeno's doubts, i.e. Zeno's paradoxes as connected to received opinions and possibly more generally to well-known views, assuming that the received opinions as presented in Al-Fārābī's *Book of Dialectic* are included among the well-known views that are the primary subject of the book. Aristotle, however, did not connect Zeno's paradox to received opinions, and if he saw the paradox as connected to widely held views, he did not, apparently, consider such a relation to be relevant for the audience of the *Topica*. In contrast, Al-Fārābī addresses Zeno's paradoxes of motion, even if he subsumes them under "the question of the halves", in the *Book of Dialectic*, apparently considering the readers of that work to connect this question to received opinions and perhaps even to widely held views more generally.

But who were the readers of Al-Fārābī's *Book of Dialectic* and the *Book of Refuting the Misleading* who would consider Zeno's paradoxes relevant for their received opinions? To my mind, it seems most likely that Al-Fārābī has in mind believers in Kalām atomism, particularly those influenced by Basrian Muʿtazila. Abū al-Hudhayl who came to be recognized as the "most influential early Muʿtazilī theologian" developed a theory of atoms and their accidents which he used to explain God's acts of creation[20]. In his view, Zeno's paradox of the halves proved the existence of indivisible units, *viz.* atoms, such that any locomotion traverses a necessarily finite number of those units[21]. Not all Muʿtazilites accepted this solution; indeed, Abū al-Hudhayl's nephew, the 9th century al-Naẓẓām rejected atomism arguing that Zeno's paradox of the halves could be solved by accepting the possibility of taking a leap, *ṭafra*, over infinitely many units of space[22].

That these views were not fringe, but mainstream is emphasized by their presence in the *Book of Beliefs and Opinions* of Saʿadia Al-Fayyumi, a contemporary of Al-Fārābī and head (*gaʾon*) of the Jewish academy at Pumbedita. Among his arguments that the world is created Saʿadia includes an argument against the infinity of time. According to Saʿadia, if time were infinite it could never be traversed. Saʿadia argues for this by considering[23] each unit of time (*al-ʾān*) to be a

20. Mourad 2018.

21. The attribution of this argument to the eighth-ninth centuries Abū al-Hudhayl is made by the eleventh century Muʿtazilī theologian Ibn Mattawayh. Still, there is no reason to assume it is not genuine or to question the importance of Zeno's paradoxes for Muʿtazilī atomism before Al-Fārābī. For a translation and analysis of Ibn Mattawayh's account of Abū al-Hudhayl's argument, see Dhanani 1994, pp. 160-161.

22. See Dhanani 1994 and Dhanani 2004. Still, it is not entirely clear that al-Naẓẓām thought that there were *infinitely* many parts between all distances. See Pines 1997, pp. 14-15, n. 37.

23. Note that Saʿadia's word for 'considering' or 'establishing' is from the root w-ḍ-ʿ, the same

point (*al-nuqṭa*) and reducing the argument to that of Zeno's paradox of the stadium. Saʿadia does not name Zeno, but rather mentions "one of the unbelievers (*al-mulḥidūn*)" who met with and debated one of "those who affirm the unity of God (*al-muwaḥidūn*)". The unbeliever mentions the paradox of traversing an infinite number of parts. We do not hear the response of the monotheist, but Saʿadia offers a number of solutions to the paradox: the first is atomism, i.e. there is but a finite number of parts to every distance or length of time; the second is the theory of the leap (*al-ṭafra*) over infinite parts; the third is that there is a infinite number of parts of time corresponding to an infinite number of parts of space[24]; and the fourth solution, which Saʿadia identifies as his own, depends on the distinction between potential and actual infinity that we find in Aristotle's *Physics*, though it receives no attribution here[25]. Saʿadia's prominent use of these arguments in the first chapter of his *magnum opus* suggests that they were well known outside of Basra in Al-Fārābī's time even among non-Muslims[26]. Further, Saʿadia's inclusion of atomism as a monotheistic solution to Zeno's unbelief tells us that monotheists, both Muslims and Jews, took Zeno's paradox seriously and that arguments against it in favor of atomism and the leap theory were considered part of mainstream religiously acceptable views. Finally, note that Saʿadia's invention of the dialogue between the infidel and the monotheist is wholly unnecessary to the argument; in its context, indeed, it would seem to have no other purpose than to bring this discussion into a context of dialectic and debate.

Al-Fārābī's treatment of Zeno's paradox in his commentaries on the *Topica* and *De Sophisticis Elenchis* is likely to be a response to the kind of dialectical discussions of Zeno that Saʿadia encountered. Like Saʿadia, Al-Fārābī prefers a solution rooted in Aristotle's *Physica*; but unlike Saʿadia, Al-Fārābī does not admit atomism as an acceptable view on theological grounds. Indeed, it might be that countering this view is at the heart of his critique of accepted views in the *Book of Dialectic*. If so, then Al-Fārābī treats Zeno's paradoxes in greater detail in the dialectical works in order to counter their uses by those he perceives as contemporary dialecticians and sophists, including especially atomists. For Aristotle, I argued above, Zeno's paradoxes raise interesting theoretical questions without causing serious doubts about basic physical principles. For Al-Fārābī, the paradoxes have indeed gained an audience that takes them as serious critiques and accordingly developed an entire non-Aristotelian scientific system to treat them. One cannot

root used to translate Aristotle's τόπος throughout the *Topica*.

24. This solution is stated very briefly in the vaguest possible terms and Saʿadia does not point out that it would allow the world to be eternal.

25. See Saʿadja b. Jûsuf al-Fajjûmî, *Kitâb al-Amânât waʾl-Iʿtiqâdât*, §36. English translation in Rosenblatt 1948, pp. 44-45.

26. Note also that Avicenna mentions the paradoxes of Zeno, attributing them to both ancient thinkers and modern, implying that they continued to be well known and much discussed in his day and in his circles. See Avicenna, *Al-Ṭabīʿīyāt, al-samāʿ al-ṭabīʿī*, 276 (trans. in McGinnis 2009, p. 276).

merely address these paradoxes in the physics since they deny the very possibility of physics as Aristotle understood it. Far better to address them in more detail and point out their fallaciousness in earlier logical works studied before one comes to study science proper.

3. Averroes

Averroes may be best known in the West for his return to Aristotle and for following Aristotle's text closely in translation. Yet he also follows Al-Fārābī in treating Zeno's paradox of motion as part of dialectic and the sophistical refutations, rather than primarily in physics (though Averroes does discuss it there too[27]). Toward the opening of Averroes' *Middle Commentary* on Aristotle's *Topica* we find a list of three benefits (منافع) of dialectic: becoming accustomed to arguments, arguing with the general public, and in grasping well-known premises necessary for the theoretical sciences (العلوم النظرية, הידיעות העיוניות). Averroes divides this third benefit into five parts (وجوه, פנים).

> "The fourth is [those principles] through which the sophistry of those who mislead people with regard to the principles of the sciences is rejected. This is like what Aristotle did in the first treatise of the *Physica* with those who denied plurality and the existence of motion"[28].

Here Averroes clearly locates Zeno's paradox, even including its treatment in the *Physica*, in dialectic and the refutation of sophistry, which he seems to consider a single enterprise.

When he treats Aristotle's short discussion of Zeno's paradox at *Topics* 160b6-10 in his *Middle Commentary*, Averroes clarifies at some length how precisely it fits in to dialectic. He says:

27. See Averroes, *Epitome in Physicorum Libros*, ed. Puig, p. 52 (Spanish translation in Puig 1987, p. 153 – see also Puig's discussion on p. 38). The Arabic of the Middle and Long Commentaries on the *Physica* is not extant. For a discussion of Zeno's paradoxes of motion as they appear in the Michael Scot's Latin translation of the *Long Commentary* on the *Physica* see Puig Montada 2018. For a discussion of how this paradox appears in an anonymous medieval Hebrew translation of the *Long Commentary* see Glasner 2001. On the Hebrew translations of Averroes' three commentaries on Aristotle's *Physica*, see Glasner 2011, pp. 183-184.

28. Averroes, *Middle Commentary on Aristotle's Topics*, ed. Butterworth / Haridi, pp. 33-34: اودحج نيذلا عم عامسلا نم ىلولأا ةلاقملا يف وطسرأ لعفب امك – هولعلا ءىدابم يف ىناطسفوسلا ةطلاغم ىقلتت امهب نأ عبارلاو ةكرحلا دوجو ترثكلا. Qalonimos ben Qalonimos's fourteenth century Hebrew translation of this line is: והרביעי שבהם ידחו הטעאות המטעי' בהתחלות החכמות כמו שעשה אריסטו במאמר הראשון מספר השמע עם אשר כפרו ברבוי ומציאות התנועה. All citations from and references to the Hebrew translation of Averroes *Middle Commentary* on Aristotle's *Topica* are from the critical edition currently being prepared by my doctoral student, Arye Rainer as part of his dissertation. Since the focus of this article is on the Hebrew tradition, I cite the Hebrew as well as the Arabic.

"One who brings a proposition that contradicts a universal premise that is supported by induction ought not bring a proposition that gives a universal refutation which is a refutation of the contrary. Rather he should bring a proposition that gives a partial refutation which is a refutation of the contradictory. For refuting the premises which proceed through universality is peevish. Thus could one be required to deny the sensibles. For example, if we saw fit to explain that every animal is moving through an induction that proceeds through animals who sense by that thing which is moving, it would not be fitting to respond with something like Zeno's argument from which it follows that nothing is moving. This argument is that in which he said that every moving thing will cross half a distance before crossing the entire distance, and half of that half before crossing the half, and half of the half of that half, and so on *ad infinitum*. So it is absurd that someone will cross an entire distance in finite time. Accordingly, nothing is moving. Arguments like this reject the sensibles, but with such arguments it is difficult to accept their contradictory. But were it not for this [contradictory], it would be impossible that sensibles would occur. However, the master of this science ought to be warned about these [arguments], though they are primarily contained in sophistic. This injunction is one that a respondent ought to employ with premises like these"[29].

This comment agrees with Aristotle at *Topica* 160b6-10 that Zeno's paradox of the stadium is difficult to contradict, but nevertheless should not be accepted. Yet, Averroes' version differs in six key respects. In the first place, Averroes has included here a concise statement of the stadium paradox which is nearly identical to his presentation of the same paradox in the *Middle Commentary* on *Physica* 239b9-14[30]. As we saw, Aristotle's *Topica* 160b6-10, in contrast, merely alludes to the problem by name. In the second, Averroes understands the whole discussion here to refer to the problem of excessive universality, a problem we en-

29. Averroes, *Middle Commentary on Aristotle's Topics*, ed. Butterworth / Haridi, pp. 228-229: وينبغي إذا أتى بقول يناقض المقدمة الكلية المثبتة بالاستقراء أن لا يأتى بقول يبطلها بالكلية – وهو الإبطال بالضد – بل يأتى بقول يبطلها به ابطالا جزئيا – وهو الإبطال بالنقيض. فإن ابطال المقدمات التى هذا شأنها بالكلية شنيع إذ يلزم عنه إنكار المحسوسات – مثال ذلك إذا أراد أن يبين بالاستقراء أن كل حيوان متحرك بأن يستقرئ حيوانات يحس من أمرها أنها تتحرك فيتلقى المجيب ذلك بمثل قياس زنين اللازم عنه أنه ولا شىء متحرك, وهو الذى قال فيه كل متحرك فإنه يقطع نصف المسافة قبل أن يقطع جميعها ونصف النصف قبل النصف ونصف نصف النصف قبل النصف وكذلك إلى غير نهاية وهو يقطع المسافة كلها فى زمان متناه أعظاما غير متناهية وذلك محال فإذن ولا شىء يتحرك. فإن أمثال هذه القياسات تدفع المحسوسات لكن هى مع هذا قياسات يصعب نقضها ولولا ذلك لم يمكن أن يعارض بها المحسوسات. إلا أن هذه الصناعة ينبغي أن يحذرها وهى بالسوفسطائى أولى. فهذه هى الوصية التى ينبغى للمجيب أن يمتثلها فى أمثال هذه المقدمات. Qalonimos' translation (para. 336) has: וראוי כאשר יביא מאמר יסתור ההקדמה הכללית המקיימת בחפוש שלא יביא מאמר שיבטלה בכללות והוא הבטול להפך אבל יביא מאמר שיבטלה בטול חלקי, והוא הבטול בסותר. כי בטול ההקדמות אשר זה דרכם בכללות מרוחק אחר שיחוייב ממנו הרחקת המוחש. דמיון זה כאשר ראינו שנבאר בחפוש בשכל חי מתנועע בשנחפש בעלי חיים שיורגש מעניינם שהם מתנועעים, הנה אין ראוי למשיב זה, בכמו היקש זנין המחוייב ממנו שהוא אין דבר מתנועע. והוא אשר אמר בו כל מתנועע הנה הוא יחתוך חצי המהלך קודם שיחתוך כלו וחצי החצי קודם החצי וחצי חצי החצי קודם חצי החצי, וכזה אל בלתי תכלית, והוא יחתוך המהלך כלו בזמן בעל תכלית וזה שקר. הנה אם כן אין דבר מתנועע. כי כמו אלו ההקשים ידחו המוחשים אבל הם עם זה הקשים יקשה סתירתם. ולולי זה אי איפשר שיחלוק בהם המוחשים, אלא שבעל זאת המלאכה שיזהר מהם. והם בהטעאה יותר ראשונים. הנה זאת היא הצואה אשר ראוי למשיב שיקיים אותה בדמיוני אלו ההקדמות.

30. See, e.g., ms. Paris, Bibliothèque Nationale, hebr. 934, ff. 109v-110r: ספק זנון ... וספקות זה האיש המפורסמים בבטול התנועה ארבע ... והספק השני הוא אשר יודע בחצאים. וזה שהוא יאמר שהמתנועע האחד כאשר חתך המהלך כלו יחוייב שיחתוך חציו קודם חתכו המהלך כולו וחצי החצי קודם החצי וזה אל בלתי תכלית הנה יחוייב שאם היתה התנועה נמצאת שיתנועע המתנועע מהלך בלתי ב"ת. Averroes identifies this as commenting on Treatise VI, chapter 11; our editions of the *Physica* locate this passage in Book VI, chapter 9.

countered at Aristotle's *De Sophisticis Elenchis* 172a8-9 and in my bad joke at the opening of this article. Third, in Aristotle's *Topica*, the accusation of peevishness (δυσκολαίνειν) was leveled at those who do not raise objections or counter-arguments; for Averroes the corresponding Arabic term, شنيع,[31] refers to the excessively universal character of the argument. Fourth, for Aristotle, as we saw, Zeno's argument went against the δόξαι; for Averroes, Zeno's argument is due to mistaken, excessively far reaching inductions. Fifth, a directed response, treating the relevant part of the discussion alone is, according to Averroes, the proper strategic response to excessive universality. Sixth and finally, for Averroes, this kind of argumentation is properly part of sophistics, i.e. sophistical refutation, while for Aristotle, its proper place was in physics.

What do these changes and additions tell us about how Averroes saw the importance of Zeno's paradox? That he does not mention that Zeno's paradox runs counter to generally accepted opinions (presumably, המפורסמות, المشهورات) here suggests that he does not consider Zeno's views to be obviously contrary to widely held views. This may be a sign that he, like Al-Fārābī before him, recognized that Zeno's paradox of the halves was important for grounding atomism and the like among the Kalām and that, moreover, such theories had attained a fairly widespread acceptance in Averrroes' day[32].

In Averroes' view, then, such Kalām atomism is apparently argued by excessively universal arguments. For Averroes, this approach is peevish and is governed by mistaken inductions. Rather than looking at all animals and inferring the existence of some thing (*ʾamr*) that makes them moving and which is connected to sense-perception, they make excessively universal arguments about the impossibility of moving at all. Atomism can be proposed as an answer to these arguments about the apparent impossibility of motion, but it will not be connected to sense-perception since atoms cannot be seen. That is, I am suggesting that Averroes' account here is designed to reject any attempt by followers of Abū al-Hudhayl to use atomism and the denial of infinite divisibility as a solution to Zeno's

31. According to *Glossarium Graeco-Arabicum*, this word more frequently translates ἄτοπος or ἀτοπία. Qalonimos here uses מרוחק, which works, but is not used exclusively with this meaning.

32. On Averroes' critique of atomism and especially the notion that atomism negates the possibility of causation, see KOGAN 1985, pp. 71-164, esp. p. 91-97. In his *Guide of the Perplexed*, Moses Maimonides, a contemporary of Averroes, lists atomism as the first of the 12 scientific premises common to the Mutakallimūn (*Dalālat-al-ḥā'irīn*, 105^{v} – transl. PINES 1963, p. 195). Maimonides addresses some problems that could be seen as related to Zeno, such as the problem of the arrow, on (ff. 106r-107v – pp. 197-198). Among the difficulties Maimonides raises is that half distances are not always possible since a given distance may have an odd number of atoms. In such a case, dividing the line into a finite number of atoms would not be sufficient to explain how a distance can be divided in to half (see p. 198). Note that Maimonides says only that this problem is related to the Kalām proofs for the first three premises he lists, but not which one. He clearly expects his readers to be familiar with this argument and other arguments mentioned here and their use in extracting what he terms the "premises of the Mutakallimūn".

paradox. Such atomism denies what has been sensed and poses something that has no basis in sensation.

Moreover, according to Averroes, the Zenoist/atomist seeks to refute the universal premise without addressing the particulars. In the example Averroes gives, someone notices that many animals move and inductively infers that all animals move. The proper refutation, says Averroes, is to bring a counterexample showing the contradictory, namely an animal that does not move. Such a refutation would result in rejecting the proposition that all animals move. What the Zenoist or Kalām atomist does, however, is to oppose the entire universal proposition, "all animals move", with its contrary: "no animal moves". In order to accept this, one would have to reject the particular premises s/he had previously accepted, namely that some particular animals move. To do this, one would have to reject what his/her senses perceived and so reject sensation itself as a valid means to attain truth.

Averroes' point, then, is methodological; Zeno's paradox of the halves is mentioned in the context of discussing how the proper way to refute an induction is by bringing a contradictory example, not a contrary to the universal proposition. Unlike Al-Fārābī, he is not interested in explaining infinity across distance and time here, but in explaining how to make and refute inductions. Since induction is discussed in Aristotle's *Topica*, this argument is properly a part of dialectic and accordingly it is discussed at some length in Averroes' *Middle Commentary* on the *Topica*. Yet, since the Zenoist's attempt to refute the induction is an error in argument, namely to bring a contrary where a contradictory is needed, its full discussion is most properly in sophistic. It is, thus, clear why Averroes felt the need to go into more detail than Aristotle did in discussing Zeno's paradox of the halves in the context of the logical *Organon*.

4. The Hebrew Tradition

The commentaries of Al-Fārābī and Averroes on dialectic and sophistical refutation would have been the primary sources for Hebrew readers interested in Aristotle's *Topica* and *De Sophisticis Elenchis*. Since neither Aristotle's *Topica* nor his *De Sophisticis Elenchis* has ever been translated into Hebrew, medieval Hebrew readers would have only had access to translations of Al-Fārābī's and Averroes' commentaries. Some few may have had access to Latin texts of Aristotle's works, but Jews did not begin study Aristotle in Latin in earnest until the sixteenth century when they began to attend the University of Padua. Even then, manuscript evidence suggests a preference for Hebrew texts until the seventeenth century.

Al-Fārābī's extensive commentary on Aristotle's *Topics*, the *Book of Dialectic*, was one of the earliest philosophical translations made into Hebrew, probably appearing in the twelfth century. The translation is unfinished and includes only the

first treatise of the book. Moreover, the choice of terms would appear to have been unfinished; numerous Arabic terms have been translated with a view to homophony, but these terms do not appear consistently. My suspicion is that the translator used homophonic terms as placeholders in a pioneering translation with the intention of replacing them with better terms. The translator replaced some but not all of these terms, before abandoning his translation, leaving the work short and with a great deal of inconsistency. It is, unfortunately, cut off right before the first mention of Zeno's paradox. Yet, the more detailed account of Zeno's problem of the halves is preserved quite clearly, as we saw, in Al-Fārābī's commentary on *De Sophisticis Elenchis*, the *Book of Refuting the Misleading*. This commentary is in the same series as the Hebrew translation and uses a very similar set of Hebrew terms. The translator of *Refuting the Misleading* remains anonymous, but the work is clearly intended to be read alongside *Kitāb al-jadal* and various *opuscula* of Al-Fārābī, which were translated into Hebrew around the same time.

Judging from the dispersion of the manuscripts, these works were read in Europe fairly continuously until well into the sixteenth century. There is even a short gloss commentary on Al-Fārābī's two works, *Dialectic* and *Refuting the Misleading*, preserved in marginal notes in two manuscripts. Unfortunately, there are no glosses where Al-Fārābī discusses Zeno's paradox of the halves in the *Refuting the Misleading*[33].

One reader of Al-Fārābī's *Book of Dialectic*, though not necessarily in its Hebrew translation, was Shem Tov Falaquera (ca. 1225-1295)[34]. In his *Epistle of the Debate*, Falaquera's Scholar responds to a Pietist's assertion that his faith cannot be questioned with the following remarks:

> "If you knew the difficult questions that [arise] by way of dialectics you would not say this. Have you not heard that among the ancient philosophers there were those who brought proof for the refutation of motion even though it is a thing perceived by the senses (this is the problem known as the problem of the halves)... The fallacy of these proofs that are dialectical cannot be recognized and cannot be refuted except by him who knows the science of demonstration and he alone can recognize the lie in them. Now, if concerning these things that are sensed, they [those practiced in dialectics] pose great difficulties for man and lead him astray, then how much more so is this the case concerning tradition?"[35]

33. The marginal notes are preserved in ms. Wien, Österreichische Nationalbibliothek, hebr. 53, ff. 20r-40v (second pagination) and ms. Paris, Bibliothèque Nationale de France, hebr. 928, ff. 24v-32v. Of these two manuscripts only the first contains the Hebrew translation of Al-Fārābī's *Refuting the Misleading* (ff. 27v-40v). In the case of the *Dialectic*, the marginal commentary has slight divergences in the two manuscripts that suggest it was copied from at least one other manuscript. Accordingly, there is reason to suspect that marginal Hebrew commentary on *Refuting the Misleading* is not unique to the Vienna manuscript.

34. See Halper 2021.

35. Shem Tov Falaquera, *Epistle of the Debate*, pp. 61-62 (transl. Harvey 1987, pp. 24-25).

While Falaquera does not tell us the solution to Zeno's paradox, he does locate it in the science of dialectic. That is, one should encounter such fallacies in dialectic (and probably also the sophistical refutations, which are a part of dialectic). The full refutation, he says, requires demonstration, i.e. science itself, but the question is dialectical in character. Moreover, like Al-Fārābī, he connects this paradox to testing tradition, even if this connection is somewhat tenuous, and notes that if dialectic can cause one to question the senses, it can cause one to question tradition as well.

Joseph Ibn Kaspi (1280 - c. 1345) not only read Al-Fārābī, but included a summary of *Refuting the Misleading* in his main logical text, *ṣeror hakesef*[36]. This work reproduces in slightly simplified form Al-Fārābī's response to Zeno's paradox in his *Refuting the Misleading*, a response which was highly indebted to Aristotle, *Physica* 233a21-31, as we saw above. Ibn Kaspi, however, includes this simplified argument in the section on fallacies of meaning rather than in the section on fallacies of linguistic expression[37]. Yet, this change is connected with Kaspi's understanding of homonymous terms and his emphasis on the ambiguity of meaning in cases like these, rather than on ambiguity of the linguistic term. In this case, he emphasizes that Zeno's argument confuses two different judgments (*dinim*) about two different properties (*tekhunot*), *viz.* infinity and finiteness about length and division. This ought to lead the Zenoists to two separate premises, but in fact they take them to be one. Still, Kaspi's account remains quite close to Al-Fārābī's and so can be understood to include a fairly extensive discussion of Zeno's paradox in the part of logic that treats sophistic.

In the second decade of the fourteenth century Qalonimos ben Qalonimos of Arles translated Averroes' *Middle Commentaries* on the *Topics* and *Sophistical Refutations* in their entirety into Hebrew. These translations were made in the style of the Ibn Tibbon family and so could be more easily read alongside the other translations of Averroes' Short, Middle and Long Commentaries on the Logical Organon[38]. Indeed, within 10 years of Qalonimos' translations of these works, Levi Gersonides (1288-1344) wrote extensive commentaries on both of them.

Unfortunately, the two manuscripts containing Gersonides' complete commentaries on the *Topics* and *Sophistical Refutations* are not in good shape and I have not yet been able to decipher them[39]. Yet, part of Gersonides' commentary

36. For an edition of this work, see Rosenberg 1984. The discussion of Zeno's paradox is on p. 288. While I shall point to some idiosyncrasies of Ibn Kaspi's Hebrew, it adheres quite closely to that of the anonymous translation found above in note 17.

37. See Charles H. Manekin's article in this volume (Manekin 2023).

38. On the edition of the *Topics* commentary, see note 28 above. An edition of the *Sophistics* commentary is still a desideratum.

39. See ms. Torino, Biblioteca nazionale universitaria, A I 14 and ms. Oxford, Bodleian Library, Mich. 64. The Turin manuscript was damaged in the fire of 1904 and the Oxford manuscript is very faint.

on the *Topics* survives in legible form in München, Bayerische Staatsbibliotek, Heb. MS 26, f. 319r-350v[40]. This manuscript is in an Italian hand and dated 1551. The text of the super-commentary is not complete and another scribe has continued the text (from 351r to 403r) with Qalonimos' translation of Averroes' *Middle Commentary* on the *Topics*. This section of the manuscript also contains numerous illustrations and is, I believe, the only illustrated text of Averroes' *Middle Commentary* on the *Topics* and *Sophistical Refutations* in any language.

Gersonides' super-commentary does not comment on every sentence, but selects sentences it finds interesting and then attaches its remarks to those. Yet it does not clearly indicate which sentences are quotations and which are the commentary, and a reader looking at this super-commentary alone would find the text somewhat mystifying. That is, the super-commentary assumes its readers to have Qalonimos' translation of Averroes' *Middle Commentary* on the *Topics* in front of them and to compare the two texts continuously.

One sentence the super-commentary focuses on is Averroes' statement that one of the benefits of dialectic is learning to refute those who "deny the existence of motion". Gersonides does not connect this statement to Zeno's paradox, but instead notes that the existence of motion and moving things are "among the principles that the physicist assumes" when he does physics. Still, he notes, this is not trivial since "the nature of moving things is different" among different moving things. Thus, "earth moves downwards, while fire moves upwards". Gersonides tells us, "Aristotle explains there [i.e. in the *Physica*] using dialectical methods that their [i.e. Zeno's and the like's] argument is of the utmost absurdity"[41]. That is, Gersonides actually reverses the pattern we have seen until now. He suggests that the proper place to address Zeno's paradoxes of motion is in the physics. Still, he recognizes that the method employed there is not demonstrative, but dialectical. In this respect, Gersonides follows Averroes, though it is possible that he was aware of Latin traditions that treated Zeno's paradoxes in the *Physica* while only alluding to them in the *Topica* and *De Sophisticis Elenchis*.

Yet, Gersonides still gives Zeno a prominent place in his discussion of how syllogisms with well-known premises are usually neither always true nor always false in his super-commentary on Averroes' *Middle Commentary* on the *Topica*. Gersonides writes:

> "You ought to know that it will not happen that that which is well-known to the general public will be false in every respect. However, it can happen among theoret-

40. At the 2022 XXVI[th] Annual SIEPM Colloquium on "Dialectic in the Middle Ages: Between Dialectic and the Foundation of Science" at Bar Ilan University, Manekin discussed this manuscript in detail, estimating that about 2/5 of Gersonides' *Commentary* on the *Topics* was extant therein.

41. Ms. München, Bayerische Staatsbibliotek, heb. 26, f. 321r-v: הכחישו התנועה ... הם התחלות בעל החכמה ... שהארץ התנועע אל המטה והאש אל המעלה ... וארסטו' ביאר להם שם בדרכים הטבעית ... ולזה יהיה טבע הנמצאות מתחלף הנצוחיים שמאמרם הוא בתכלית הבטול.

> ical scientists that they will reject sensibles because of an absurd opinion that arises as true in their thought. Zeno and many other ancients, for example, did this when they denied many of the sensible things"[42].

Gersonides, accordingly, joins Al-Fārābī in considering Zeno's view to be among the widely held views, if only among those well known to theoreticians. He may have the Kalām in mind, but he may also have had in mind any number of other later developments physics, medicine or even magic that could lead people who consider themselves theoreticians to reject what their own eyes behold. We may suppose, though, that since these well-known opinions are those of theoreticians, they are better addressed in physics than in dialectic and Gersonides accordingly elaborates them only there.

5. Conclusion

Aristotle saw the *Physica* as the most appropriate place to treat Zeno's paradoxes of motion. Still, he alluded to the paradoxes, or at least some of them, in the *Topica* and *De Sophisticis Elenchis*. These allusions, however, were quite general and implied that fuller treatments were to be found elsewhere. Yet for Al-Fārābī, the proper place to address Zeno's paradoxes was precisely in commentaries on the *Topica* and *De Sophisticis Elenchis*. This is because he saw those paradoxes as relating to widely-held opinions, presumably those held and promoted by Kalām theologians. Averroes, however, treated Zeno's paradoxes in his commentaries on Aristotle's *Physica*, but also went into them in fairly extensive detail in the his Middle Commentaries on the *Topica* and *De Sophisticis Elenchis*. While Al-Fārābī focused on the physical theory that would refute Zeno's paradoxes, Averroes's *Middle Commentary* on the *Topica* focused on the logical argumentation for making and refuting inductions. Jewish students of the *Topica and De Sophisticis Elenchis* inherited from the Muslim predecessors the notion that Zeno's paradoxes are dialectical fallacies and so should be treated in the context of the *Topica and De Sophisticis Elenchis*. Still, Gersonides seems to have made some effort to direct his readers away from examining Zeno's paradoxes in the context of the *Topica*, and to return them to the *Physica*.

42. Ms. München, Bayerische Staatsbibliotek, heb. 26, ff. 319v-320r: וראוי שתדע שהמפורסם אל ההמון לא יקרה שיהיה כוזב בכל, אבל יקרה במפורסם אצל | בעל מלאכה מהמלאכות העיוניות שידחה המוחש מפני דעת בטל עלה במחשבתו שהוא צודק כמו שעשה זנין וזלותו רבים מהקודמים בהכחישם רבים מהדברים המוחשים.

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Abstract: Following the Arabic tradition, medieval Hebrew commentaries on Aristotle's *Topica* and *De Sophisitics Elenchis* understood Zeno's paradoxes of motion as dialectical fallacies related to widely-held opinions or incorrect inductive arguments. Following Al-Fārābī and Averroes, Hebrew Aristotelian commentators include discussions of Zeno's paradoxes of motion in their commentaries on *Topica* and *De Sophisitics Elenchis*. Aristotle's own discussions of Zeno's paradoxes in those works, however, merely allude to the difficulties without presenting solutions. Indeed, they point elsewhere, most likely to the *Physica* where Aristotle provides a detailed account of those paradoxes and their solutions. The shift in emphasis in the discussions of Zeno's paradoxes of motion as dialectical paradoxes in the *Topica* and *De Sophisticis Elenchis* in the works of Al-Fārābī and Averroes was likely due to the importance of those paradoxes for *Kalām* atomism. Hebrew commentators inherited this approach, even though they did not operate in the context of the *Kalām*.

Keywords: Logic; Aristotle; Zeno; Al-Farabi; Averroes; Dialectic.

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Notes